NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

What math do elementary teachers really need to know?

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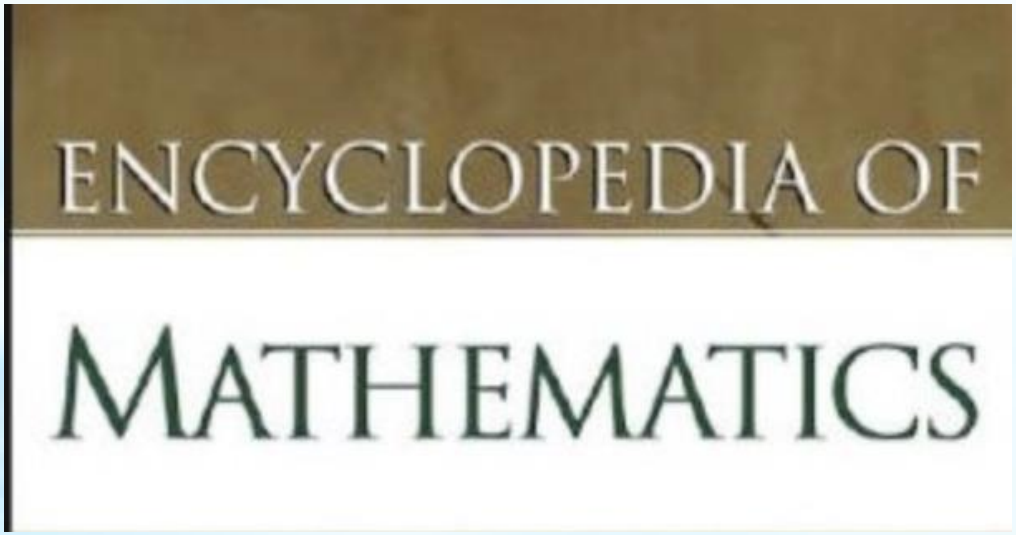
August 2020

One Two

The easy answer



What this talk is all about

- Not so much about fun problems, but more about the nitty, gritty everyday, but making it better.
- Because I think that's where we make it or break it!

What math do teachers need? Is it..

- How to do the procedures?
- Multiple ways to do them?
- Multiple manipulatives/tools/visuals to use?
- Learning trajectories?

Is it…

- Why the procedure works?
- How one idea links to other math ideas?
- When a particular piece of math is used in everyday situations or, more broadly, why is it worth knowing?

Can you get away with…

- Good pedagogical moves?

Are there good pedagogical moves?

Of course, and good teaching is built on them.

Even if you know all the math in the world, you still have to know how to engage students, help them think for themselves, help them articulate their thoughts, etc.

Can you get away with a good resource?

NO!!!

The point of having a live teacher, not a robot, is to be responsive.

So let’s get real!

What does it look like at different grade levels with standard topics?

Grade 2 subtraction

Typical standards:

- Solve problems involving taking from, comparing, adding to with unknowns in all positions, using drawings and equations
- Subtraction facts

Grade 2 subtraction

- Use strategies based on place value, properties and operation relationships and explaining them
- Subtracting like values, sometimes by composing and decomposing
- Mentally adding 10 or 100

Unknowns in all position?

- What is different about thinking about?

12 + 23 = []	23 – 12 = []
12 + [] = 35	23 – [] = 12
[] + 12 = 35	[] - 12 = 23

Is there such a thing as…

a subtraction problem?

- Is this a subtraction problem?
- I had $12 and needed $23. How much more do I need?

Is there such a thing as…

a subtraction problem?

- Is this?
- I had some money and spent $12 and have $23 left. How much did I have?

What does comparing ..

have to do with subtraction?

I thought subtraction was take away.

Consider:

I have 24 tatoos.

My brother has 15.

How many more do I have?

Subtraction facts

Pretty clear EXCEPT

Does it mean instantaneous recall or relatively quick recall?

Can it take you 3 seconds to figure out 14 – 8 using a strategy or is that too long?

Strategies based on place value,properties,….

What are “efficient enough” strategies?

Is the standard algorithm more efficient than others?

How long do we “tolerate” other strategies?

For example

Let’s consider 71 – 38.

What options do we have?

Hundred charts (71 – 38)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Hundred charts (71 – 38)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Hundred charts (71 – 38)

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31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
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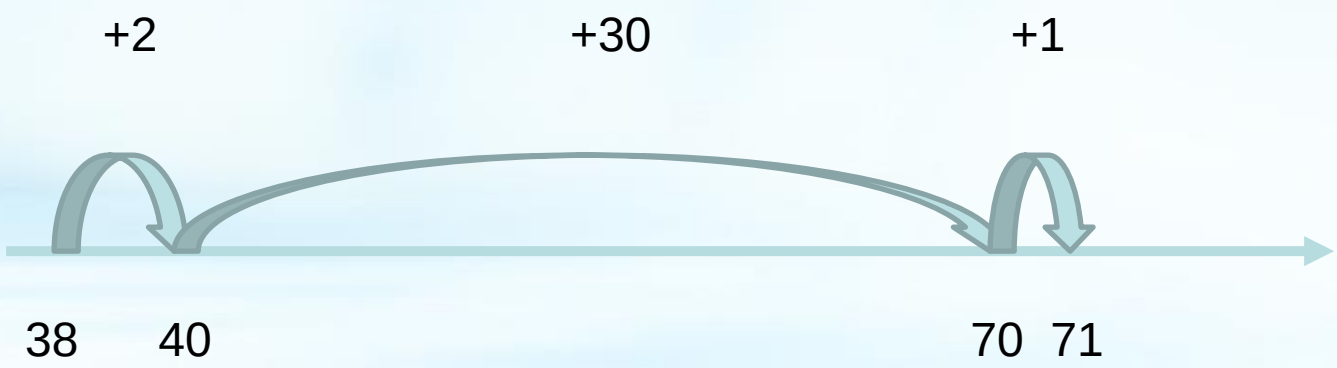
Open number line (71 – 38)



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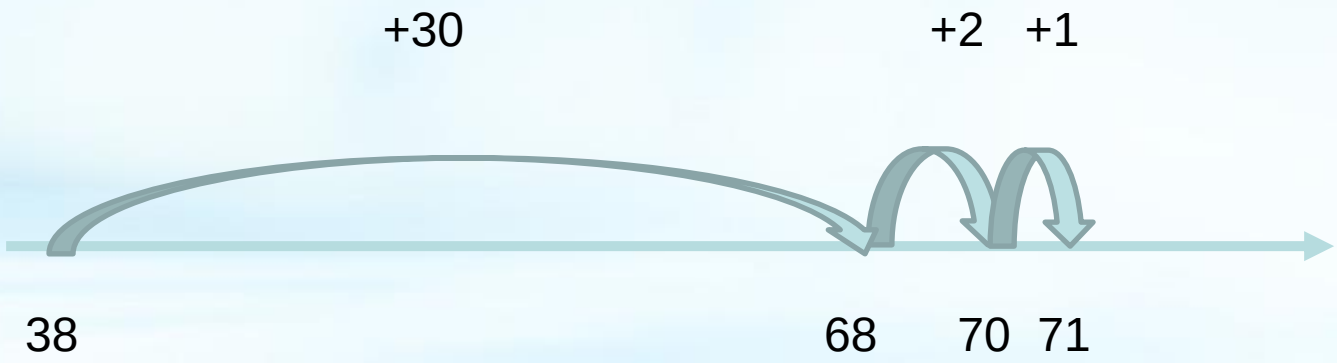

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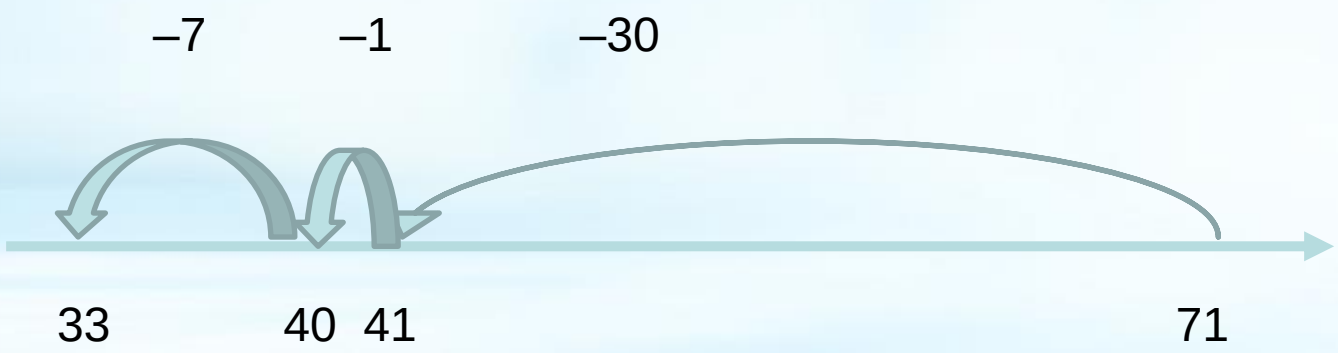

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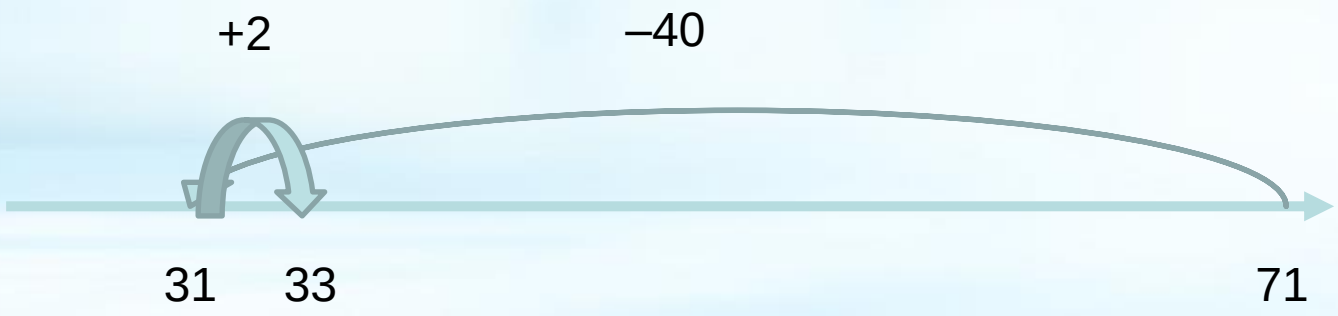
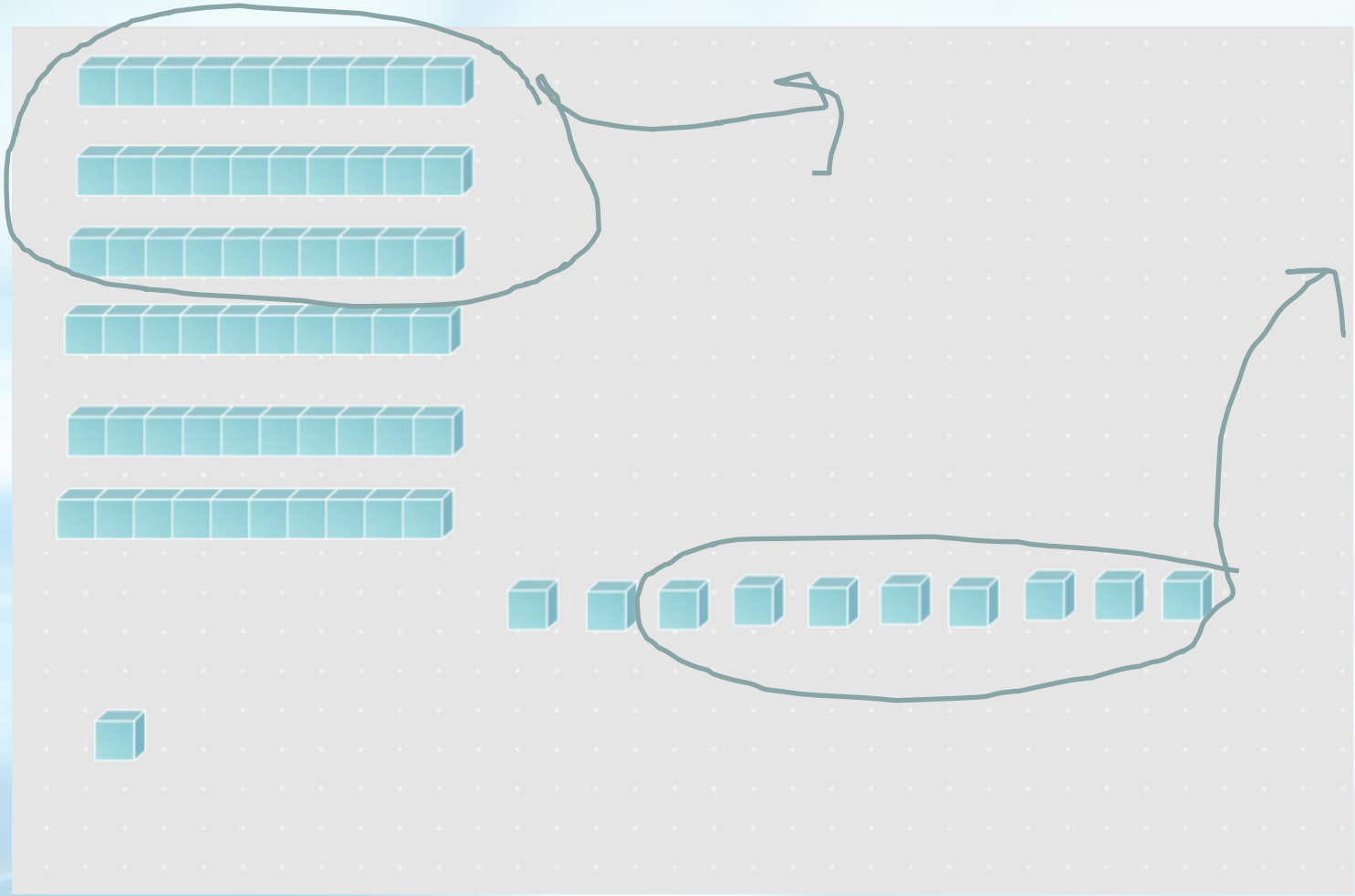


Base ten blocks (71 – 38)



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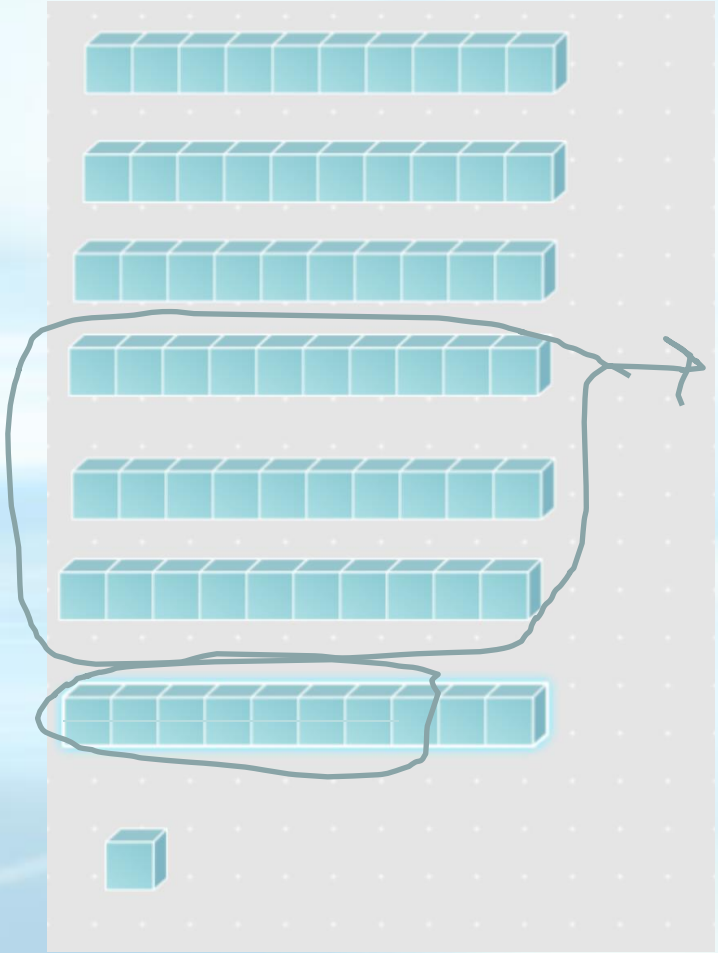

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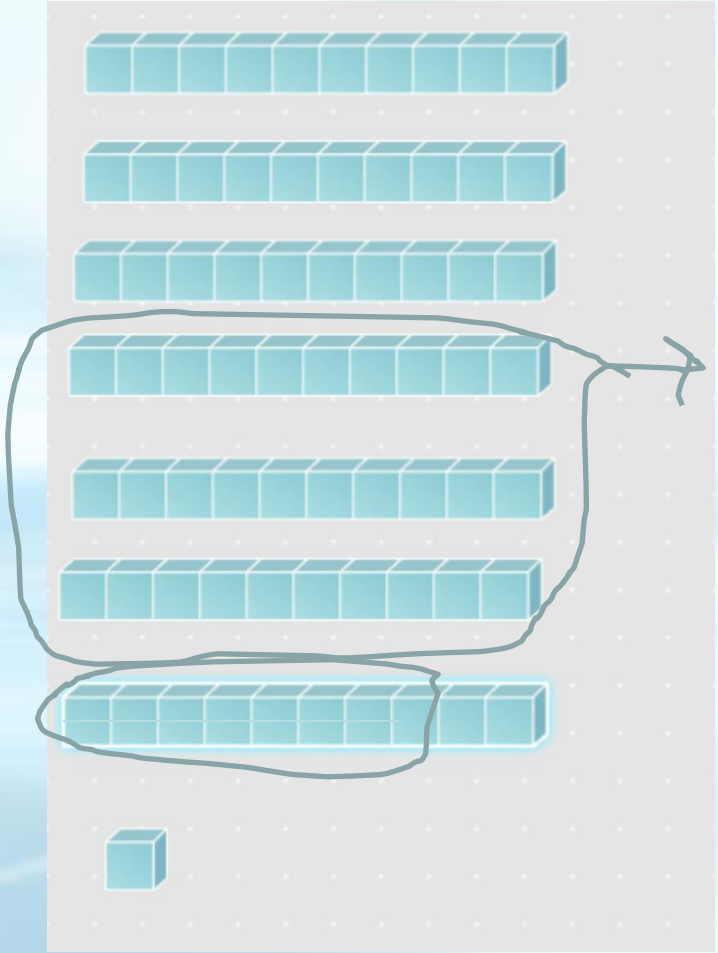

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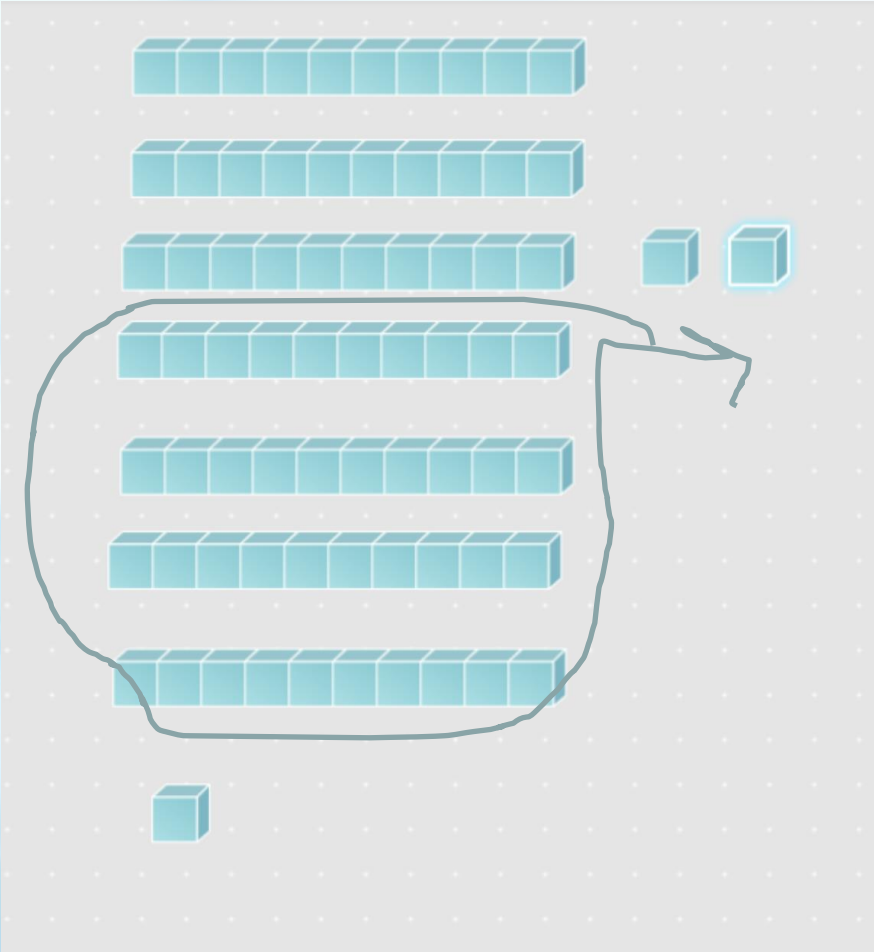


Symbolically

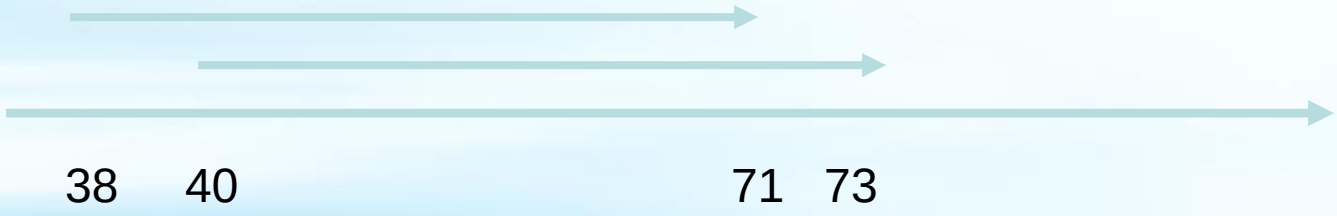
$$
\begin{array}{r}
6 \\
\cancel{7}^{1}1 \\
-\underline{3\ 8} \\
33
\end{array}
$$

Symbolically

$$
\begin{array}{r}
6 \\
\cancel{7}\,1 \\
-\underline{3\ 8} \\
33 \\
\underline{(10-8+1)}
\end{array}
$$

Symbolically

$$71 - 38 = 73 - 40 = 33$$



Symbolically

71 – 30 = 41

41 – 8 = 33 [do it in parts]

Symbolically

$$\begin{array}{r} 38 \\ +\ 33 \\ \hline 71 \end{array}$$

Linking to other math ideas

- The link between subtraction and addition
- The link between subtraction and counting back and counting forward

Everyday uses

- Should these be adult uses or kid uses of subtraction?

So what do you think?

- Do you think teachers should know all of what I just showed you?
  Which parts could we let go?

Grade 4 fractions

Typical standards

- Equivalent fraction by multiplying top and bottom by same amount [visual first]
- Compare by creating common denominators or numerators or comparing to a benchmark like 1/2

Grade 4 fractions

- a/b as a copies of 1/b
- Decompose into a sum of fractions with same denominator
- Add and subtract mixed numbers with like denominators
- Solve problems involving + and – of fractions with like denominators

Grade 4 fractions

- Consider a/b as a x 1/b to multiply fractions by whole numbers
- Solve problems involving multiplication of fractions by whole numbers

Equivalence

- Is it enough to know that you can draw a picture for a particular fraction?

  E.g. 2/3 = 4/6 because

Or do you need to know why it works?

Equivalence

- Is it enough to be able to get equivalent fractions or should a teacher know enough to help students realize that numerators and denominators for equivalent fractions for 2/5 are always a multiple of 3 apart and why?
- E.g. 6/15 OR 20/50

Comparison

- Why is getting common numerators useful? How is it useful?

- How does a student compare to a benchmark like ½? Do they get an equivalent fraction or use a numerator/denominator relationship?

Comparing to 1/2

- How do you know that 18/41 is less than ½ without getting an equivalent fraction?

- Does the teacher realize it's because 18 is less than half of 41?

Comparing to Other benchmarks

- So if you were comparing, e.g. 4/17 to 3/8, you might think that 4/17 is less than 1/4 since 4 x 4 = 16 and 3/8 is not less than ¼ since 3 is more than ¼ of 8.

Adding fractions with the same denominator

- Why don’t you add the denominators, but only the numerators?

- It should be something like 5 ducks + 2 ducks = 7 ducks (we don’t say duck-ducks) so 5 eighths + 2 eighths = 7 eighths.

Adding mixed numbers

- Should you make them improper fractions first or not?
- Why yes? Why no?
- Consider, e.g. $3\frac{2}{7} + 4\frac{4}{7}$

Linking fractions to other math ideas

- Why can we write a fraction to show dividing, e.g. 12 ÷ 5 as 12/5?

- How do fractions relate to decimals? percents? Why does the relationship matter?

Linking fractions to everyday uses

- What sorts of fair share problems do children really encounter?
- When else do children use fractions?

So again…

- Which of these thing do you think is superfluous?

- Which should teachers know?

Grade 6 Rate and Ratio

Typical standards

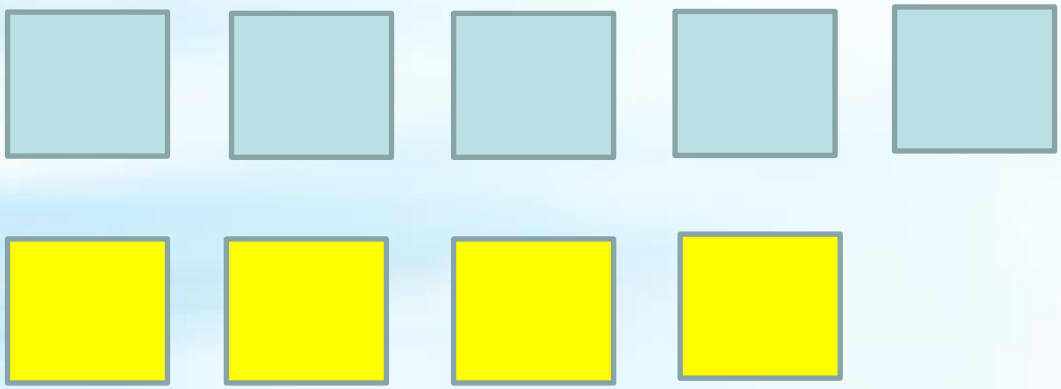
- Using ratio language
- Notion of unit rate related to ratios
- Equivalent ratios

Grade 6 Rate and Ratio

- Solving unit rate problems
- Determining a percent of a quantity
- Using ratio to convert measurement units

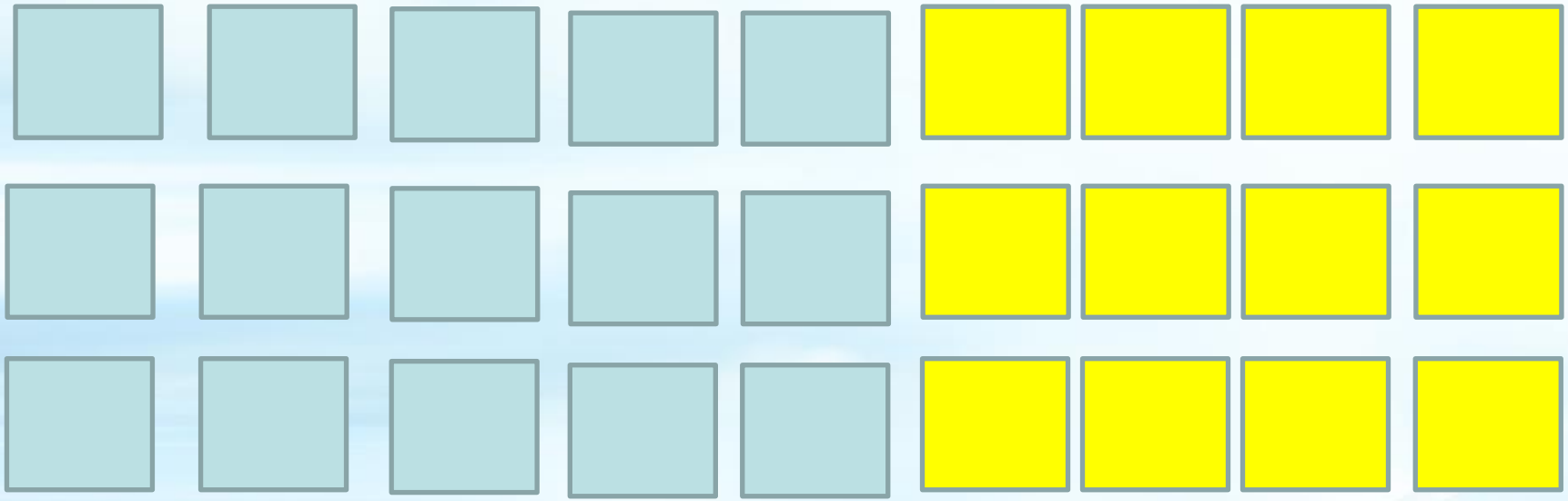
Ratio language

- Is this really showing a ratio of 5:4 or is it just showing 5 and 4?



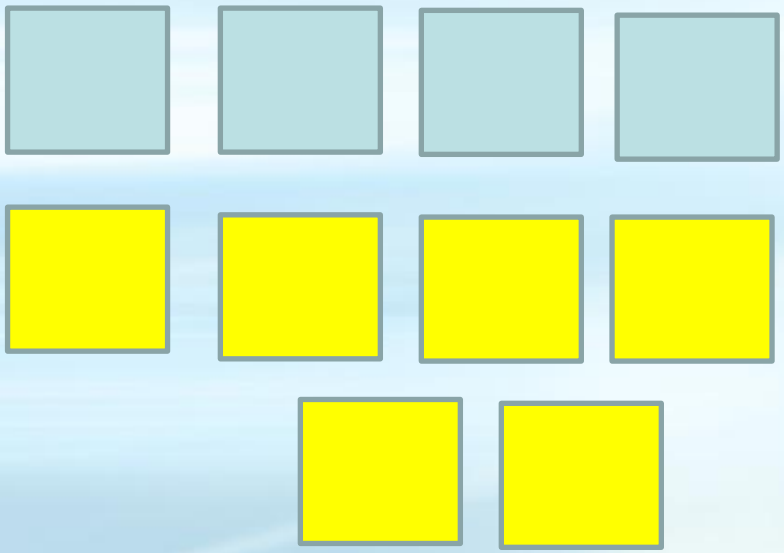
Ratio language

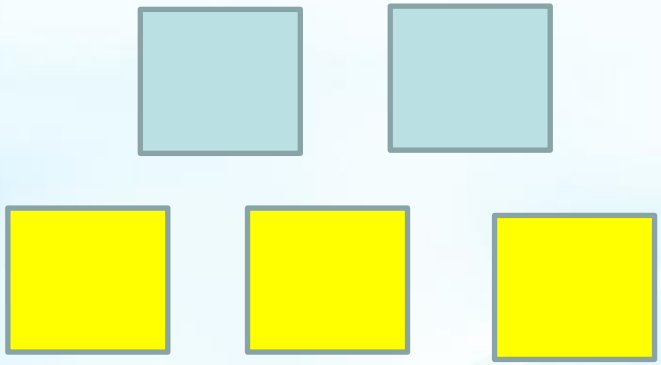
- What about this one?



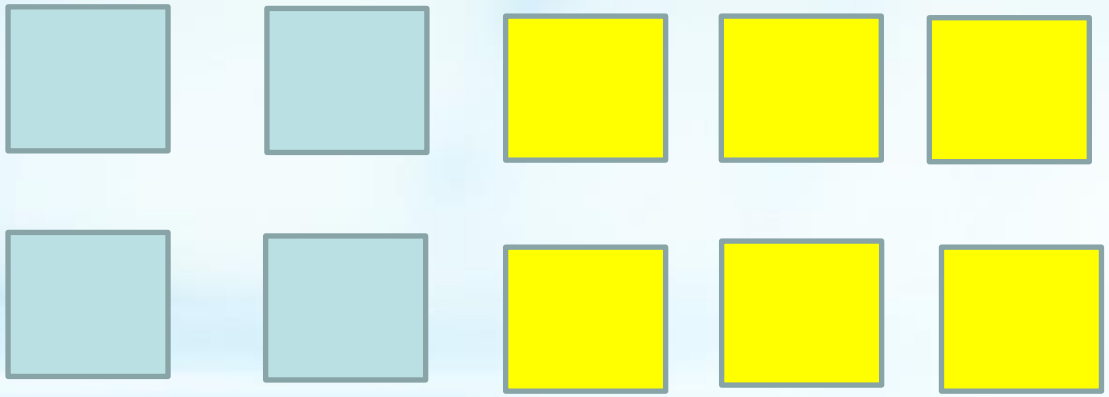
Equivalent ratios

- Why are the ratios 4:6 and 2:3 equivalent?
- Does this picture make it clear?





Compare to this



Tables of values

- How do ratio tables work? For example, suppose you are looking for an equivalent ratio to 8:42 where the first term is 6.
- Why can you “add columns”?

8	**4**	**2**	**6**			
42	21	10.5	31.5			

Unit rates

- When is a unit rate a useful tool and when it is less useful?

- 4 boxes of pasta cost $7.16.
- How much should 8 cost?
- 6 cost?
- 5 cost?

Unit rates

- Can a unit rate be “2 boxes of pasta”?

- 4 boxes of pasta cost $7.16.
- How much should 6 cost?

A Visual model

- How could a double number line help you figure out the number of children in a restaurant if the ratio of children: adults is 6:10 and you know there are 96 patrons.

A Visual model

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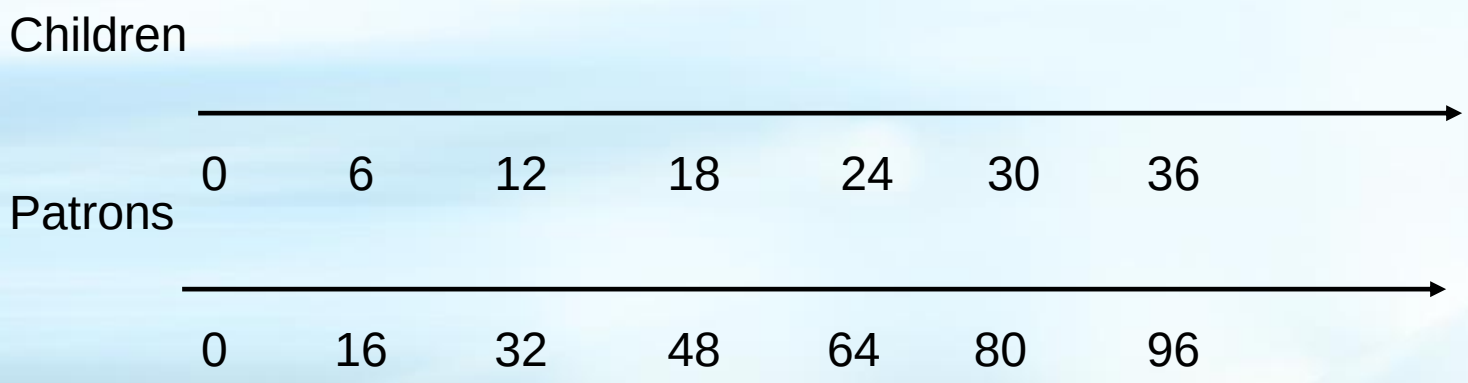
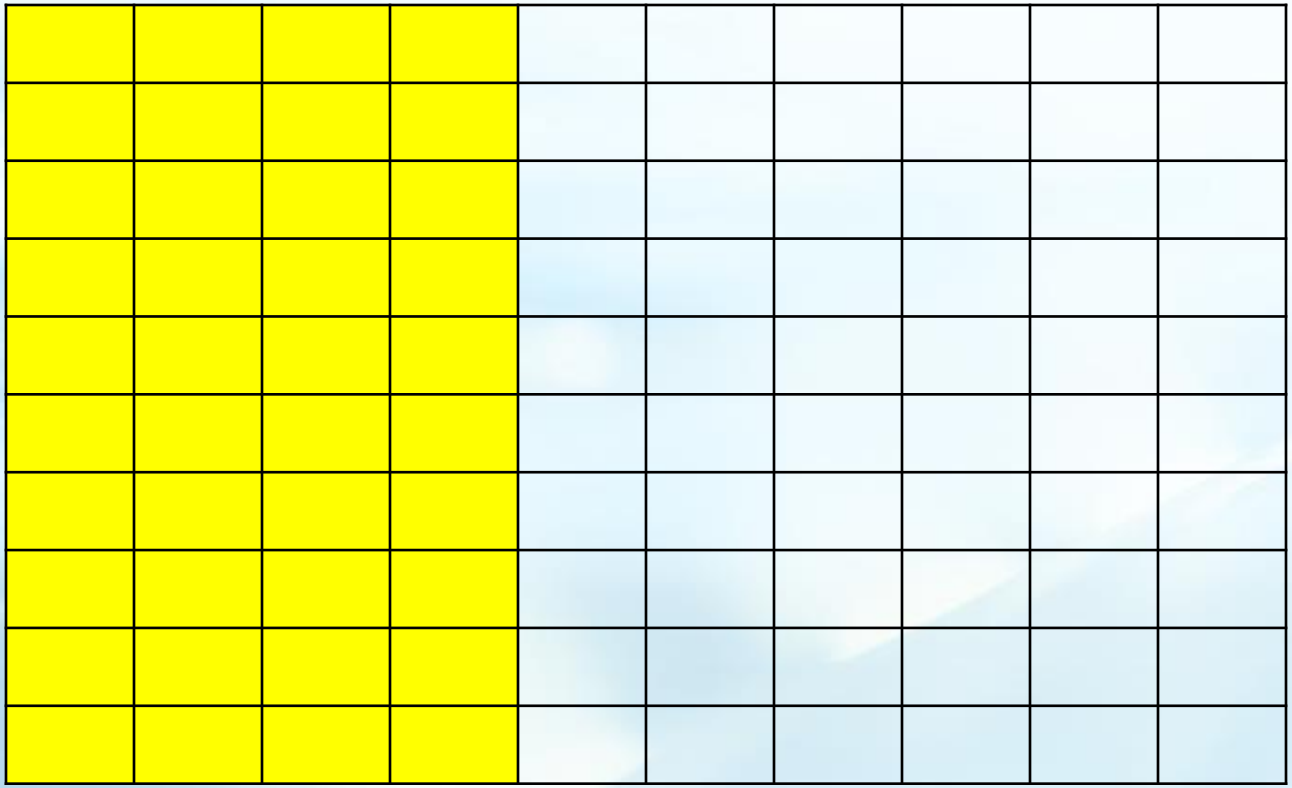


A Visual model for Percent

- How could a 10 x 10 grid help you figure out what number 28 is 40% of?



Relating percents

- What's a smart way to figure out 15% of something mentally?
- Which other percents are relatively easy to do mentally (if the numbers are not too crazy)?

Relating percents

- Suppose you know that A is 40% of B.
- What other percent statements can you make?

- E.g. A÷2 is 20% of B.
- A is 20% of 2B.
- 2A is 40% of 2B.

Percent problems

- Why is 35% of 23 = 0.35 x 23? Where did the times come from?

- Why is 35% of 23 = 35 x 23 hundredths?

Percent problems

- When does it make more sense to use a percent and when does it make more sense to use a fraction equivalent?

- E.g. What is 75% of 444?

Percent problems

- When you are trying to figure out the ”old price” when you know the sale price and the discount, why do you divide?

- E.g. A jacket was discounted by 30%. Now it costs $42. What was the original price? [Why is it $42 ÷ 0.7?]

Should students and teachers know…

That when we solve problems involving ratios or percents, we are invariably using equivalent ratios?

For example

The ratio of flour to sugar in a recipe is 6:1.

How much flour do you need if you use 1 ½ cups of sugar?

You are solving 6:1 = [] : 1 ½.

Relating ratios to other math

- Are ratios and fractions the same or not?
- Can every ratio situation (even part-to-part) be described using fractions?
- How is ratio related to multiplication or division or is it?
- How does ratio play into place value thinking?

When do we use ratios?

- When do children use percents?
- ratios?

What is unnecessary?

- Are some of the ideas presented not critical for teachers to know?

Grade 8 Expressions & Equations

- Exponent laws
- Square root and cube root
- Scientific notation with appropriate units to estimate

Grade 8 Expressions & Equations

- Graph proportional relationships recognizing slope as unit rate
- Recognize slope doesn’t change within the line

Grade 8 Expressions & Equations

- Solve linear equations in one variable using equivalent equations
- Solve pairs of simultaneous line equations graphical and algebraically
- Solve problems based on simultaneous linear equations

Exponent laws

- So why is $4^{-3} = \frac{1}{64}$?

Square root/cube root

- Why do square roots of consecutive numbers or cube roots get so close eventually?
- E.g. $\sqrt{2000}$ and $\sqrt{2001}$ are only about 0.01 apart.

Graphing proportional relationships

- Why is every relationship between two variables where f(2x) always equals 2f(x) a line through the origin?

Solving linear equations using equivalent equations

- When you solve, e.g. $-3x + 4 = 10$, you might next write $-3x = 6$.
- Why is that okay?
- What else could you have done next instead?
- Why do you do it?

Solving linear equations

- Is an equation like $4x + 5 = 2(2x + 2.5)$ an equation?
- Can you solve it?

Solving linear equations using equivalent equations

- Is it important to be able to model equations with algebra tiles?
- For example, how could you show why $-3x + 4 = 10$ has a solution of $x = -2$ with tiles?

$-3x + 4 = 10$

Simultaneous linear equations

- Why don’t we solve all simultaneous linear equations graphically? You can always draw graphs with technology.

Simultaneous linear equations

- How would you come up with two equations that meet at (5,–11)?
- You could graph, but….

Simultaneous linear equations

- You could look at (5,–11) and notice:
- $y = -(2x + 1)$ and $y = x - 16$ and you're done.

When do simultaneous linear equations come up?

- So often, e.g.
- Mathematically:
- I am thinking of two numbers with a difference triple the sum.

When do these come up?

- Everyday life:
- I can buy 2 donuts and a coffee for …. but it cost …. for 3 donuts and 3 coffees.
- How much was each?

The story for me…

- A teacher has a **professional obligation** to know how (in lots of ways), to know why, and to know when.

- This is as true in Grade 1 as Grade 8.

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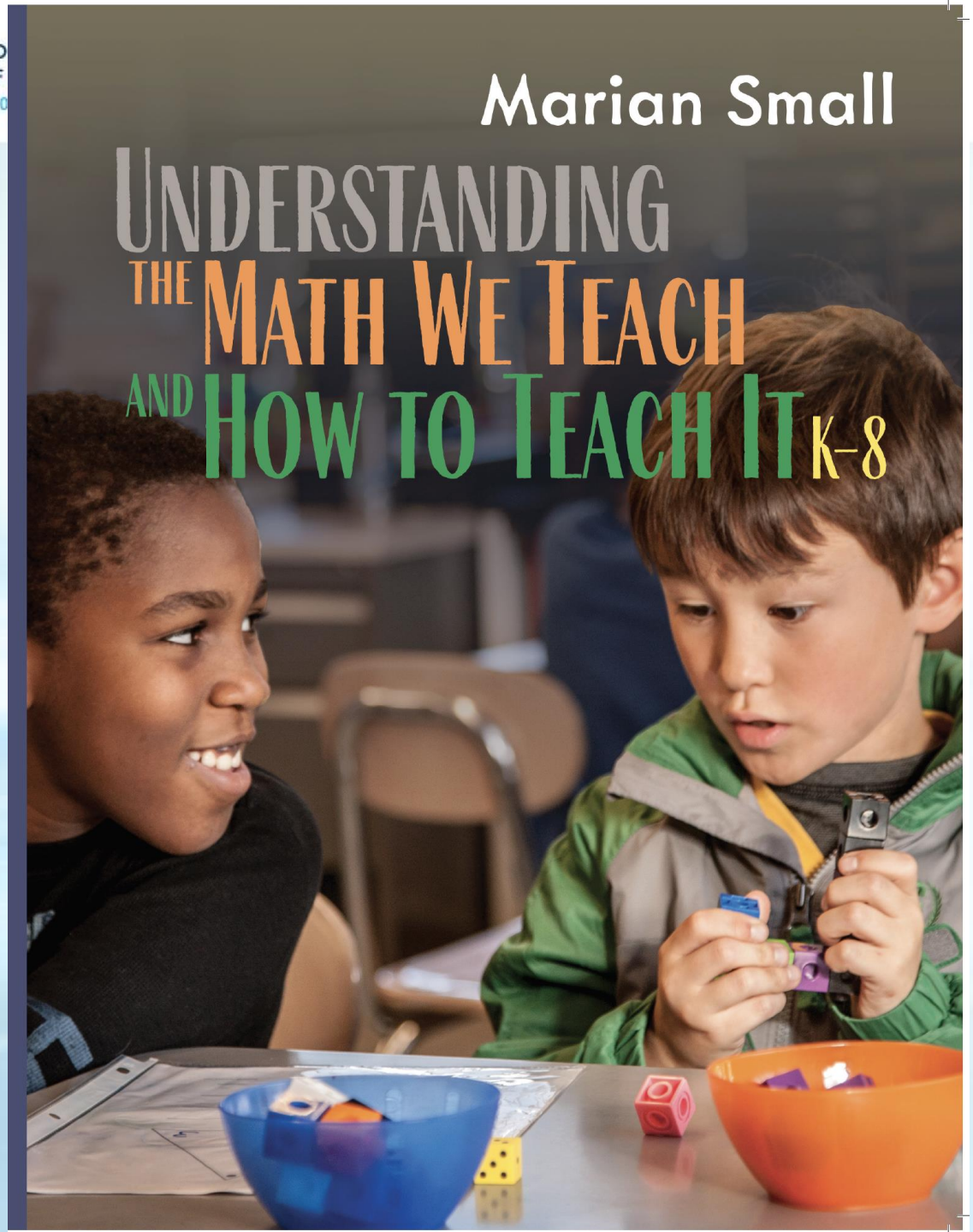

One
Two