Mathematical Discourse: “Talk” is Only Part of the Equation

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Mathematics Learning Environments
(Face-to-Face, Virtual, Hybrid)
Mathematics Education Community

“The discourse of a classroom—the ways of representing, thinking, talking, agreeing, and disagreeing—is central to what and how students learn mathematics” (NCTM, 2007, Mathematics Teaching Today, p. 46).

Mathematical discussion is viewed as “a primary mechanism for developing conceptual understanding and meaningful learning of mathematics” (NCTM, 2014, Principles to Action, p. 30).
Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. **Construct viable arguments and critique the reasoning of others**
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning
What are features of a discourse-rich mathematics learning environment?
Why is Mathematical Discourse important?

Access and Assess
✓ What students know
✓ How students know it
✓ Use of math language
✓ Ability to formulate and critique argument

<table>
<thead>
<tr>
<th>Learning Mathematics Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engage with Mathematics</td>
</tr>
<tr>
<td>Conceptual Learning</td>
</tr>
<tr>
<td>Mathematical Argumentation</td>
</tr>
<tr>
<td>Critical Reasoning</td>
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<tr>
<th>Social and Cognitive Support</th>
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<tbody>
<tr>
<td>Safe Learning Environment</td>
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<tr>
<td>Supports Risk Taking</td>
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<tr>
<td>Addresses Power and Authority</td>
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<tr>
<td>(Who’s taking? Who’s talking most often?)</td>
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From Increased Participation...           
To Conceptual Math Talk

Learning Environment

UCONN
NEAG SCHOOL OF EDUCATION
What do I draw on?
5 Practices for Orchestrating Mathematics Discussions

**Setting Goals and Selecting a Task**

**Anticipating** student responses prior to the lesson

**Monitoring** students’ work on and engagement with the tasks

**Selecting** particular students to present their mathematical work

**Sequencing** students’ responses in a specific order for discussions

**Connecting** different students’ responses and connecting the responses to key mathematical ideas

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**NCTM:** 100 Days of Professional Learning

**Title:** Orchestrating Mathematical Discussions: Overcoming Challenges (Grade 6-8)

**Speaker:** Margaret (Peg) Smith

**Date:** April 8, 2020
What challenges have you experienced....?

...engaging ALL students?
... getting students to listen to each other?
... getting students to use the language of mathematics?
.. understanding what students are attempting to convey?
... assessing students’ thinking?
What do you assess most often?

students’ solutions
or
students’ thinking
What do we (as teachers) learn from students’ responses?

<table>
<thead>
<tr>
<th>Response 1</th>
<th>Response 2</th>
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<tbody>
<tr>
<td>17 x 18</td>
<td>“To find 17 x 18, I multiplied 17 times 20 and got 340. Then I subtracted 34 or 2 groups of 17, so the answer is 306</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
</tr>
<tr>
<td>136</td>
<td></td>
</tr>
<tr>
<td>+ 17</td>
<td></td>
</tr>
<tr>
<td>306</td>
<td></td>
</tr>
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</table>
What are different modes for communicating mathematics?

List as many different ways as possible.
Multiple Literacy in Mathematics

Ways to Give and Make Meaning

Vocabulary (Words)
Symbols
Reading - Interpreting
Writing - Producing
Speaking - Listening
Mathematics Tasks: Number

1. Mi az összege az összes számjegyet a sorrendben 1, 2, 3, 4, 5, 6, 7, ... 99, 100?

2. මාදිලිය ය ායා ගන්න 56 23 34 45 56

3. Determine the greatest common divisor of the elements of the set \{n^{13} - n \mid n \in \mathbb{Z}\}. 
Means of Communicating in Mathematics Learning Environments

➢ Common Everyday Language
➢ Mathematics verbal language (using words)
➢ Symbolic language (using mathematics symbols)
➢ Visual representations (using graphs, models, pictures)
➢ Gestures
➢ Unspoken “Taken as Shared” assumptions
➢ Quasi-Mathematical Language
  (e.g., “student utterances that may not be coherent”)
Mathematics - A Foreign Language

Semantics
• Median of a set of numbers
• Median of a triangle

Syntax
• A number is 4 more than a number x
• \( y = x + 4 \)

Symbols
• +, ≤, >, ÷, \( \sum \)
• \( y \leq x + 3, \angle ABC \)

Vocabulary
• Hypotenuse
• Trapezoid
• Table, median
➢ English Words with Different Meaning in Math
  ▪ e.g., positive/negative, table, rational, limit

➢ Technical vocabulary
  ▪ e.g., hypotenuse, divisor, trapezoid

➢ Terms with multiple meaning
  ▪ e.g., Square (a geometric figure, a square of a number)
  ▪ e.g., Base (of an exponential expression, of a triangle)

➢ Compound phrases that represent new concepts
  ▪ e.g., least common denominator, square root
Terms that are homonyms with common English words
- e.g., plain vs. plane, arc vs. ark, sum vs. some

Words with different meanings in different content registers or context
- Radical (social studies vs. math)
- Solution (science vs. math)
- Mean (malicious behavior vs. average)

Types of Mathematics Vocabulary
Multiple Terms may convey the same mathematical concepts

- **Addition**
  - *combine, plus, sum, increased by*

- **Subtraction**
  - *less, minus, decrease, difference, less than*

- **Proof**
  - *justification, argument*

- **Probability**
  - *likelihood, chance*
Redundancy in Verbalizations

- Why is it important for students to recognize the multiple verbalizations?
- How might these different verbalizations influence the assessment of student learning?
- How might these different verbalizations influence the assessment of EB knowledge?
- What can be done to support student in making meaning for these different verbalization?

- 12 – 5
- 12 subtract 5
- 5 less than 12
- 12 less 5
- 12 take away 5
- The difference of 12 and 5
Paraphrases of statements

Triangle A is as large as Triangle B
Triangle A and B are equal in size
Triangle A and Triangle B are the same size

Four divided into nine equal nine-fourths
Nine divided by four equals nine-fourths
If nine is divided by four, nine-fourths results
Mr. Brown in redoing is lawn. How much sod must Mr. Brown purchase for a lawn space that is 8 ft long by 4 ft wide?

- “Sod” implies to cover the space
Mathematics is a technical language that conveys information using words, symbols, notations, icons, graphs, & representations.

Mathematical Discourse involves the transfer and exchange information through speaking, listening, reading, writing, interpreting, producing ...
What other challenges might an students experience in a discourse-rich mathematics learning environment?
Instructional Goals

1. Content Development
   ❖ Develop proficiency in subject matter content

2. Language Development
   ❖ Proficiency using the language of mathematics
   ❖ Its vocabulary, grammatical structure, and symbols

In sum, provide students opportunities to use the language of mathematics, in all of its forms productively.
Implicit messages

Teacher statements:

“If you don’t do this carefully, you will get the wrong answer.”

“Be sure to explain your thinking, so that I understand how you are approaching this program.”
Teacher statements:

“Can someone identify the mistake that Martha made?”

“Martha is almost there; can anyone support her way of thinking?”
Implicit messages

Teacher statements:

Mark and Paul, the two of you used similar strategies. Which one of you would like to explain how you figured it out?

“That was powerful thinking, please explain how you got that again.”
Conversational vs. Lecture

FROM *Detached*

“Today we will learn about ....”

TO *Inviting*

“What do you think we’ll have if....”

Use Active Voice
Be Direct
Colloquial Expressions

• May not be familiar to students, such as emergent bilinguals
  • e.g., “In the red”

Can you think of other colloquial expressions that might be used in the mathematics classroom?
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