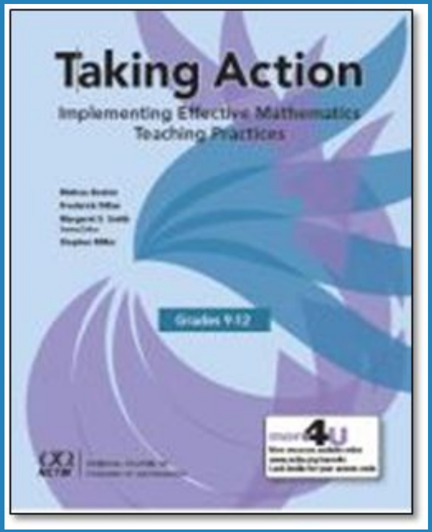
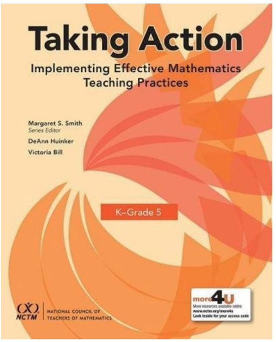
Taking Action:
Focus on High School

Link to Handouts

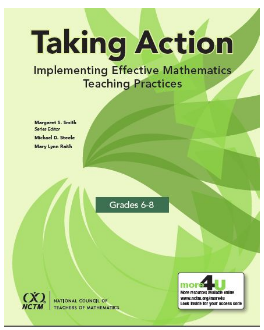


Melissa Boston,
Duquesne University, Pittsburgh, PA
NCTM Board of Directors (2020-2023)

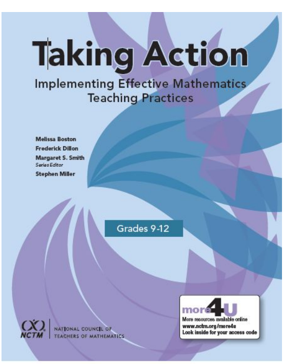
TAKING ACTION SERIES



Huinker, DeAnn and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017.



Smith, Margaret, Michael Steele and Mary Lynn Raith. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8.* Reston, VA: National Council of Teachers of Mathematics, 2017.



Boston, Melissa, Frederick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12.* Reston, VA: National Council of Teachers of Mathematics, 2017.

Effective Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Poll

- A. Administrator or Instructional Coach
- B. Elementary Teacher
- C. Middle School Teacher
- D. Teacher of Algebra 1 or 2
- E. Teacher of Geometry
- F. Teacher of Statistics
- G. Teacher of Trigonometry or Pre-Calculus

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How familiar are you with the Effective Teaching Practices and the Taking Action Books?

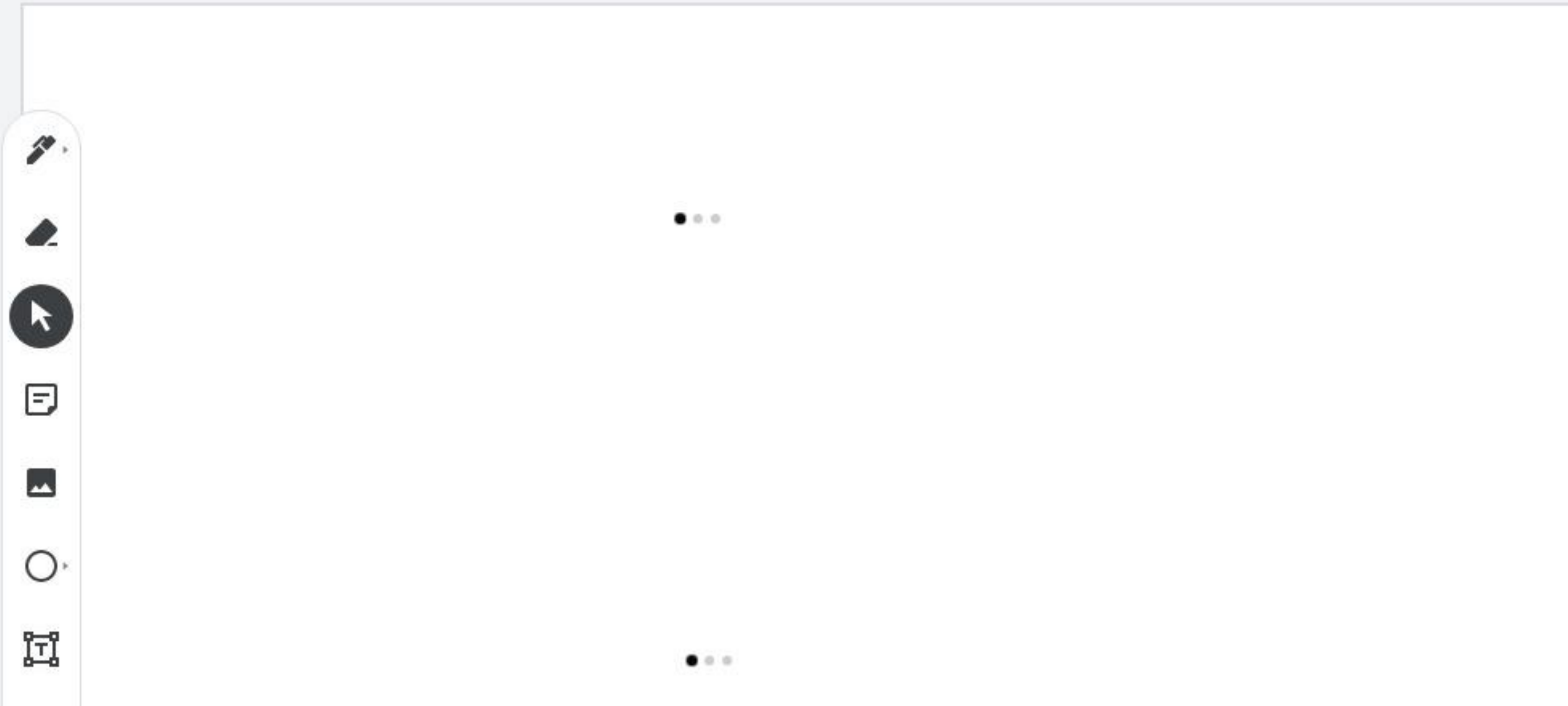
Not familiar with the Effective Teaching Practices

Very familiar with the Effective Teaching Practices

Not familiar with the "Taking Action" book(s)

Very familiar with the "Taking Action" book(s)

Web Content



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Effective
Mathematics Teaching Practices

1. Establish mathematics goals to focus learning.
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Our Time Together

Explore the ETP of "Elicit and Use Evidence of Students' Thinking" by:

- Engaging with the "Bike and Truck" Task
- Watching the "Bike and Truck" Video

Explore the ETP of "Build Procedural Fluency from Conceptual Understanding" by:

- Considering the role of tasks such as the "Bike and Truck" Task and the "Pay it Forward" Task a
- Considering the role of Sequences of Tasks

Elicit and Use Evidence of Student Thinking

Evidence should:

- Provide a window into students' thinking;
- Help the teacher determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

> *Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.*
>
> (Wiliam 2007, pp. 1054; 1091)

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Elicit and Use Evidence of Student Thinking

Research studies related to eliciting and using evidence of student thinking focus on two key areas:

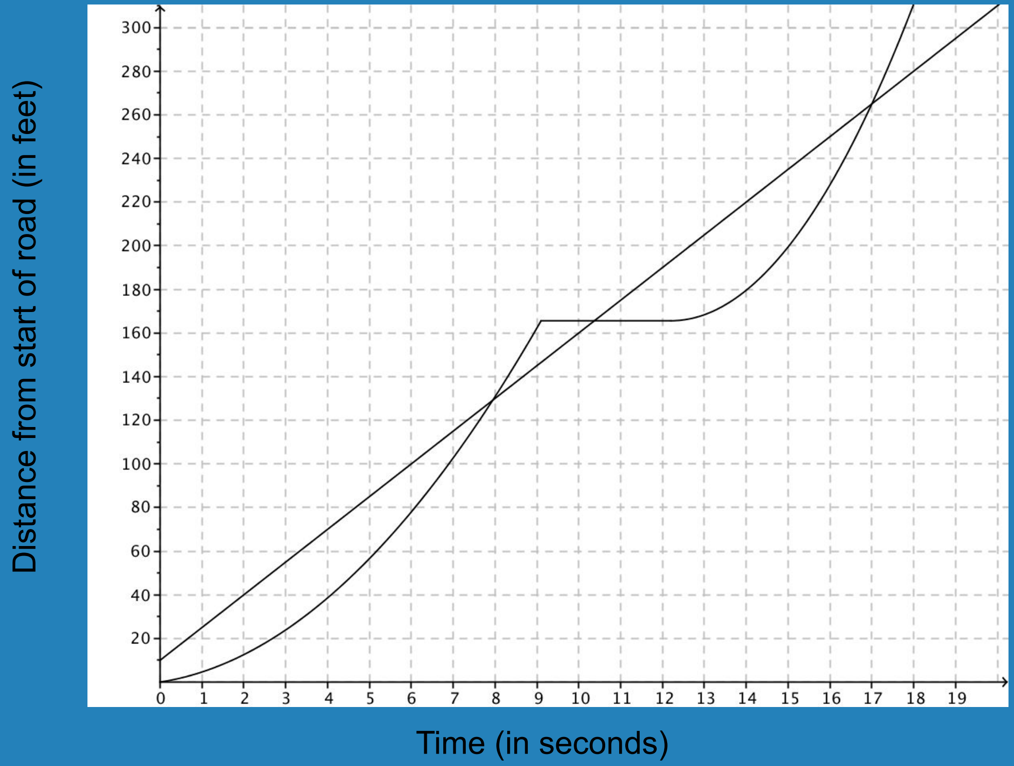
(1) how teachers interpret and make sense of student thinking, and

(2) how teachers make use of what they know and understand about student thinking before, during, and after a lesson.

Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12, NCTM, 2017

Bike and Truck Task

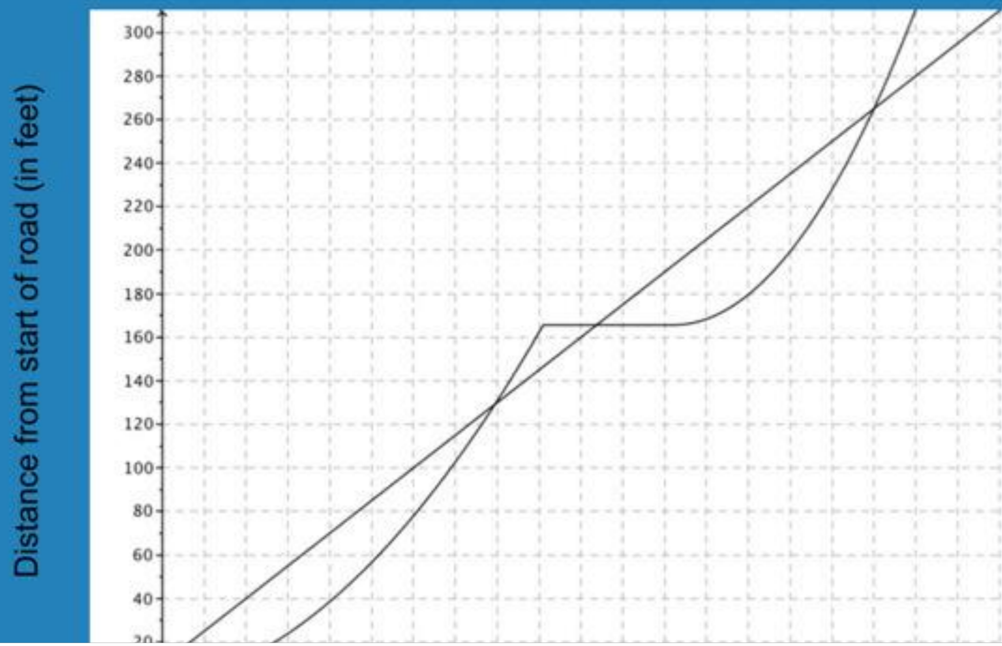
A bicycle traveling at a steady rate and a truck are moving along a road in the same direction. The graph below shows their positions as a function of time. Let B(t) represent the bicycle's distance and K(t) represent the truck's distance.



INSTITUTE for LEARNING

Draw It

A bicycle traveling at a steady rate and a truck are moving along a road in the same direction. The graph below shows their positions as a function of time. Let B(t) represent the bicycle's distance and K(t) represent the truck's distance.



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Bike and Truck Task

1. Label the graphs appropriately with B(t) and K(t). Explain how you made your decision.
2. Describe the movement of the truck. Explain how you used the values of B(t) and K(t) to make decisions about your description.
3. Which vehicle was first to reach 300 feet from the start of the road? How can you use the domain and/or range to determine which vehicle was the first to reach 300 feet? Explain your reasoning in words.
4. Jack claims that the average rate of change for both the bicycle and the truck was the same in the first 17 seconds of travel. Explain why you agree or disagree with Jack and why.

Bike and Truck Task

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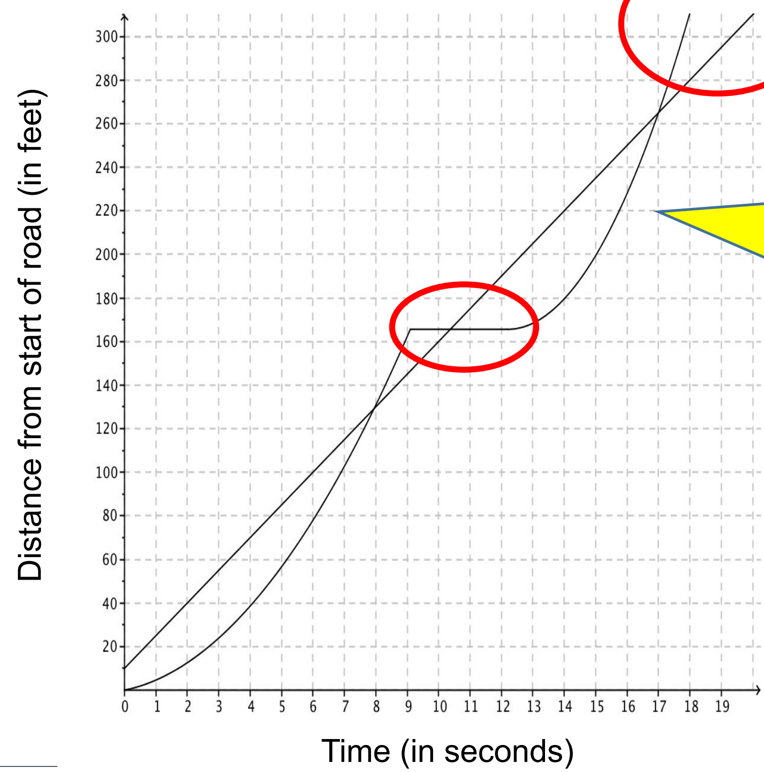

What misconceptions might students have?

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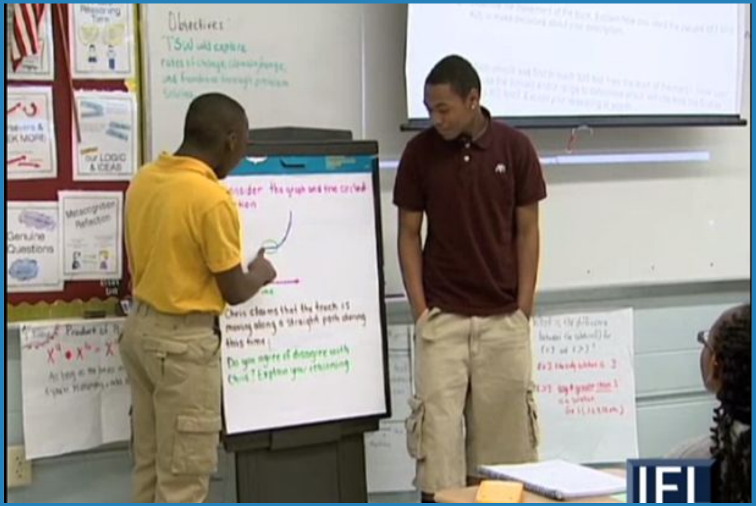
Bike and Truck Task
Video Context

School: Tyner Academy
Principal: Carol Goss
Teacher: Shalunda Shackelford
Class: Algebra 1
Number of Students: 26



At the time the video was filmed, Shalunda Shackelford was a teacher at Tyner Academy in the Hamilton County School District. The lesson features students in an Algebra 1 class.

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Bike and Truck Video 1

- To what extent did Ms. Shackelford elicit students' thinking?
- To what extent did (or could) Ms. Shackelford use the evidence to inform her instruction?

Collaborate Board

Bike and Truck Video

To what extent did Ms. Shackelford elicit and use students' thinking? To what extent did Mrs. Shackelford use evidence of student thinking to inform instruction?

Elicit and Use Evidence of Student Thinking

Thoughtfully planned questions that elicit important aspects of student thinking can lead to important mathematical ideas being made public, and a teacher must then plan to bring those aspects of student thinking together in a discussion that surfaces key ideas and builds understandings for all students.

Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12, NCTM, 2017

Putting the Effective Teaching Practices Together

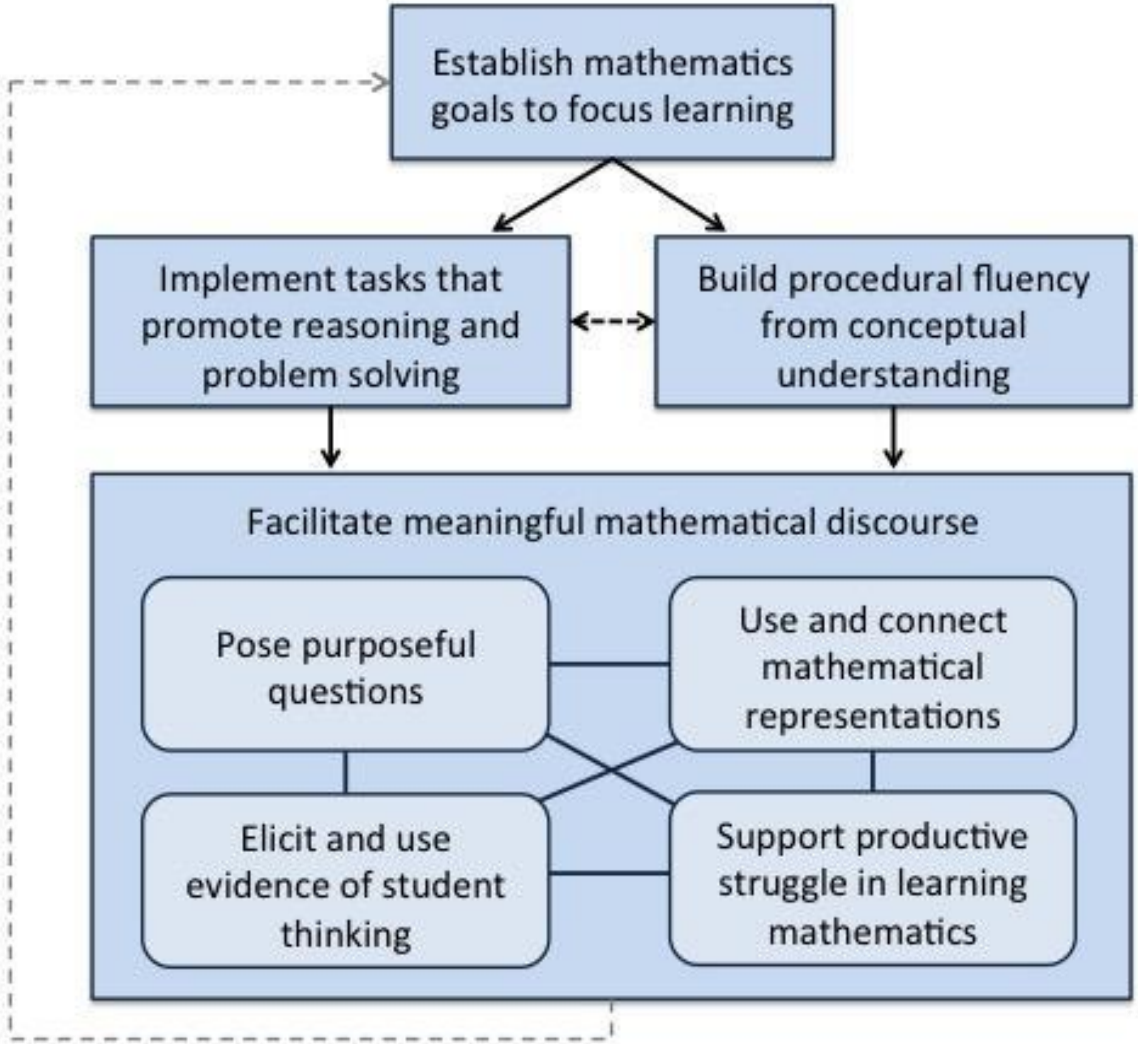

Figure 10.1 in "Taking Action…Grades 9-12" (Boston, Dillon, Smith & Miller, 2017).

Build Procedural Fluency from Conceptual Understanding

Mathematics classrooms should:

- Provide students opportunities to use their own reasoning strategies and methods to solve problems;
- Press students to explain and discuss why the procedures they are using work for particular problems;
- Use visual models to support students' understandings of general methods

To use mathematics effectively, students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results. (Martin, 2009, p. 165)

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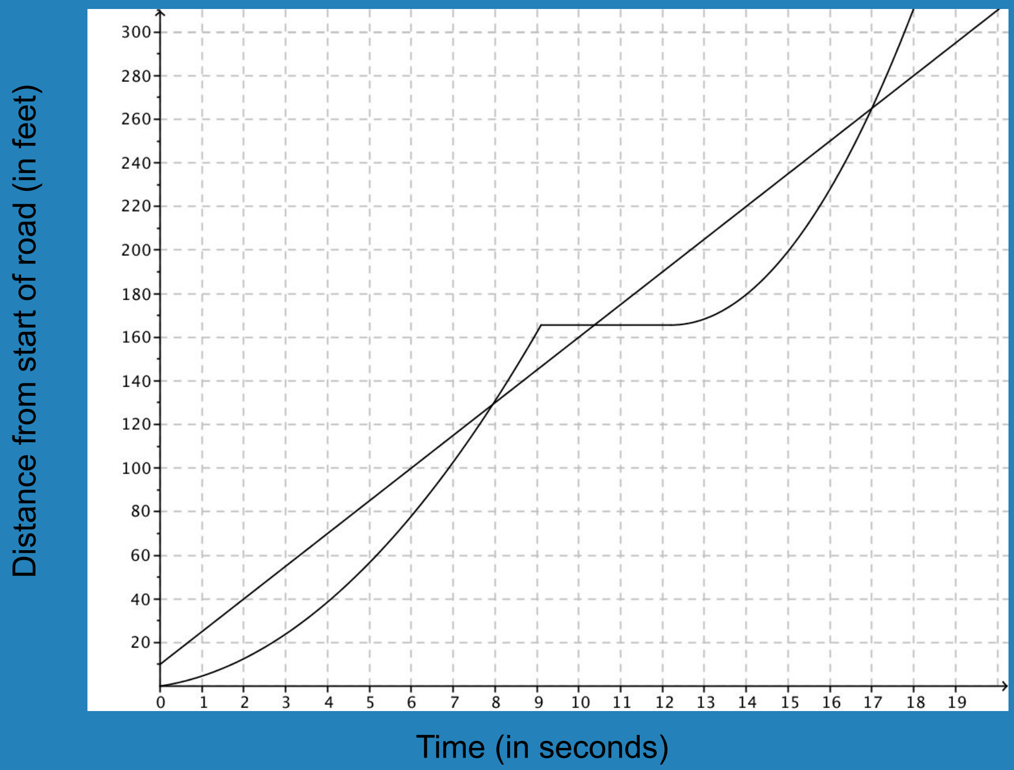
Bike and Truck Task, Part 2

Between what two seconds did the truck drive the fastest? How do you know?



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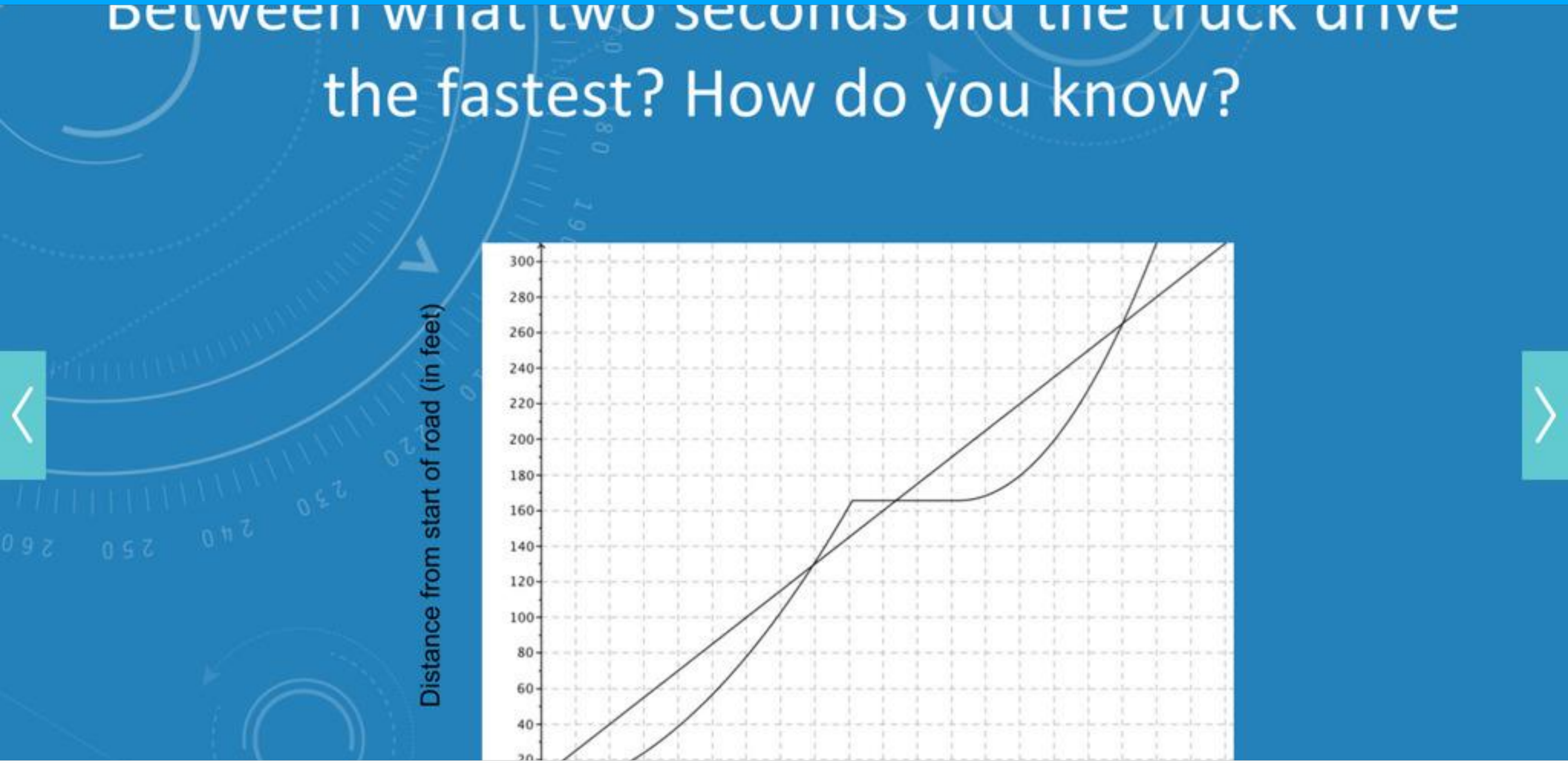
Draw It



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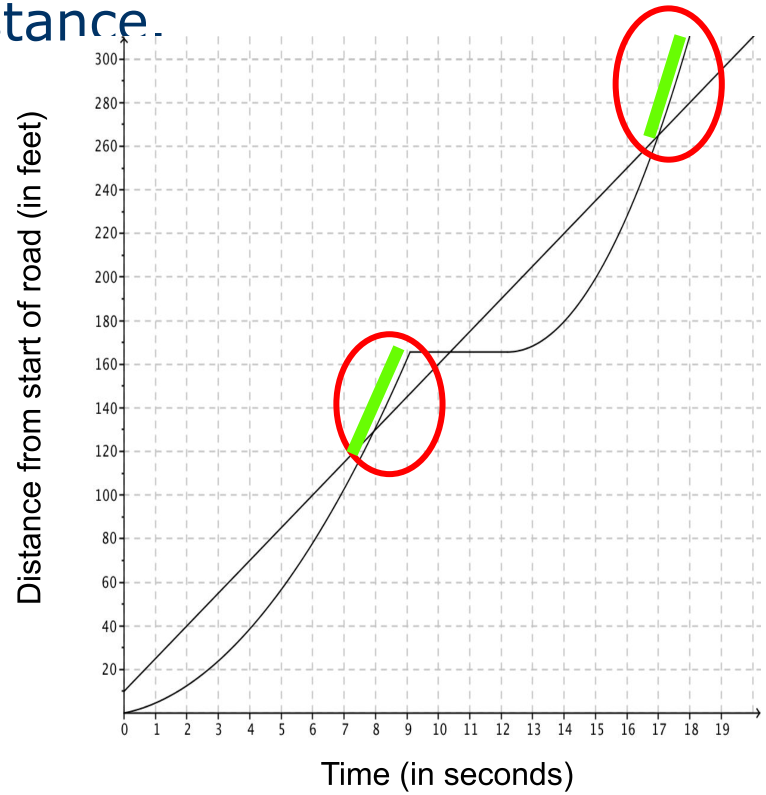

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Bike and Truck Task

Between what two seconds did the truck drive the fastest? How do you know?

- Steepness of Graph – covers more distance in a shorter amount of time.
- Use Numerical Value – change in distance compared to (or over) change in time.
- Slope Formula

Do you need to know a formula for slope or rate of change to answer this question?

How does this question help you make sense of the formula for slope or avg rate of change?

Bike and Truck Video 2

The teacher poses, “Between what two seconds did the truck drive the fastest? How do you know?”

How does this experience help students develop a conceptual understanding of rate of change?

Collaborate Board

Bike and Truck Video Clip 2

How does this experience help students develop a conceptual understanding of rate of change?

How to Edit

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From Video 2

"Between what two seconds did the truck drive the fastest? How do you know?"

"What happens after 17 seconds? Is the bike or truck traveling faster?"

How do these questions help you make sense of the procedure for finding slope or avg. rate of change?

Okay. This showing that the truck went 40 feet in only 1 second and the bike went 40 feet in 2 seconds. So the truck traveled faster.

2:12 to 2:30 on the videoclip: As the student explains, the following calculations are visible on the overhead:

$$\frac{160-120}{9-7} = \frac{40}{2} = \frac{20}{1}$$

[Truck; 7 to 9 seconds]

$$\frac{300-220}{18-16} = \frac{80}{2} = \frac{40}{1}$$

[Truck; 16 to 18 seconds]

$$\frac{300-260}{18-17} = \frac{40}{1}$$

[Truck; 17 to 18 seconds]

$$\frac{300-260}{19-17} = \frac{40}{2}$$

[Bike; 17 to 19 seconds]

Sequences of Tasks that Promote Procedural Fluency (Briars, 2016)

1. Develop conceptual understanding by building on students' informal knowledge. Provide experiences where students engage only conceptually on the basis of their prior knowledge.

Engage students in a situation (mathematical or contextual) in which the procedure *makes sense*.

Boston, Dillon, Smith, and Miller. 2017, p. 56-57.

“Pay It Forward” Task

Trevor decides to do a good deed for three people and then each of the three people would do a good deed for three more people and so on. He believed that before long there would be good things happening to billions of people.

At stage 1 of the process, Trevor completes three good deeds. How does the number of good deeds grow from stage to stage? How many good deeds would be completed at stage 5? Describe a function that would model the Pay It Forward process at *any* stage.

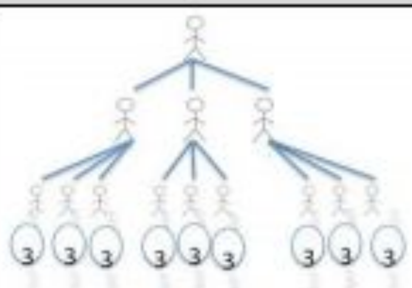
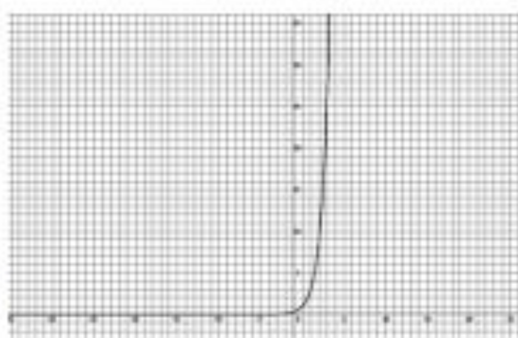
Sequences of Tasks that Promote Procedural Fluency (Briars, 2016)

2. Support students in ***developing informal strategies to solve problems***, where they use their own invented strategies and shortcuts and engage with a variety of strategies created by their peers.

Allow students to develop and share informal, invented strategies.

Boston, Dillon, Smith, and Miller. 2017, p. 56-57.

“Pay It Forward” Student Work

<table>
<tr><th>Group 1
(equation - incorrect)</th><th>Group 2
(table like Group 6 & 7 and equation)</th><th>Group 3
(diagram like Group 4 and table)</th></tr>
<tr><td>$y = 3x$
At every stage there are three times as many good deeds as there were in the previous stage.</td><td>$y = 3^x$</td><td><table>
<tr><th>x (stages)</th><th></th><th>y (deeds)</th></tr>
<tr><td>1</td><td>3</td><td>3</td></tr>
<tr><td>2</td><td>3 x 3</td><td>9</td></tr>
<tr><td>3</td><td>3 x 3 x 3</td><td>27</td></tr>
<tr><td>4</td><td>3 x 3 x 3 x 3</td><td>81</td></tr>
<tr><td>5</td><td>3 x 3 x 3 x 3 x 3</td><td>243</td></tr>
</table></td></tr>
<tr><th>Group 4
(diagram)</th><th>Group 5
(table like Group 6 & 7 and graph)</th><th>Groups 6 and 7
(table)</th></tr>
<tr><td>
So the next stage will be 3 times the number there in the current stage so 27 x 3. It is too many to draw. You keep multiplying by 3.</td><td></td><td><table>
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<tr><td>5</td><td>243</td></tr>
</table></td></tr>
</table>

Sequences of Tasks that Promote Procedural Fluency (Briars, 2016)

3. Provide opportunities for students to ***refine informal strategies to develop fluency with standard methods and procedures (algorithms or formulas)***. Engage students in considering how to make invented strategies or shortcuts more efficient, ***explicitly comparing and contrasting different methods to solve the same problem.***

Prompt students to "look for and make use of structure and to "look for and express regularity in repeated reasoning" (sound familiar?).

Boston, Dillon, Smith, and Miller. 2017, p. 56-57.

<u>Sequences</u> of Tasks that Promote Procedural Fluency

What might we expect students to be able to do when presented with a problem like "Petoskey Population", after they have had the opportunity to develop an understanding of exponential functions?

The population of Petoskey, Michigan, was 6,076 in 1990 and was growing at the rate of 3.7% per year. The city planners want to know what the population will be in the year 2025. Write and evaluate an expression to estimate this population.

Year	x = number of years after 1990	y = Population	
1990	0	6,076	× 1.037
1991	1	(6,076) × 1.037 = 6,301	× 1.037
1992	2	(6,301) × 1.037 = 6,534	× 1.037
1993	3	(6,534) × 1.037 = 6,776	× 1.037
1994	4	(6,776) × 1.037 = 7,027	
. . .		. . .	
2025	35	(6,076) × 1.037^{35}	

Fig. 4.1. Possible table for

Look for and express regularity in repeated reasoning

Look for and make use of structure

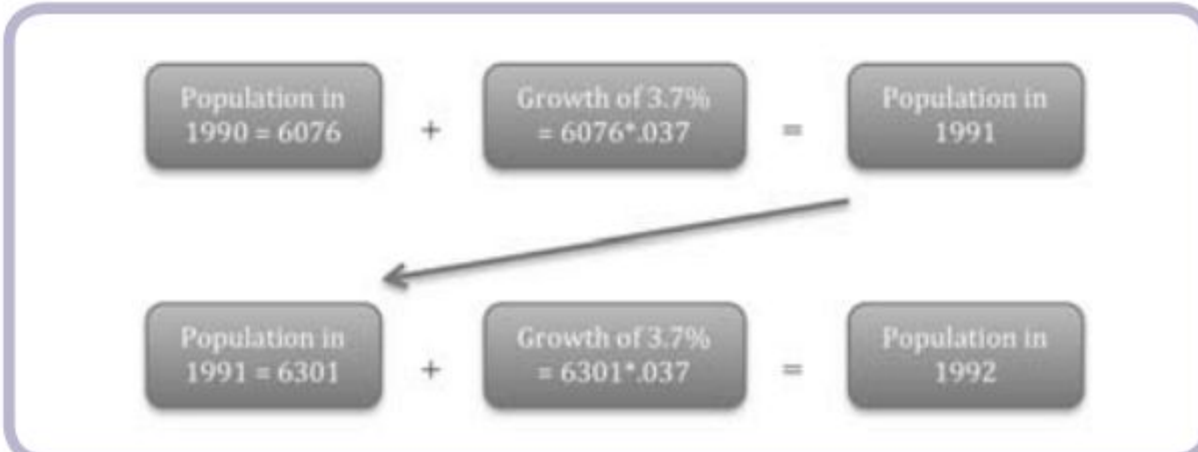


Fig. 4.3. Possible diagram for the Petoskey Population Task

1991: $(6{,}076 + 6{,}076 \times .037) = (6{,}076) \times 1.037 = 6{,}301$

1992: $(6{,}076) \times 1.037 \times 1.037 = 6{,}534$

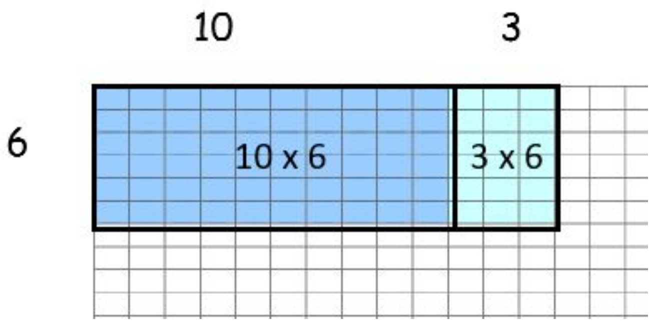
1993: $(6{,}076) \times 1.037 \times 1.037 \times 1.037 = 6{,}776$

Procedural Fluency

- Students would recognize it as an as exponential function in the form $y = a(b^x)$ even if it was in a list of different types of functions
- Students would understand that for each year an additional factor of 1.037 would be multiplied.
- Students would be able to set up the correct function because of their understanding of what exponential functions represent and how they work.
- Students would not interpret b^x as b times x

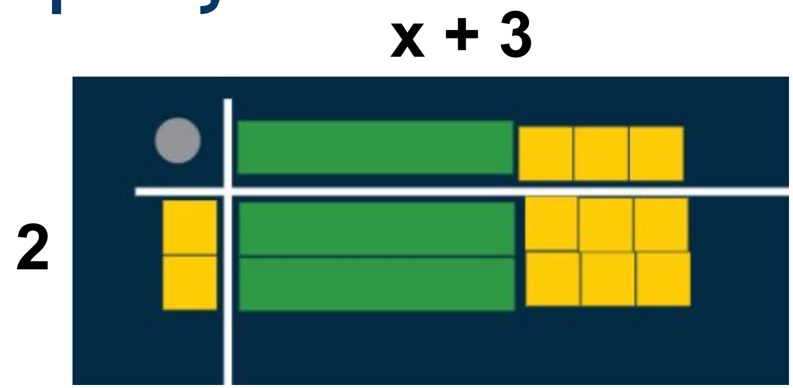
Sequence of Tasks over Time

Distributive Property

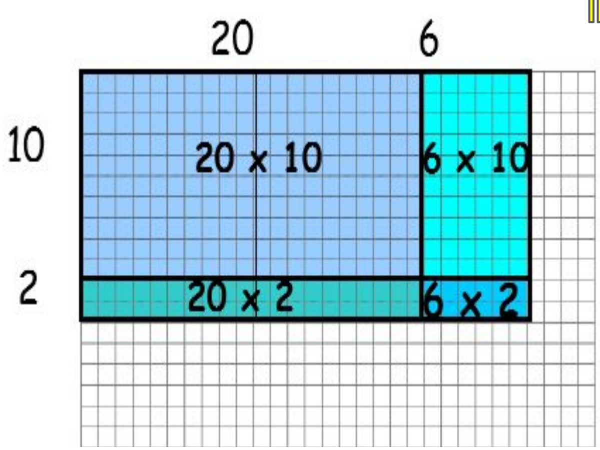


10s and 1s Partitioning

http://didax.com/apps/algebra-tiles/

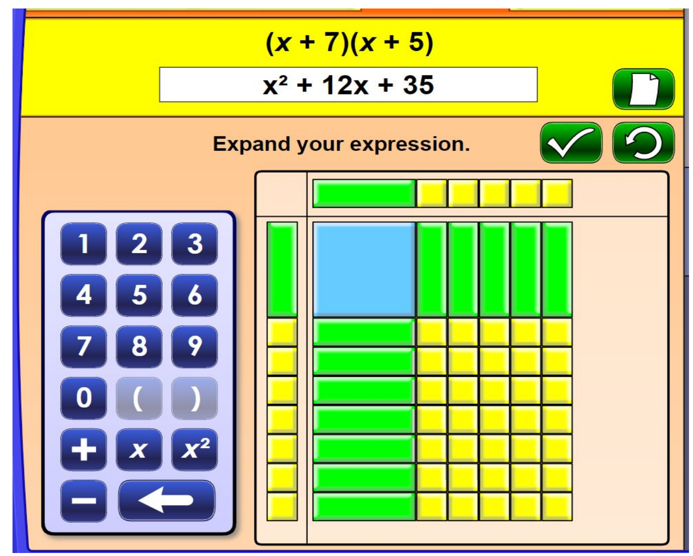


Expanding Variable Expressions



Partial Products

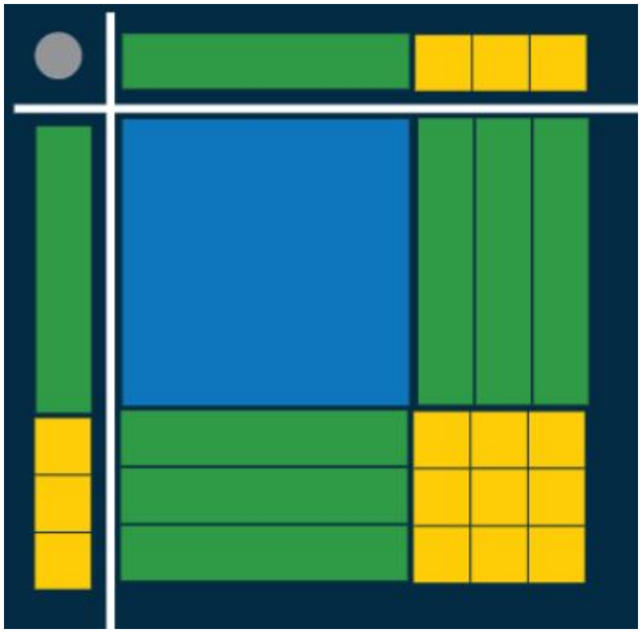
https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Algebra-Tiles/



Multiplying Binomials

Sequence of Tasks over Time

Perfect Squares

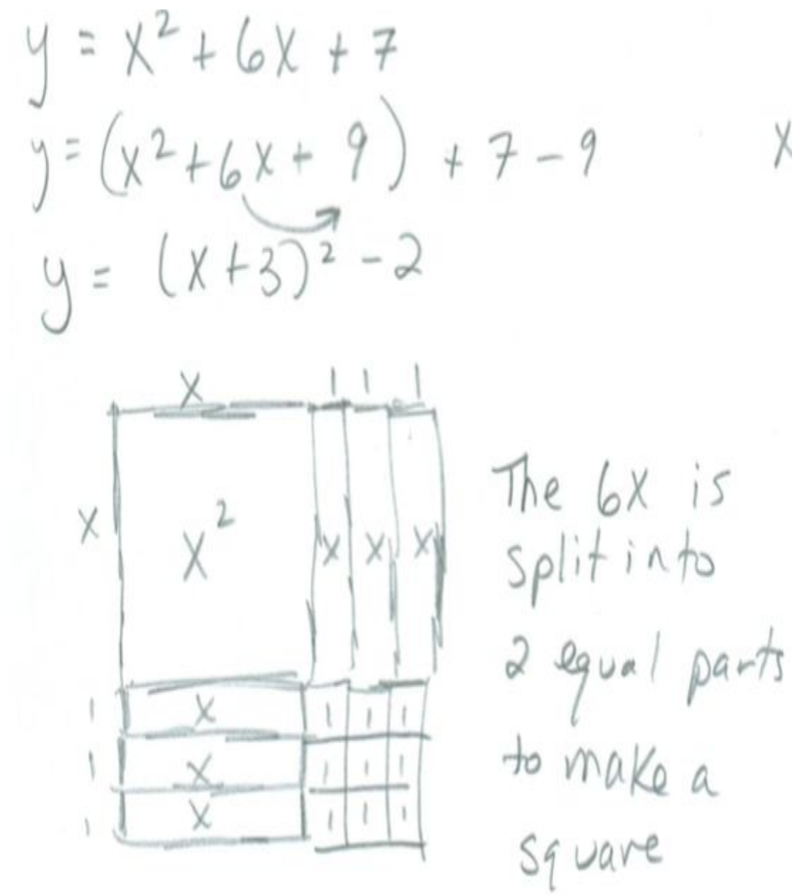


$(x + 3)^2$

Square the first term
Multiply the terms and double
Square the last term

http://didax.com/apps/algebra-tiles/

Completing the Square



Build procedural fluency from conceptual understanding Teacher and student actions	
What are *teachers* doing?	What are *students* doing?
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate. Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems. Striving to use procedures appropriately and efficiently.

How do *students* feel about instruction reflective of the Effective Teaching Practices??

Researchers' Ratings

High Academic Rigor:

- Tasks & Implementation
- Questioning
- Students' Responses
- Mathematical Residue

Rich Discussions:

- Participation > 50%
- Teacher Press
- Student Providing

Highly correlated with

Students' Ratings

High ratings on all "7 Cs" from Tripod:

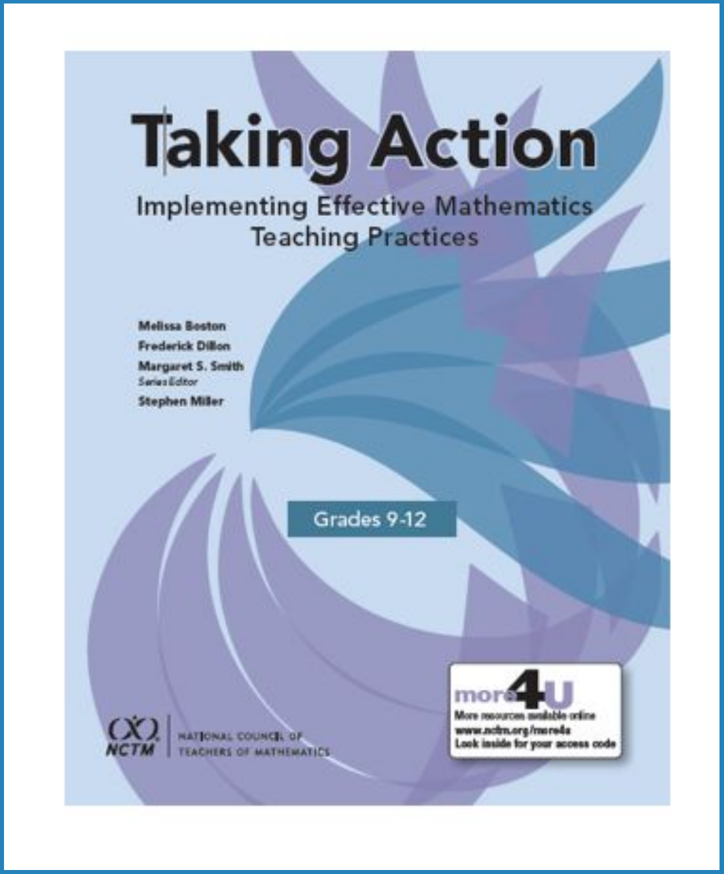
- Caring
- Conferring
- Captivating
- Clarifying
- Consolidating
- Challenging
- Classroom Management

Boston & Parke, 2019

More about *Taking Action*

Each Practice has a case study and includes a link to videos and/or samples of student work.

Each Chapter ends with suggestions for "taking action" in the classroom.



Thank You!

Melissa Boston
bostonm@duq.edu

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