Self-Assessment in Math...how?

NCTM 100 Days of Learning
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Evaluation as Learning

• “The analytical work of finding and evaluating mistakes inspires deep learning. One cannot understand a mistake without a rich understanding of its root…”

• Teachers are mistakenly depriving students the opportunity to learn from reviewing their own work.

• Is there a difference between Computational, Procedural, or Mechanical Errors?

I (the teacher) am learning a whole lot from grading!
BUILDING STUDENT AGENCY THROUGH SELF-ASSESSMENT

QUESTIONS TO GUIDE YOU THROUGH THE LEARNING CHALLENGE

@TheLearningPit

Concept
How many concepts are you thinking about right now? Which one is the most interesting, relevant or puzzling? Let's call your concept, "x". Why did you choose that one?

Questions
What is x? How do we know what x means? What's the difference between x and concept y? When would x be good/bad/different/relevant/irrelevant? Is it possible to always/never use x? What if x means y?

Eureka!
How clear is your understanding of x now? What makes x more complex than you first thought? What advice could you give someone who hasn't reached the eureka point yet?

Consider
How could you apply concept x to another situation? What strategies did you use to go through the pit that could also work next time?

Construct
When you compare all the ways you found to describe concept x, which one works best? If you were to rank all your ideas about concept x, which one(s) would you prioritise?

Cognitive Conflict
What problems can you find with concept x? If you are saying that concept x means y, then does y always mean x? In what ways does your definition of x come unstuck?
How do we get our students to engage with the learning process, using evaluation and feedback to identify their own level of understanding, driving deeper connections?
do---evaluate---reflect---revise---connect
# Self-Paced Learning

## Exponential and Logarithmic Functions Targets and Tasks

<table>
<thead>
<tr>
<th>Date</th>
<th>Targets</th>
<th>Concept Building (group work)</th>
<th>Concept Practice (individual work)</th>
</tr>
</thead>
</table>
|      | Target 8: Recognize and use exponential behavior and the graph of an exponential function to model a scenario. | 1. Desmos: What Comes Next? (40 min) | 4. Aleks Target 8  
- Graphing an exponential function and its asymptote: \( f(x) = a(b)^x \)  
- Finding domain and range from the graph of exponential functions  
- Writing an exponential function rule given a table of ordered pairs  
- Finding the initial amount and asymptote given a graph of an exponential function  
- Choosing an exponential model and using it to make a prediction |
| Goal: Finish Target 8 Assessment by 1/30-1/31 | - I can differentiate between exponential growth and decay and use this understanding to model a scenario.  
- I can recognize and state the qualitative features of the graph of \( y = a(b)^x \) for \( 0 < b < 1 \) and \( b > 1 \) including asymptotes, end behavior, domain and range. | 2. Desmos: Choose one of the following activities: (40 min)  
- Game Set Flat  
- Predicting Movie Ticket Prices | |
|      | Target 9: Find the inverse of a function and explain its components algebraically and graphically. | 3. Desmos: Exponential Growth and Decay (40 min) | 5. Keep on Truckin’ (80 min) |
| Goal: Finish Target 9 Assessment by 2/12-2/13 | - I can find the inverse of a function graphically and algebraically, stating the domain and range for both the function and its inverse.  
- I can use a function and its inverse to model a scenario. | 6. Aleks Target 9  
- Horizontal line test  
- Finding, evaluating, and interpreting an inverse function for a given linear relationship  
- Graphing the inverse of a function given its graph | |
|      | Target 10: Create and construct logarithmic functions graphically and algebraically. | 7. Log Logic (80 min) | 10. Aleks Target 10  
- Evaluating logarithmic expressions  
- Solving an equation of the form \( \log_a x = c \)  
- Using properties of logarithms to evaluate expressions  
- Expanding a logarithmic expression  
- Writing an expression as a single logarithm  
- Solving a multi-step equation involving a single logarithm |
| Goal: Finish Target 10 Assessment by 3/5-3/16 | - I can evaluate a logarithmic expression to determine its numerical value.  
- I can graph a logarithmic function including asymptotes, end behavior, domain and range.  
- I can use the relationship between exponential and logarithmic functions to explain the reasoning behind the laws of logarithms. | 8. Graphing Logarithmic Functions (60 min) | 9. Investigating Logs (60 min) |
Pandemic

How do viruses spread through a population?

How do viruses spread through a population? From ebola to bird flu, humans are surrounded by deadly viruses.

In this lesson, students use exponential growth and logarithms to model how a virus spreads through a population and evaluate how various factors influence the speed and scope of an outbreak.

Students will

- Write an equation to represent exponential growth; describe the effect of each parameter on the overall model
- Solve exponential equations using a variety of methods, including logs

Before you begin

The more difficult questions in this lesson require using the formula for the sum of a finite geometric series or a spreadsheet to perform the same calculations recursively. This lesson provides an opportunity for students who have learned about the formula for the sum of a finite geometric series to apply it.

Common Core Standards

Content Standards

F.LE.2, F.LE.4, F.BF.1, F.BF.2, F.BF.5, F.LE.5

Mathematical Practices

MP.1, MP.3, MP.7

Downloads

- Student Handout
- Exemplar Responses

Mathalicious

Many factors influence how a virus spreads. Let's start with the scenario in which an infected person passes the virus to three new people each week. Choose one modification below and explain how it will affect the outbreak.

Population: 7,700,000,000

Infected per Person: 3

Immune: 8%

After Infection:

- Stay
- Heal
- Die

Quarantine: None

Week: 28
Target 6: Rational Thinking

Three rational functions are given below. Identify the degrees of the numerators and denominators. Use the rational function and its graph to look for patterns that will help you identify horizontal or oblique (slant) asymptotes. These asymptotes are known as End Behavior Asymptotes.

<table>
<thead>
<tr>
<th>Function</th>
<th>Degree of Numerator</th>
<th>Degree of Denominator</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) = \frac{2x}{x^2 + 1} )</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( g(x) = \frac{x - 1}{x^2 + 2x - 4} )</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( h(x) = \frac{x^3 - 4}{(x + 1)^2} )</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

How do the degrees of the numerator and denominator affect the End Behavior Asymptotes? Fill in the boxes with the following: <, >, and =.

- If the degree of the numerator \( n \) is less than the degree of the denominator \( m \), then the x-axis is the horizontal asymptote.
- If the degree of the numerator \( n \) is equal to the degree of the denominator \( m \), then the horizontal asymptote is the line \( y = \frac{a}{b} \).
- If the degree of the numerator \( n \) is greater than the degree of the denominator \( m \), then there is no horizontal asymptote. (There is a slant diagonal or oblique asymptote.)
Discovery-based learning facilitates more light bulb moments, which builds student confidence.
Collaboration
LEARNING by Reflecting

WHAT?

DESCRIPT WHAT HAPPENED
OUTLINE TIMELINE

SO WHAT?

DESCRIPT WHY IT IS IMPORTANT?
OUTLINE THE IMPACT OR MEANING IT HAS FOR YOU

NOW WHAT?

DESCRIPT THE IMPACT OF WHAT YOU HAVE DONE
OR LEARNED WILL HAVE ON YOUR FUTURE WORK
OUTLINE WHAT YOU WILL DO TO CONTINUE TO
LEARN

Schia Rosenthal-Tolizano • globallyconnectedlearning.com
Why incorporate reflection in a math classroom?

- To empower students to drive their own learning and to engage students in the process of identifying their own strengths and opportunities for growth by reflecting on their progress.
Today's Date: 11/5
Concepts and Level of Understanding

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**I can** Statements

- I am starting to understand, but I am still confused.
- I am learning, but I don't have it yet. I can do it if I look at an example or ask for help!
- I understand what I am doing and I can explain it to someone else.

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Teacher Comment:

- I can identify and differentiate between discontinuities (removable vs nonremovable) in a rational function.
- I understand the removable hole discontinuity usually will be a line crossing an axis, but the vertical asymptote has two reflecting curves that will never touch the other axis.
- I can use the factored form of a rational function to determine its zeros.
- I understand that I can equal it to zero to get the answer.

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Teacher Comment:

- I can use the factored form of a rational function to determine its zeros.
- I am having trouble finding the difference between determining the zeros and the vertical asymptotes because I feel they both equal the denominator to zero, so I am confused on why they are different from each other.

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Teacher Comment:

- I can use the factored form of a rational function to determine its zeros.
- I am using the graphing calculator to help with finding the vertical asymptotes.
- I am using the factored form to help find the zeros.
- I understand that I can equal it to zero to get the answer.

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Student Thoughts- Daily Reflections

I think that it does because I look at what I can work on and sometimes I think that I didn't make any progress, but when I write my reflection, I realize that I did make some progress. Sometimes I think that I understand it, but when I'm writing my reflection, I realize that I still need to work on some things.

No I do not think daily reflections help me understand where I am at in the class. I do not like writing them because often I am still confused at what the I can statements are really saying or what they mean.

It really helps me look back and reflect and see if I maybe am still having trouble on a certain concept and helps me ask better questions next time it comes around. It also helps me realize how much I know and understand and helps me gain confidence in math and solving future problems.

I personally don't think daily reflections help because I forget a lot of what I have done and don't enjoy thinking about vocabulary outside of working in solving a math problem. It makes me doubt myself and feel like I do not know what is going on when I do.
EMPOWERING STUDENTS TO OWN THE ASSESSMENT PROCESS
1. The length of a rectangular park is 1 mile longer than twice the width. The area of the park is 55 square miles. What is the width of the park?

\[ A = lw \]
\[ 55 = (2w + 1)(w) \]
\[ 55 = 2w^2 + lw \]
\[ 0 = 2w^2 + lw - 55 \]
\[ \text{csolve}(2x^2 + lx - 55 = 0, x) \]
\[ x = 5.5 \text{ or } x = -5.5 \]

The correct answer is 5.5.

2. A stone was thrown from the top of a cliff 60 meters above sea level. The height of the stone above sea level t seconds after it was released is given by \( h(t) = -5t^2 + 20t + 60 \) meters.

a. Find the time taken for the stone to reach its maximum height. Write your answer in a complete sentence. (Hint: the maximum height is at the vertex.)

The vertex \((2, 80)\) tells us the time when the stone reaches its maximum height.

b. What was the maximum height above sea level reached by the stone? Write your answer in a complete sentence.

The maximum height is 80 feet. It took 2 seconds for the stone to reach its maximum height.

c. Give a possible lead coefficient for this polynomial. Explain your choice.

A leading coefficient could be positive (+) because the graph ends up going to the right side of the graph and is an odd function. (The end behavior is positive.)

d. In factored form, write a possible function that represents \( f(x) \).

\[ f(x) = (x + 10)(x - 3) \]

1. Consider the graph of the function \( f(x) \).

a. What is the end behavior of \( f(x) \)?

As \( x \to -\infty \), \( f(x) \to -\infty \)

As \( x \to \infty \), \( f(x) \to \infty \)

b. What is a possible degree of \( f(x) \)? Explain how you determined this.

The possible degree of \( f(x) \) could be 3 and any odd number (coefficient). This is based on the number of maximum and minimum heights.

c. Give a possible lead coefficient for this polynomial. Explain your choice.

A leading coefficient could be positive (+) because the graph ends up going to the right side of the graph and is an odd function. (The end behavior is positive.)
## ASSESSMENT REFLECTION EXAMPLE

### Given the graph of a polynomial, I can write a function to model its behavior.

<table>
<thead>
<tr>
<th>I am challenged by...</th>
<th>I was challenged by...</th>
</tr>
</thead>
<tbody>
<tr>
<td>I know, what do you know?</td>
<td>this problem because I</td>
</tr>
<tr>
<td>Initially chose the wrong answer.</td>
<td></td>
</tr>
<tr>
<td>I think I was challenged by.</td>
<td></td>
</tr>
<tr>
<td>This problem because I did not.</td>
<td></td>
</tr>
<tr>
<td>Fully comprehend the factors.</td>
<td></td>
</tr>
<tr>
<td>With a multiplicity of 3 not cubic,</td>
<td></td>
</tr>
<tr>
<td>It is positive and it just goes through the x-axis. My mistake was.</td>
<td></td>
</tr>
<tr>
<td>Not noting that (x-1)^3 was.</td>
<td></td>
</tr>
<tr>
<td>Correct because at that point the graph acted cubic.</td>
<td></td>
</tr>
<tr>
<td>I also should have realized that (x-1)^3 had the multiplicity at 3.</td>
<td></td>
</tr>
<tr>
<td>It bounced on the graph.</td>
<td></td>
</tr>
</tbody>
</table>

### I am proud of my growth/understanding in...

<table>
<thead>
<tr>
<th>What do you know?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zeros: (-3, 0), (0, 0), (2, 0).</td>
</tr>
<tr>
<td>Multiplicity: m2, m1, m1.</td>
</tr>
<tr>
<td>Function: f(x) = (x+3)(x-0)^2(x-2).</td>
</tr>
</tbody>
</table>

Circle one of the following and explain why you chose the given statement.

#### Self- Evaluation

<table>
<thead>
<tr>
<th>I can do this on my own and explain my solution path to others.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I can do this on my own, but I am not yet confident enough to answer questions or explain it to others.</td>
</tr>
<tr>
<td>I can do this on my own, but I am still making computational or minor errors.</td>
</tr>
<tr>
<td>I can do this with the help of an example.</td>
</tr>
<tr>
<td>I need more time to understand this.</td>
</tr>
</tbody>
</table>

I feel like I can do problems with this concept on my own, but I do make errors with interpreting the graph. In the first one, it was hard to tell that the x=1 was acting cubic. I feel that I can sometimes mess up with leading coefficient.
What were some of your most powerful learning moments and what made them so?

What concept from this unit do you think that you will retain the most? What about that concept made it ‘stick’?

What about your thinking, learning, or work brought you the most satisfaction? Why?

How did you develop your understanding of the Target 12 'I Can' Statements? Where did you find an 'aha moment'? Explain.

Where did you encounter struggles, and what did you do to deal with it? Explain.

How have you demonstrated mastery of the Target 12 'I Can' Statements? Explain the process you used to understand a concept. (Don't say you completed the Desmos activities and Aleks; you want to elaborate.)

What concept from this unit do you think that you will retain the most? What about that concept made it ‘stick’?
When writing the assessment reflections, I am able to understand what specifically caused me to make mistakes and what areas I have mastered.

I don’t like them, especially when I get a 100 on the assessment and I get points off on my reflection for not explaining my math.

I feel that the reflections after I have taken the assessment help me understand where I actually stand in my understanding of the topic. Past years, I just took the test and never looked back. Now, the reflections force me to dig deep and explain what I know. This is really valuable for other tasks to see what I can do better next time to build a better understanding of future topics. It also helps me remember the task instead of just putting it in the very back of my mind.

I am not a fan I do not really enjoy writing them it seems almost like busy work. I feel like I doesn’t really help me understand in the end how I grew and understand math.
Thank you so much for joining us!

If you have any follow up questions, please feel free to email us at…

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Empowering students to own the assessment process
Mistakes Allow Thinking to Happen

- Why Mistakes Matter in Creating A Path
- 9 Ways To Help Students Learn Through Mistakes
- The Truth About Making Mistakes: Helping Students Discover the Benefits
- This Is Why You Should Be Proud of Making Mistakes

"The greatest mistake you can make in life is to be continually fearing you will make one."

Elbert Hubbard

![Metacognition Cycle Diagram](image)