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Self-Assessment in Math…how?

NCTM 100 Days of Learning

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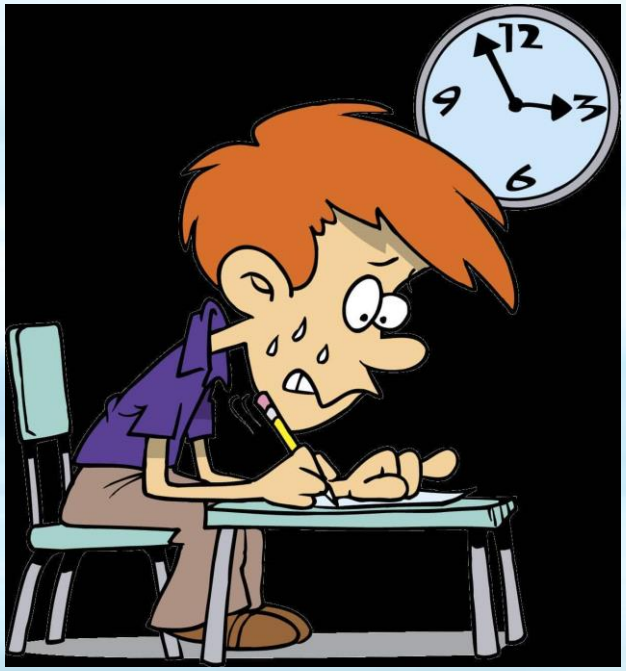
12
9
3
6


C+
1+1=11
NO

Evaluation as Learning

- "The analytical work of finding and evaluating mistakes inspires deep learning. One cannot understand a mistake without a rich understanding of its root…"
- Teachers are mistakenly depriving students the opportunity to learn from reviewing their own work.
- Is there a difference between Computational, Procedural, or Mechanical Errors?

I (the teacher) am learning a whole lot from grading!

BUILDING STUDENT AGENCY THROUGH SELF-ASSESSMENT

How do we get our students to engage with the learning process, using evaluation and feedback to identify their own level of understanding, driving deeper connections?

do---evaluate---reflect---revise---connect





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SELF-PACED LEARNING

Exponential and Logarithmic Functions Targets and Tasks

Date	Targets	Concept Building (group work)	Concept Practice (individual work)
Goal: Finish Target 8 Assessment by 1/30-1/31	Target 8: **Recognize and use exponential behavior and the graph of an exponential function to model a scenario.** • I can differentiate between exponential growth and decay and use this understanding to model a scenario. • I can recognize and state the qualitative features of the graph of $y = a(b)^x$ for $(0 < b < 1)$ and $(b > 1)$ including asymptotes, end behavior, domain and range.	1. Desmos: *What Comes Next?* (40 min) _____ 2. Desmos: Choose one of the following activities: (40 min)_____ • *Game Set Flat* • *Predicting Movie Ticket Prices* 3. Desmos: *Exponential Growth and Decay* (40 min) _____	4. Aleks Target 8 • Graphing an exponential function and its asymptote: $f(x) = a(b)^x$ • Finding domain and range from the graph of exponential functions • Writing an exponential function rule given a table of ordered pairs • Finding the initial amount and asymptote given a graph of an exponential function • Choosing an exponential model and using it to make a prediction
Goal: Finish Target 9 Assessment by 2/12-2/13	Target 9: **Find the inverse of a function and explain its components algebraically and graphically.** • I can find the inverse of a function graphically and algebraically, stating the domain and range for both the function and its inverse. • I can use a function and its inverse to model a scenario.	5. Keep on Truckin' (80 min) _____	6. Aleks Target 9 • Horizontal line test • Finding, evaluating, and interpreting an inverse function for a given linear relationship • Graphing the inverse of a function given its graph
Goal: Finish Target 10 Assessment by 3/5-3/6	Target 10: **Create and construct logarithmic functions graphically and algebraically.** • I can evaluate a logarithmic expression to determine its numerical value. • I can graph a logarithmic function including asymptotes, end behavior, domain and range. • I can use the relationship between exponential and logarithmic functions to explain the reasoning behind the laws of logarithms.	7. Log Logic (80 min)_____ 8. Graphing Logarithmic Functions (60 min)_____ 9. Investigating Logs (60 min)_____	10. Aleks Target 10 • Evaluating logarithmic expressions • Solving an equation of the form $\log_b a = c$ • Using properties of logarithms to evaluate expressions • Expanding a logarithmic expression • Writing an expression as a single logarithm • Solving a multi-step equation involving a single logarithm

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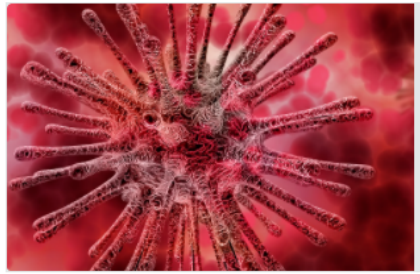
SPARKING CURIOSITY

Pandemic

How do viruses spread through a population?

Plan | Teach | Videos



How do viruses spread through a population? From ebola to bird flu, humans are surrounded by deadly viruses.

In this lesson, students use exponential growth and logarithms to model how a virus spreads through a population and evaluate how various factors influence the speed and scope of an outbreak.

Students will

- Write an equation to represent exponential growth; describe the effect of each parameter on the overall model
- Solve exponential equations using a variety of methods, including logs

Before you begin

The more difficult questions in this lesson require using the formula for the sum of a finite geometric series or a spreadsheet to perform the same calculations recursively. This lesson provides an opportunity for students who have learned about the formula for the sum of a finite geometric series to apply it.

Common Core Standards

Content Standards

F.LE.2 F.LE.4 F.BF.1 F.BF.2 F.BF.5 F.LE.5

Mathematical Practices

MP.4 MP.3 MP.7

Downloads

Student Handout

Exemplar Responses

Mathalicious

Many factors influence how a virus spreads. Let's start with the scenario in which an infected person passes the virus to three new people each week. Choose one modification below and explain how it will affect the outbreak.

Population:

7,700,000,000

Infec. per Person: 3

Immune: 8%

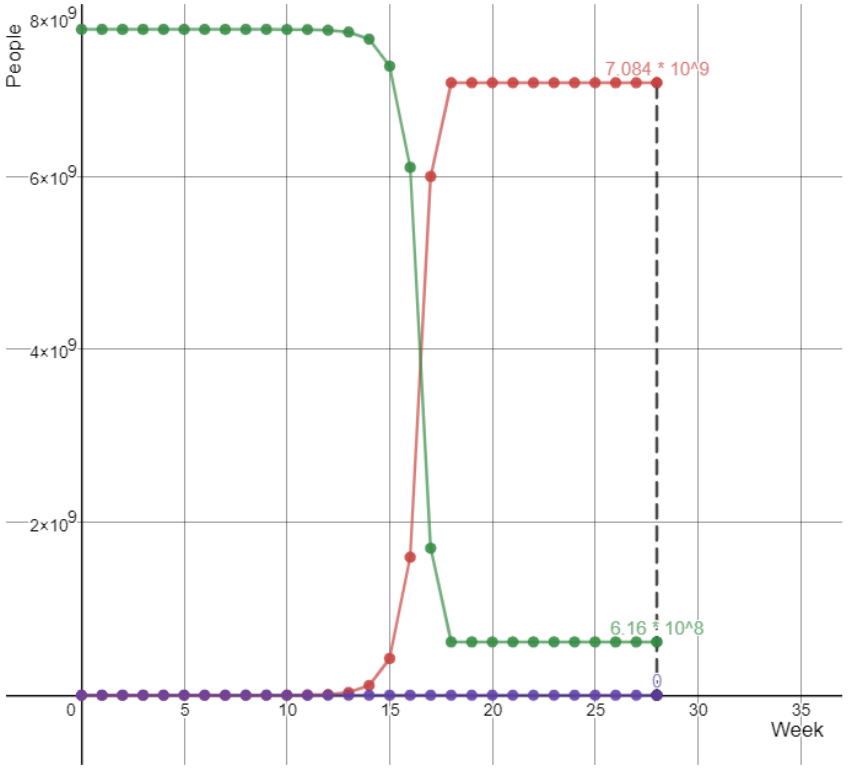
After Infection:

Stay | Heal | Die

Quarantine: None

Week: 28

Healthy | Infected | Dead



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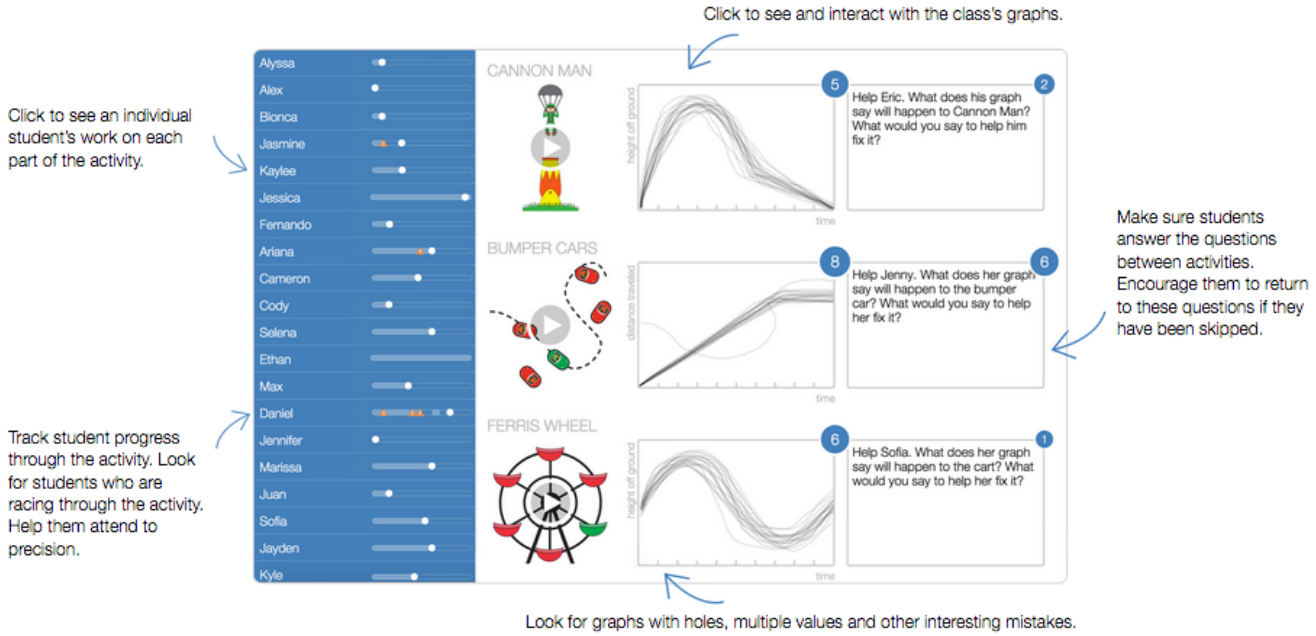
DISCOVERY-BASED ACTIVITIES

THE TEACHER EXPERIENCE

While students are working, use the teacher view to identify students to talk with. For example:



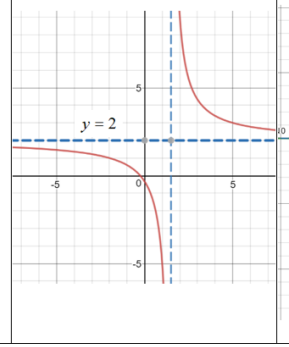
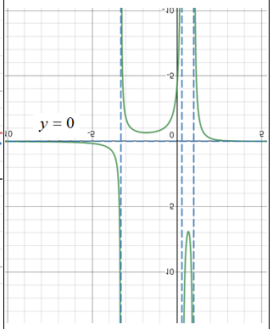
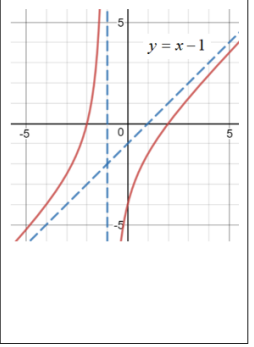
[Mathematics Vision Project](https://www.mathematicsvisionproject.org/)
https://www.mathematicsvisionproject.org/

[Desmos](teacher.desmos.com)
teacher.desmos.com

Target 6: Rational Thinking

Name: _____ Pd: _____

Three rational functions are given below. Identify the degrees of the numerators and denominators. Use the rational function and its graph to look for patterns that will help you identify **horizontal or oblique (slant) asymptotes**. These asymptotes are known as **End Behavior Asymptotes**.

$f(x)=\frac{ax^n+\ldots}{bx^m+\ldots}$	$f(x)=\frac{4x+1}{2x-3}$	$g(x)=\frac{x-4}{x^3+2x^2-4x+1}$	$h(x)=\frac{x^2-4}{(x+1)}$
Degree of **Numerator** n			
Degree of **Denominator** m			
Graph of Rational Functions			

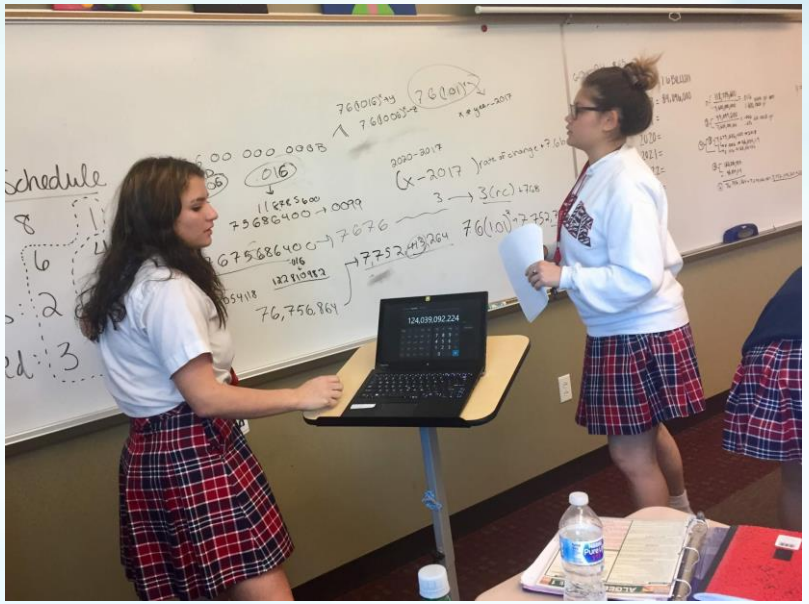
$$f(x)=\frac{ax^n+\ldots}{bx^m+\ldots}$$ ← nth degree polynomial ← mth degree polynomial

How do the degrees of the numerator and denominator affect the **End Behavior Asymptotes**? Fill in the boxes with the following: $<, =, and >$.

If...	n _____ m	, then the x-axis is the horizontal asymptote.
If...	n _____ m	, then the horizontal asymptote is the line $y=\frac{a}{b}$
If...	n _____ m	, then there is no horizontal asymptote. (There is a slant diagonal or oblique asymptote.)


Discovery-based
learning
facilitates more
LIGHT BULB MOMENTS,
which builds
student confidence.
FREE TO
DISCOVER

Collaboration




LEARNING
by Reflecting
NOW WHAT?
DESCRIBE THE IMPACT OF WHAT YOU HAVE DONE OR LEARNED WILL HAVE ON YOUR FUTURE WORK
OUTLINE WHAT YOU WILL DO TO CONTINUE TO LEARN
SO WHAT?
DESCRIBE WHY IT IS IMPORTANT?
OUTLINE THE IMPACT OR MEANING IT HAS FOR YOU
WHAT?
DESCRIBE WHAT HAPPENED
OUTLINE TIMELINE
Silvia Rosenthal Tolisano- globallyconnectedlearning.com

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Why incorporate reflection in a math classroom?

- To empower students to drive their own learning and to engage students in the process of identifying their own strengths and opportunities for growth by reflecting on their progress.

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DAILY REFLECTION

Today's Date	Concepts and Level of Understanding				Teacher Comment
	"I can" Statements	I am starting to understand, but I am still confused.	I am learning, but I don't have it yet. I can do it if I look at an example or ask for help!	I understand what I am doing and I can explain it to someone else.	
11/5	I can identify and differentiate between discontinuities (removable vs nonremovable) in a rational function.	I understand that the removable hole discontinuity usually will be a line crossing an axis, but the vertical asymptote has two reflecting curves that will never touch the other axis			
	I can use the factored form of a rational function to determine its zeros.		understand that I can equal it to zero to get the answer		What do you set equal to zero?
	I can identify and differentiate between discontinuities (removable vs nonremovable) in a rational function.	I worked on activities one and two today. I was very confused with the back side of the first activity. I am having trouble understanding all the rules and terms from the lesson like what a vertical asymptote means. I can identify if its removable or non removeable from a graph but not from the initial factored function.			
	I can use the factored form of a rational function to determine its zeros.		I understand on the second worksheet how to find h(x)		
11/11	I can identify and differentiate between discontinuities (removable vs nonremovable) in a rational function.				
	I can use the factored form of a rational function to determine its zeros.		I can do this but I need more practice to fully understanding without asking for help. Worksheet three really helped with graphing the rational function and finding the vertical asymptotes and I am starting to understand it more fully.		
11/13	I can identify and differentiate between discontinuities (removable vs nonremovable) in a rational function.				
	I can use the factored form of a rational function to determine its zeros.		I am having trouble finding the difference between determining the zeros, and the vertical asymptote because to find them you equate the denominator to zero, so I am confused on why they are different from each other.		denominator =0. This would be your holes and VA. Roots/Zeros: Set numerator = 0.

Student Thoughts- Daily Reflections

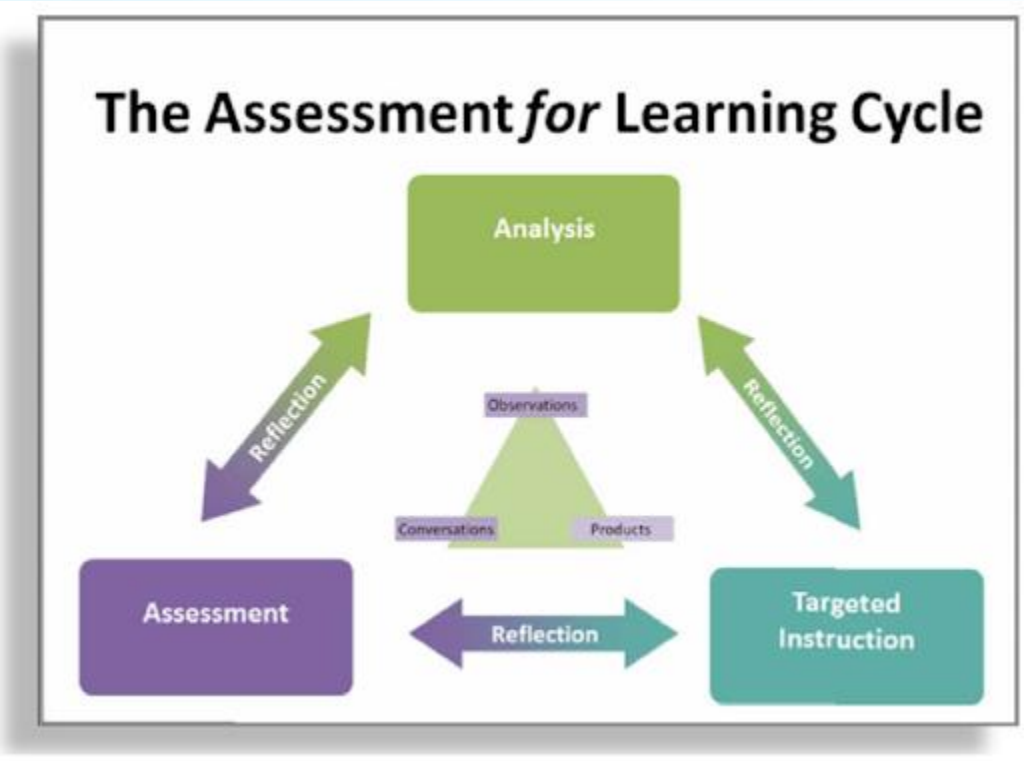
I think that it does because I look at what I can work on and sometimes I think that I didn't make any progress, but when I write my reflection, I realize that I did make some progress. Sometimes I think that I understand it, but when I'm writing my reflection, I realize that I still need to work on some things.

No I do not think daily reflections help me understand where I am at in the class. I do not like writing them because often I am still confused at what the I can statements are really saying or what they mean.

It really helps me look back and reflect and see if I maybe am still having trouble on a certain concept and helps me ask better questions next time it comes around. It also helps me realize how much I know and understand and helps me gain confidence in math and solving future problems.

I personally don't think daily reflections help because I forget a lot of what I have done and don't enjoy thinking about vocabulary outside of working in solving a math problem. It makes me doubt myself and feel like i do not know what is going on when I do.

EMPOWERING STUDENTS TO OWN THE ASSESSMENT PROCESS



SELF-ASSESSMENT EXAMPLES

Show all of your work.

1. The length of a rectangular park is 1 mile longer than twice the width. The area of the park is 55 square miles. What is the width of the park?

$A = lw$ ✓

$55 = (2w+1)(w)$

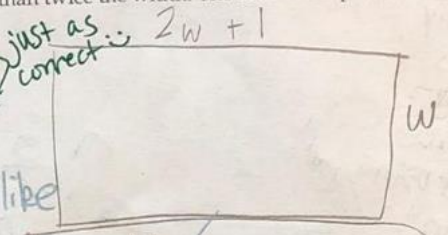
$55 = 2w^2 + 1w$

$0 = 2w^2 + 1w - 55$

csolve $(2x^2 + x - 55 = 0, x)$

$x = -5.5$ or $x = 5$

I didn't use the quadratic formula like you did just as correct ✓



$w = 5$ ft ✓

$L = 2(5) + 1 = 11$ ft

2. A stone was thrown from the top of a cliff 60 meters above sea level. The height of the stone above sea level t seconds after it was released is given by $h(t) = -5t^2 + 20t + 60$ meters.

a. Find the **time** taken for the stone to reach its maximum height. Write your answer in a complete sentence. *(Hint: the maximum height is at the vertex)*

the vertex is (2, 80) forgot to write the sentence good note!

t	0	1	2	3
Y	60	75	80	75

$h(2) = -5(2)^2 + 20(2) + 60$

$= -20 + 40 + 60$

$h(2) = 20 + 60$

$h(1) = -5(1)^2 + 20(1) + 60$ $h(1) = 80$

$h(1) = -5 + 20 + 60$ $h(1) = -5(3)^2 + 20(3) + 60$

$h(1) = -45 + 60 + 60$

$h(1) = 75$

0 secs 1 2 3 4 5 6

It takes two seconds for the stone to reach maximum height.

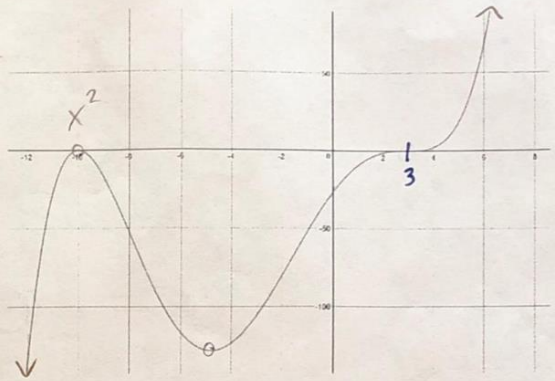
saves the work time with using vertex formula.

$t = \frac{-20}{2(-5)} = \frac{20}{10} = 2$ seconds

b. What was the **maximum height** above sea level reached by the stone? Write your answer in a complete sentence.

The maximum height is at 80ft after 2 secs. I need to show my work, even though it is above in the first portion. ✓

1. Consider the graph of the function $f(x)$.



a. What is the end behavior of $f(x)$?

As $x \to -\infty$, $f(x) \to -\infty$ ✓

As $x \to \infty$, $f(x) \to \infty$

b. What is a possible degree of $f(x)$? Explain how you determined this.

possible degree of f(x) could be 3 and any odd number (coefficient) higher. This is based on the number of maximum and minimum heights.

5

I did mention this but, I was thinking of putting 5 as the degree because of my x-intercepts. ✓

c. Give a possible lead coefficient for this polynomial. Explain your choice.

A leading coefficient could be positive (+) because the graph ends up going to the right side of the graph and is an odd function. (the end behavior is positive)

d. In factored form, write a possible function that represents $f(x)$.

$f(x) = x(x+10)^2(x-3)^3$

not needed

my degrees weren't correct here the degree was 5 right there but I realized ✓

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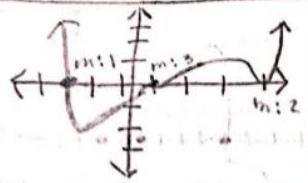
ASSESSMENT REFLECTION EXAMPLE

Given the graph of a polynomial, I can write a function to model its behavior.

I am challenged by...
I Know, What Do You Know?

11)

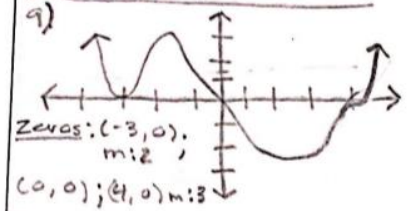


D) $f(x) = 0.01(x-1)^3(x-4)^2(x+2)$

I was challenged by this problem because I initially chose the wrong answer. I think I was challenged by this problem because I did not fully comprehend that factors with a multiplicity of 3 act cubic, m:2 bounce, and m:1 just go through the x-axis. My mistake was not noting that $(x-1)^3$ was correct because at that point the graph acts cubic. I also should've realized that x=4 had the m:2 b/c it bounced on the graph.

I am proud of my growth/understanding in ...
What Do You Know?

9)



Zeros: (-3, 0), m:2 ; (0, 0); (4, 0) m:3

Function: $f(x) = x(x+3)^2(x-4)^3$

I am proud of using my understanding of the effects that multiplicity has on the graph. I was able to get this problem correct because I did this problem after the previous one. I was able to use my understanding that a factor/zero with the multiplicity of 2 will bounce when graph, multiplicity of one will go straight through the x-axis, and a zero/factor with a multiplicity of 3 will act as a cubic on the graph.

Circle one of the following and explain why you chose the given statement.

Self- Evaluation
I can do this on my own and explain my solution path to others.
I can do this on my own, but I am not yet confident enough to answer questions or explain it to others.
I can do this on my own, but I am still making computational or minor errors.
I can do this with the help of an example.
I need more time to understand this.

I feel like I can do problems with this concepts on my own, but I do make errors with interpreting the graph. In the first one, it was hard to tell that the x=1 was acting cubic. I feel that I can sometimes mess up with leading coefficients as well.

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SAMPLE ASSESSMENT REFLECTION QUESTIONS

What were some of your most powerful learning moments and what made them so?

What about your thinking, learning, or work brought you the most satisfaction? Why?

How did you develop your understanding of the Target 12 'I Can' Statements? Where did you find an 'aha moment'? Explain.

What concept from this unit do you think that you will retain the most? What about that concept made it ‘stick’?

Where did you encounter struggles, and what did you do to deal with it? Explain.

How have you demonstrated mastery of the Target 12 'I Can' Statements? Explain the process you used to understand a concept. (Don't say you completed the Desmos activities and Aleks; you want to elaborate.)

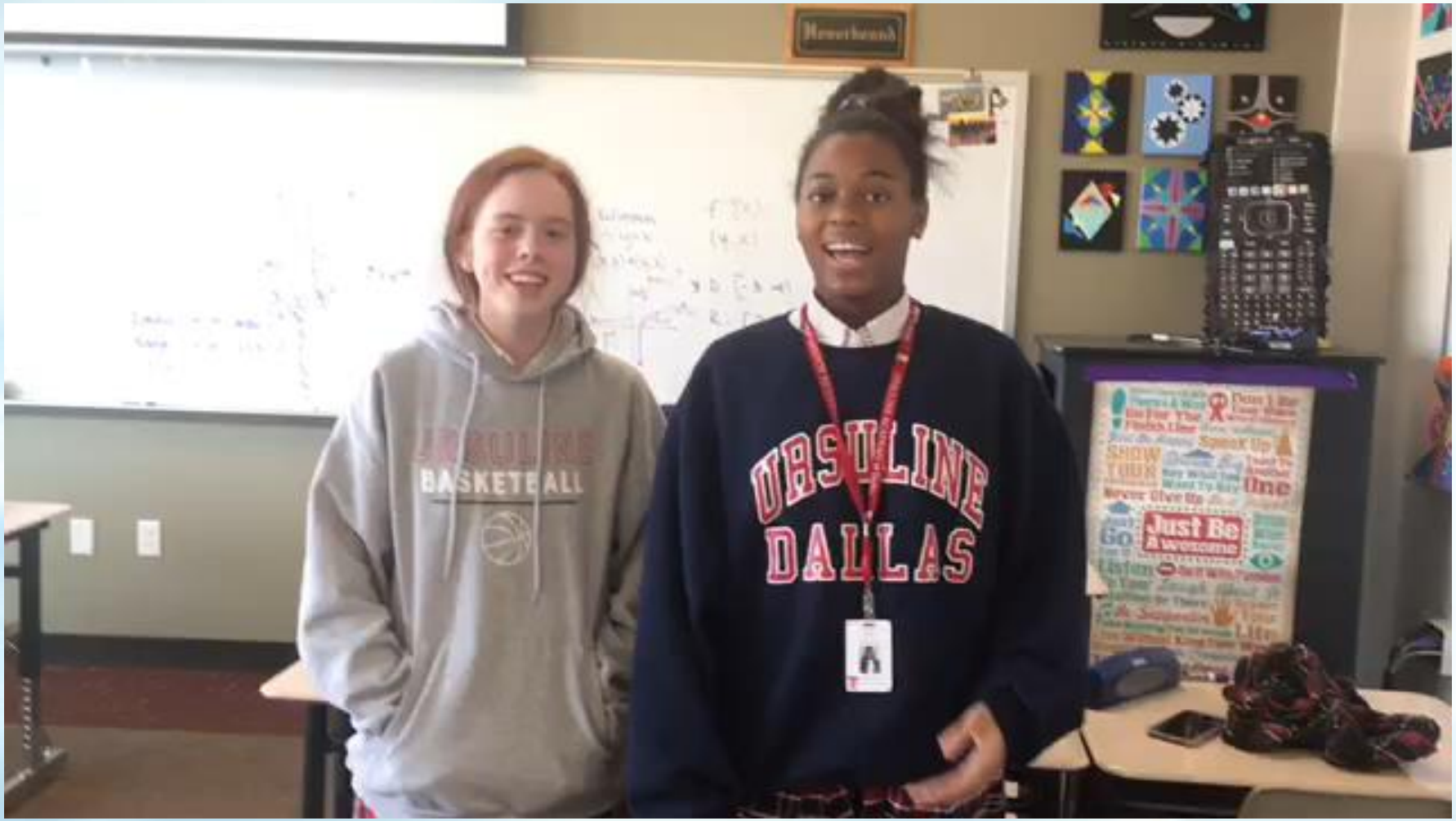
Student Thoughts- Assessment Reflections

When writing the assessment reflections, I am able to understand what specifically caused me to make mistakes and what areas I have mastered.

I feel that the reflections after I have taken the assessment help me understand where I actually stand in my understanding of the topic. Past years, I just took the test and never looked back. Now, the reflections force me to dig deep and explain what I know. This is really valuable for other tasks to see what I can do better next time to build a better understanding of future topics. It also helps me remember the task instead of just putting it in the very back of my mind.

I don’t like them. especially when I get a 100 on the assessment and I get points off on my reflection for not explaining my math.

I am not a fan I do not really enjoy writing them it seems almost like busy work. I feel like I doesn't really help me understand in the end how I grew and understand math.


BASKETBALL
URSULINE
DALLAS
Just Be Awesome

Thank you so much for joining us!

If you have any follow up questions, please feel free to email us at…

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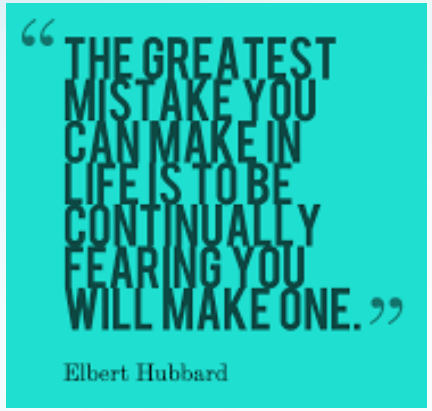
EMPOWERING STUDENTS TO OWN THE ASSESSMENT PROCESS

Mistakes **A**llow **T**hinking to **H**appen



- Why Mistakes Matter in Creating A Path
- 9 Ways To Help Students Learn Through Mistakes
- The Truth About Making Mistakes: Helping Students Discover the Benefits
- This Is Why You Should Be Proud of Making Mistakes