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Focus on Middle School:

Encouraging Discourse and Discussion

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Which One Doesn't Belong?

Type your answer and justification into the chat!

$\frac{1}{2}$	$\frac{5}{3}$
$\frac{2}{10}$	$\frac{2}{5}$

NUMBER 7

from Erick Lee

Source: www.wodb.ca

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Pear Deck: Interactive Slide

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Poll

What is Your Roll?

A. Teacher

B. Administrator

C. Math Coach

D. Other

Poll

How is your school structuring learning this year?

A. Hybrid

B. All Virtual

C. Face to Face

This Session

- In our session, we will look at some sample middle school problems and student work.
- Engage in this as both a teacher and a student!
  - When looking at student work, consider how you would use questioning to further their learning.
  - When looking at the sample problems, think about how your students would respond to the tasks.
- We will look at:
  - The CCSS Standards for Mathematical Practice
  - 8 Effective Mathematics Teaching Practices
- Opportunities for feedback

Student Feedback

- "When feedback is delivered such that it is timely, specific, understandable, actionable, students assimilate the language used by the teacher into their own self-talk" –Hattie, Fisher, and Frey, 2017
- Timely feedback is vital for facilitating discourse, supporting productive struggle, eliciting evidence of student thinking, and posing purposeful questions.

- How can we give this timely feedback in the virtual setting?

Type some ideas into the chat!

Virtual Station Work

- Divide the class into small groups (perhaps in a pre-scheduled breakout)
- Students complete a rotation:
  1. Review:
     - Practice Problems from previous learning
     - Use small (breakout) group and worked answer key to compare solutions to an assignment.
  2. Teacher works with a small group to hear and respond to a group solving a problem collaboratively
  3. Students work on new learning—this is a good opportunity for students to begin a low-floor high-ceiling task on their own, or a flipped lesson.

Accountability

- How can we ensure the students are working when we send them off to a breakout?

Share some ideas in the chat!

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True for You or Not Yet?

A. I am familiar with 8 Effective Teaching Practices.

B. I plan lesson activities to develop reasoning and problem solving in my students.

C. My *2020* classroom is structured in a way that promotes student discussions, including justifying and generalizing.

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8 Effective Mathematics Teaching Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking.

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics.
5. Use appropriate tools strategically
6. Attend to precision (including communication)
7. Look for and use structure
8. Look for and express regularity in repeated reasoning

Learning Goal (Activity 1)

- Students will understand the meaning of rational numbers. Students will be able to compare and order rational numbers in different forms, as well as understand how benchmark numbers can help when making comparisons.

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Desmos Card Sort

STUDENT SCREEN PREVIEW

1 of 2 Next

Sort the rational numbers into groups

$\frac{6}{11}$

45%

$\frac{2}{5}$

0.49

92%

0.64

108%

10^{-2}

$\frac{1}{12}$

51.9%

$\frac{13}{26}$

10%

0.99

$\frac{7}{8}$

0.3%

0.082

$\frac{44}{45}$

Student Sample #1

Sort the rational numbers into groups

6 CARDS
$\frac{13}{26}$
$\frac{6}{11}$
$\frac{1}{12}$
$\frac{2}{5}$
$\frac{7}{8}$
$\frac{44}{45}$

5 CARDS
10^{-2}
0.49
0.99
0.64
0.082

6 CARDS
10%
108%
92%
51.9%
0.3%
45%

Student Sample #2

Sort the rational numbers into groups

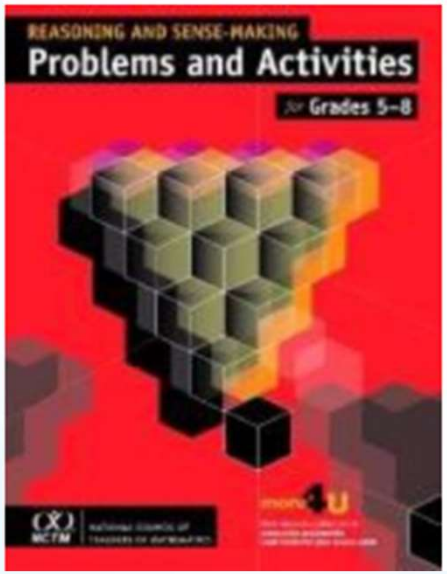
7 CARDS
0.64
51.9%
$\frac{6}{11}$
45%
$\frac{2}{5}$
0.49
$\frac{13}{26}$

5 CARDS
$\frac{1}{12}$
0.082
0.3%
10%
10^{-2}

5 CARDS
0.99
92%
108%
$\frac{44}{45}$
$\frac{7}{8}$

Operations with Decimals

- SMP Focus:
  - *Reason abstractly and quantitatively*
  - *Use appropriate tools strategically*



Source: Reasoning and Sense-Making Problems and Activities for Grades 5-8, 2011

Learning Goal (Activity #2)

Students will understand multiplication and the meanings of decimal numbers themselves, applying computational estimation accurately. Students will be able to apply this estimation in checking work and in real-world problems.

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Operations with Decimals

	A: Multiply by		B: Multiply by		C: Multiply by	
Pick a Whole Number	0.05	0.1	0.48	0.51	0.98	1.07
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	0

The multipliers in A are closest to:

The multipliers in B are closest to:

The multipliers in C are closest to:

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Getting Students to Work!

- Students were given a link to a programmed GoogleSheet and asked to “play” with some numbers. They entered whole numbers into the yellow boxes and made observations about the products and the multipliers (0.05, 0.1, 0.48, 0.51, 0.98, 1.07).

Multiplication By Decimals

File Edit View Insert Format Data Tools Add-ons Help

	A	B	C	D	E	F	G
1							
2		A: Multiply by		B: Multiply by		C: Multiply by	
3	Pick a Whole Number	0.05	0.1	0.48	0.51	0.98	1.07
4	5	0.25	0.5	2.4	2.55	4.9	5.35
5	10	0.5	1	4.8	5.1	9.8	10.7
6	4	0.2	0.4	1.92	2.04	3.92	4.28
7	8	0.4	0.8	3.84	4.08	7.84	8.56
8							
9	The multipliers in A are closest to:						
10	The multipliers in B are closest to:						
11	The multipliers in C are closest to:						

Questions

- In general, what happens when you multiply a whole number by a decimal close to 0?
- In general, what happens when you multiply a whole number by a decimal close to ½?
- In general, what happens when you multiply a whole number by a decimal close to 1?

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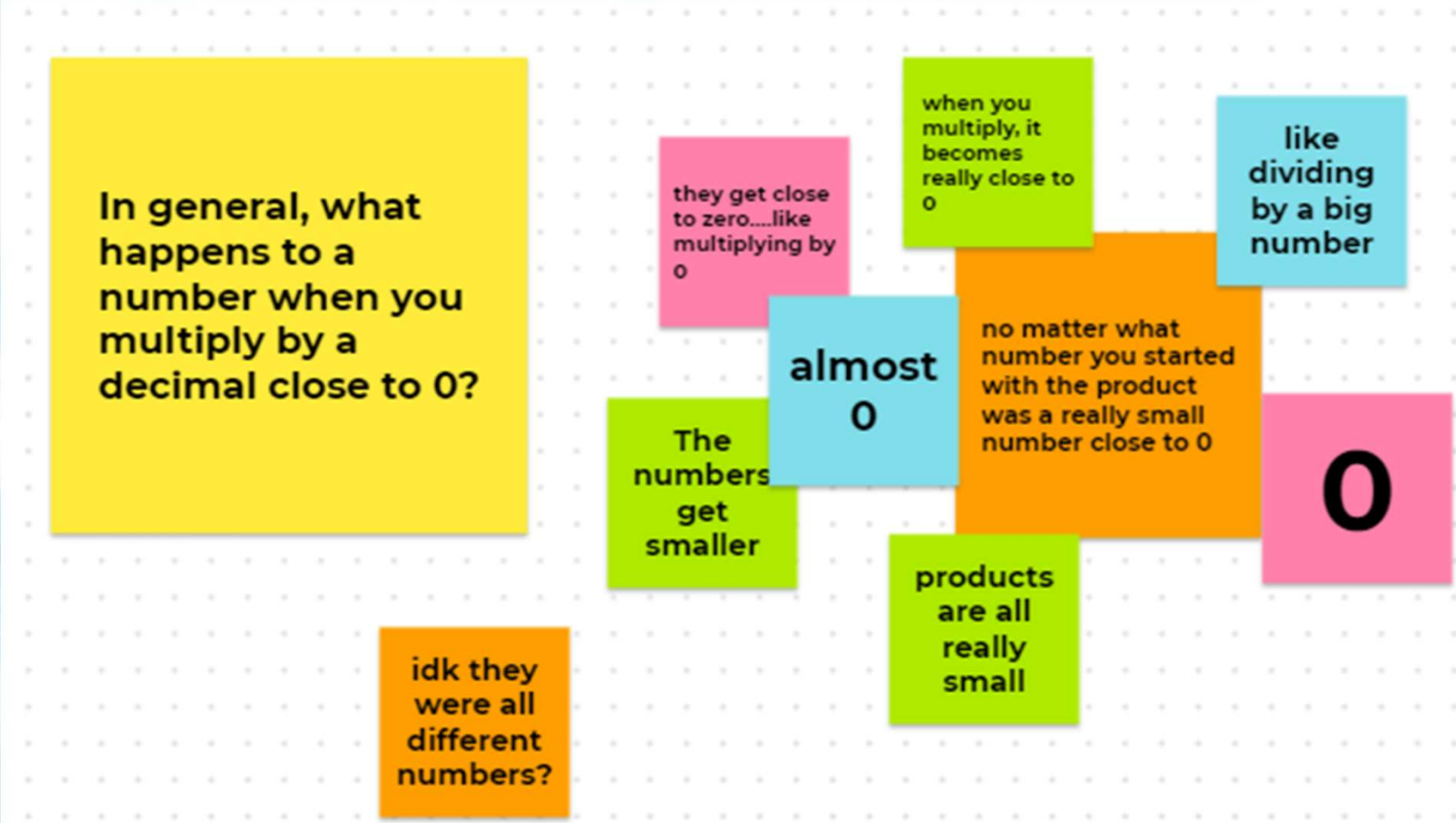
Sample Jamboard Question 1:



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Sample Jamboard Question 2:

In general, what happens to a number when you multiply by a decimal close to 1/2?

just divide the number in 2

its about 1/2

If you have 20 for example, and you multiply it by 0.49 the answer will be like 10....but less than 10.

divided by 2

the answer is more or less than half of the original number. Depends on if the original number was more or less than 1/2.

You multiply the first number to the decimal close to a half. The answer is going to be about half of the first number.

number divided by 2

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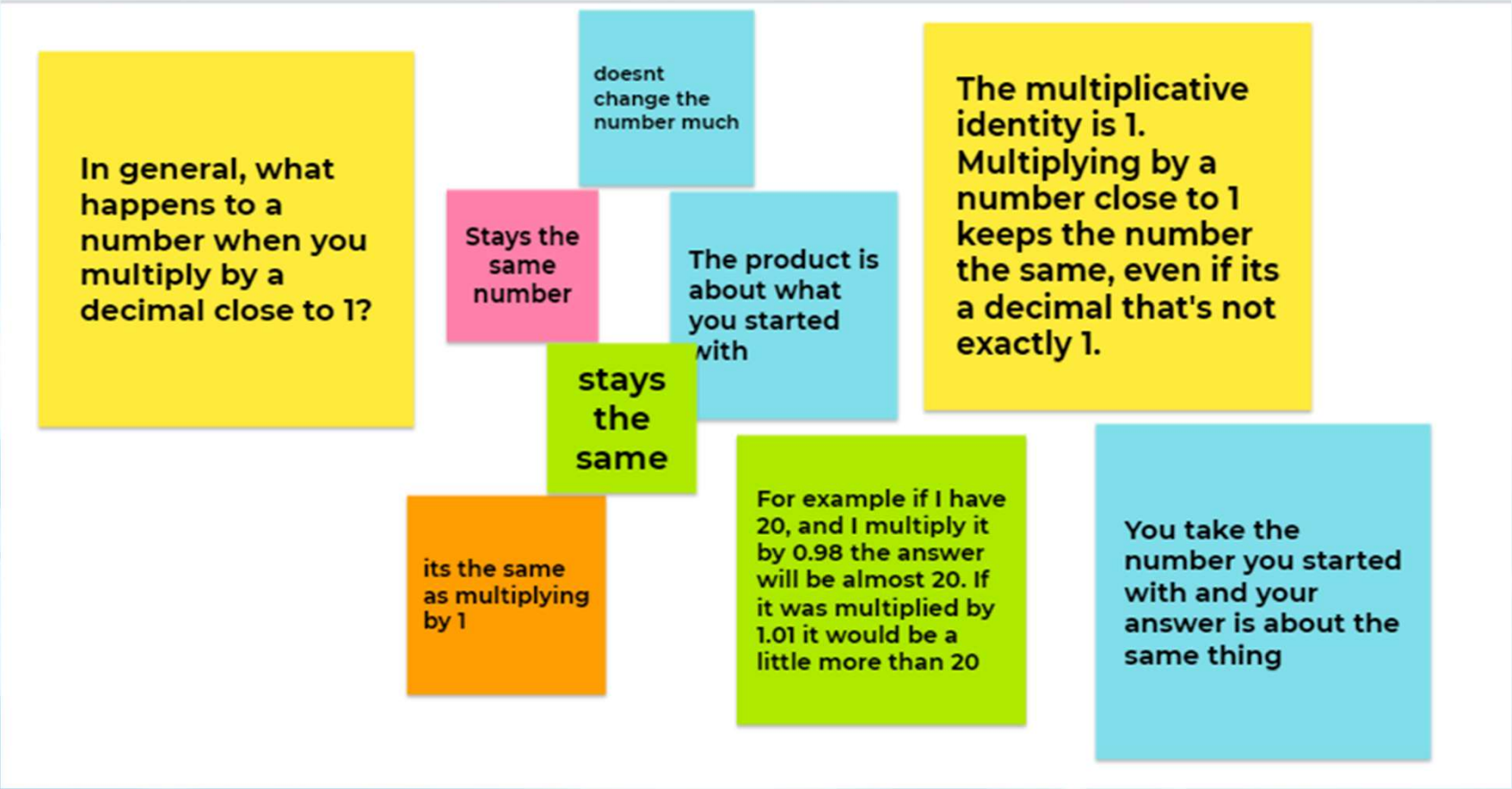
Sample Jamboard Question 3:



Moving the learning forward

- Using what you have learned about multiplying by decimals close to 0, ½, and 1, estimate the product for each of the following, be prepared to explain your thinking:
- 19.76 x 0.47 is about ____ because…
- 1.09 x 27.8 is about _____ because…
- 0.55 x 101.67 is about _____ because…
- 39.8 x 0.012 is about _____ because…

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Activity 3

- SMP Focus:
  - *Look for and use structure*
  - *Look for and express regularity in repeated reasoning*
- Problem was solved in a small group breakout

Mathematics Assessment Project
ASSESSING 21ST CENTURY MATH
Welcome to the Mathematics Assessment Project

Source: Mathematics Assessment Project:

https://www.map.mathshell.org/

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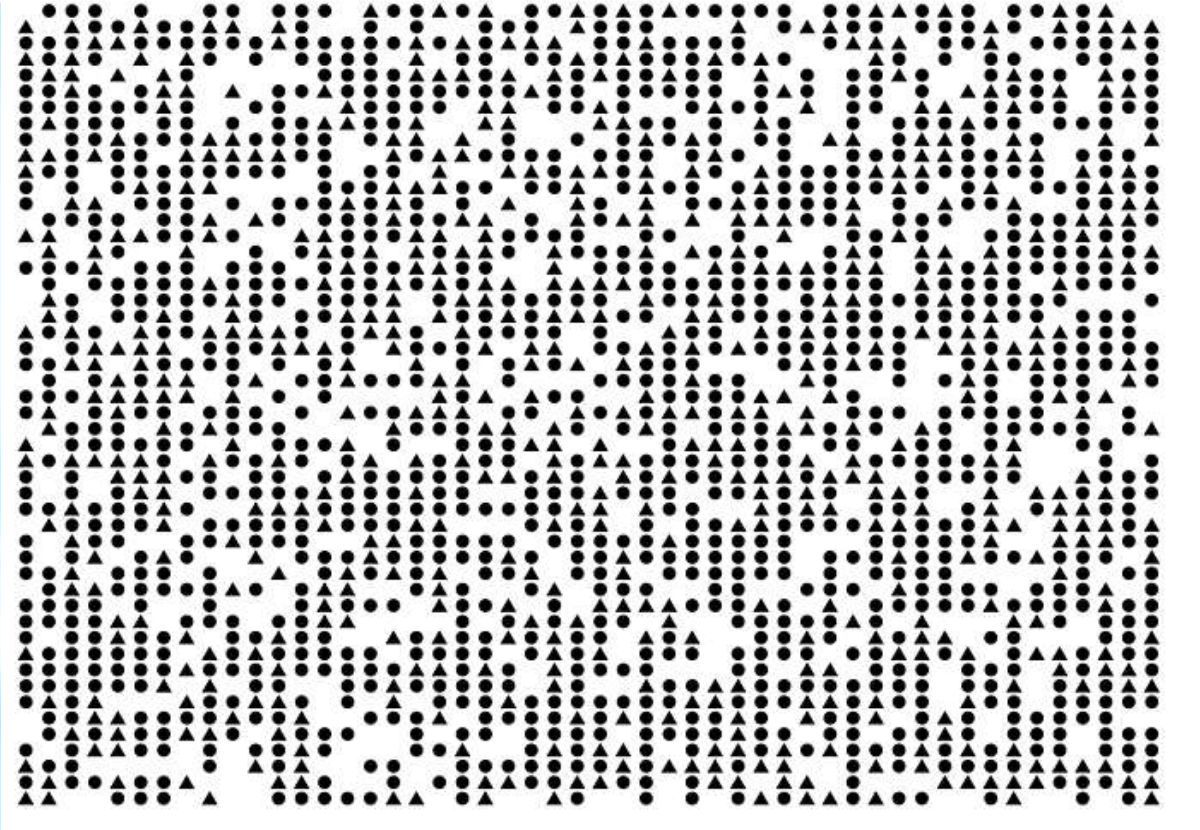
COUNTING TREES



This is a map of a tree farm. Circles represent old trees and triangles represent young trees.

What questions come to mind?

Source:
https://www.map.mathshell.org/tasks.php?unit=ME02&collection=9

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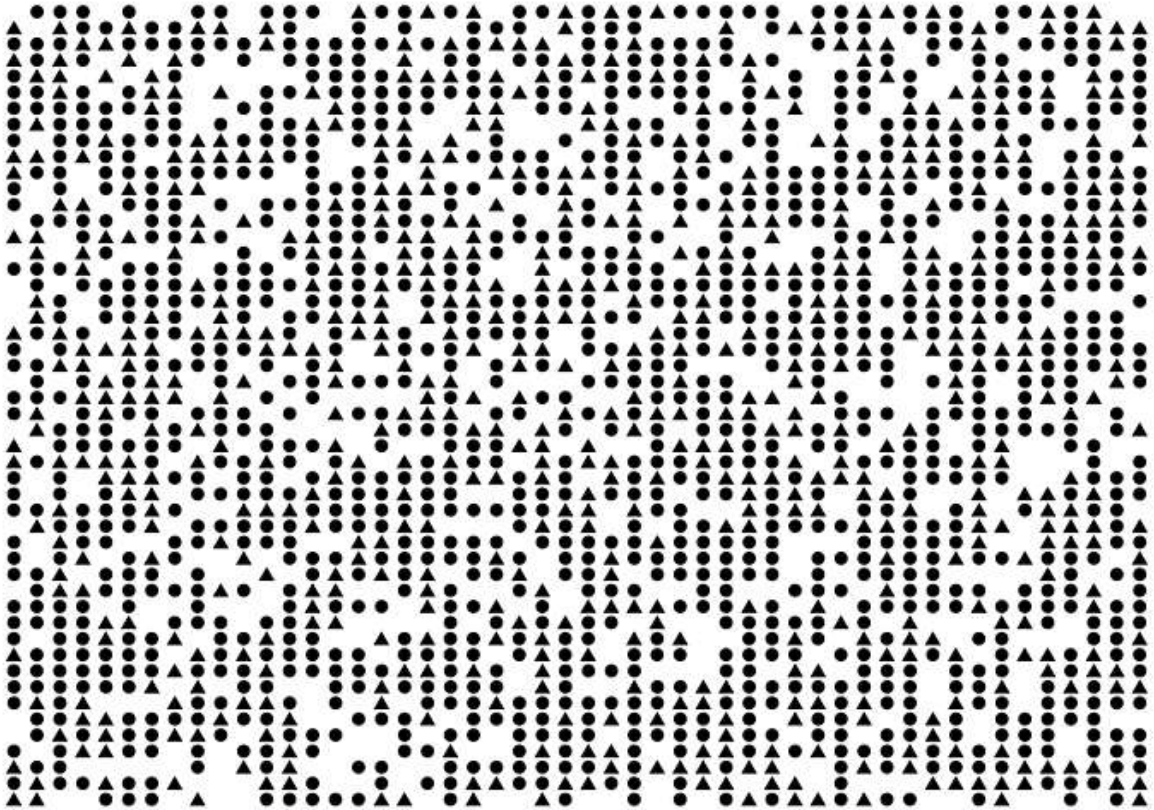
COUNTING TREES



This diagram shows some trees in a tree farm.

The circles ● show old trees and the triangles ▲ show young trees.

Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

Source:
https://www.map.mathshell.org/tasks.php?unit=ME02&collection=9

Learning goal (Activity 3)

Students will understand when and how to apply sampling to draw inferences about a set, analyzing proportional relationships and using them to solve real-world mathematical problems.

How students got to work:

- This was done during a virtual station work day.
- Students talked together to decide on a strategy for counting the trees (worked in pairs).
- Some printed the handout, then took a picture, while others looked at it online in a shared screen.

Group Sample Work #1

There are 11 triangles in first column
There are 27 circles in the first column.
There are 50 columns total.
11*50=550 young trees
27*50=1,350 old trees

There are 11 triangles and 29 circles in the first row. 50 rows total.
11*50=550 young trees
29*50=1,450 old trees.

We think there are 550 young trees and there are between 1,350 and 1,450 old trees.

- Why did you count both rows and columns?
- Why was the number of total columns and rows important? How did you find this number?
- How can you narrow down the number of old trees?
- How do you know your answer is accurate?

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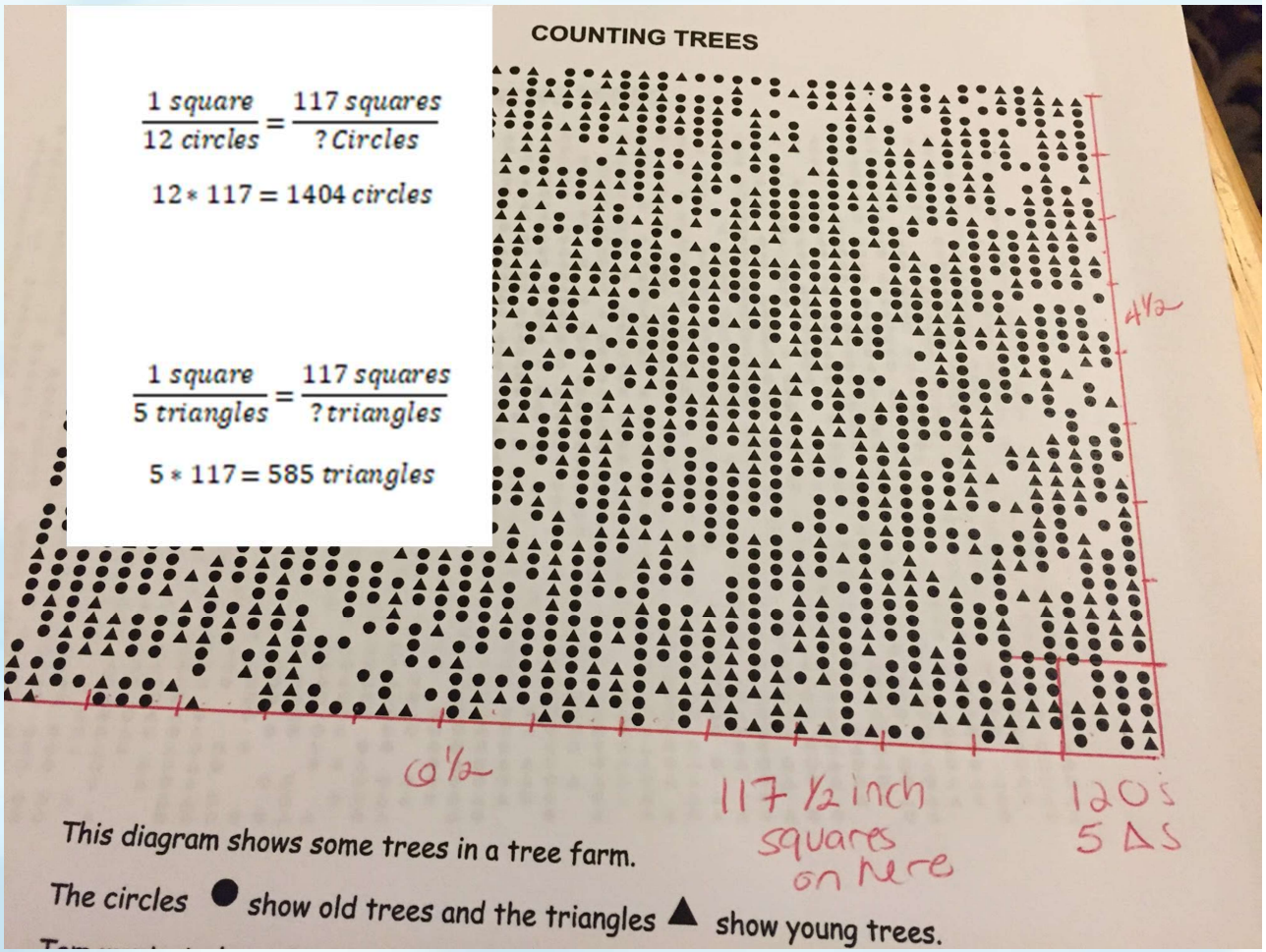
Group Sample Work #2

- Why did you choose a half inch square?
- Could drawing the lines or measuring sloppily affect your method?
- How did you find out there would be 117 of them?
- What could make your estimate more accurate?



Closure for Counting Trees

- Students returned to whole group, teacher screen-shares work as each group describes their work to the others.
- Students in other groups used a rubric to assess the solutions, and noted how similar their final answers turned out!

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Activity 4

- SMP Focus:
  - *Make sense of problems and persevere in solving*
  - *Construct viable arguments and critique the reasoning of others*

Mathematics Assessment Project
ASSESSING 21ST CENTURY MATH
Welcome to the Mathematics Assessment Project

Source: Mathematics Assessment Project:

https://www.map.mathshell.org/

A Day Out

Mr Richards, a teacher from Bosworth School, plans to take 30 students on a school trip. Here are the places they could visit.

Growlets Zoo	Prison Museum	Space Science Show
		
36 miles from Bosworth Entrance fee $2.50 per person	30 miles from Bosworth Entrance fee $6 per person	10 miles from Bosworth Entrance fee $10 per person

The class vote on which place to visit.
Here are the results:

Name	First Choice	Second choice
Olivia	Zoo	Space show
Grace	Space show	Prison museum
Jessica	Prison museum	Zoo
Ruby	Zoo	Space show
Emily	Space show	Prison museum
Sophie	Prison museum	Zoo
Chloe	Prison museum	Space show
Lucy	Prison museum	Space show
Lily	Space show	Prison museum
Ellie	Space show	Prison museum
Ella	Zoo	Space show
Charlotte	Space show	Prison museum
Katie	Space show	Prison museum
Mia	Zoo	Space show
Hannah	Zoo	Space show

Name	First Choice	Second choice
Jack	Prison museum	Zoo
Thomas	Zoo	Prison museum
Joshua	Zoo	Prison museum
Oliver	Space show	Prison museum
Harry	Prison museum	Zoo
James	Zoo	Space show
William	Space show	Space show
Samuel	Zoo	Prison museum
Daniel	Zoo	Space show
Charlie	Prison museum	Prison museum
Benjamin	Space show	Zoo
Joseph	Zoo	Prison museum
Callum	Zoo	Prison museum
George	Prison museum	Space show
Jake	Space show	Prison museum

- What questions come to mind?

Source: https://www.map.mathshell.org/download.php?fileid=1151

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Lily	Space show	Prison museum
Ellie	Space show	Prison museum
Ella	Zoo	Space show
Charlotte	Space show	Prison museum
Katie	Space show	Prison museum
Mia	Zoo	Space show
Hannah	Zoo	Space show

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Oliver	Space show	Prison museum
Harry	Prison museum	Zoo
James	Zoo	Space show
William	Space show	Space show
Samuel	Zoo	Prison museum
Daniel	Zoo	Space show
Charlie	Prison museum	Prison museum
Benjamin	Space show	Zoo
Joseph	Zoo	Prison museum
Callum	Zoo	Prison museum
George	Prison museum	Space show
Jake	Space show	Prison museum

Here are some further facts about the trip.

The bus company charge $6 per mile. The school fund will pay the first $200 of the trip. Teachers will go free. Each student will pay the same amount.

Using Sample Work

- After giving students some time to reflect and think about a plan for making the choice, consider providing some sample work for them to critique.
- Ask students, “Which student’s method is the **most fair** way to determine where the class will go?”
  - Was done in breakout rooms with 3-4 people
  - First, all students look through all the samples
  - Next, I asked each student to choose a sample and make sense of how it was solved—including double checking the sample math. Then, explain the method to the group

Mars Sample Student Response: Chloe

first choice	Second choice
Zoo = $\frac{12}{30}$	Zoo = $\frac{\quad}{30}$
Space Show = $\frac{10}{30}$	Space Show = $\frac{\quad}{30}$
Prison museum = $\frac{8}{30}$	Prison museum = $\frac{\quad}{30}$

Zoo most popular

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Mars Sample Student Response: Olly

First choice 2 points
Second choice 1 point
} Tally

zoo	𝍸 𝍸 𝍸 𝍸 𝍸 IIII	29
Prison museum	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	30
Space Show	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I	31

1) I made a tally and for there first choice they got 2 point and there second choice the got 1 point and the result I got is the space show was most popular.

2) 30 people - £10 entree per person

30 × 10 = 300 -
200
£100

The school fund pays for the first 200.

Teachers go free!

10 miles
× £16 pound per mile
£160

£160

Pupils pay - 5.33^ each

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Mars Sample Student Response: Robyn

1) 200 - 12 - 5
Space Show - 10 - 11
Prison museum - 8 - 14

30 pupils
6 x 36 - 200
6 x 30 - ~~Space Show~~ Prison-M.
6 x 10 - Space Show.

6 x 36 = 216
6 x 30 = 180
6 x 10 = 60

So I think that they should go to the Space Science show because it is cheaper to go there 21 people said either in the 1 or second choice, therefore it is a good decision, however it is £10 to get in the show which is more expensive ~~that~~ than the others.

2, each person needs £10 to get in. £200 pounds is paid by the school but it is £10 to get in for each person & £60 to get there so its. 30 x 10 = 300
+ 60 = 360
So children have to pay the extra £160
£160 ÷ 30 (children) = 5.333

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Mars Sample Student Response: Emily

ZOO

Entrance = £2.50 x 30 = £75
miles = 36 miles x £6 = £216

£218.50 each

Prision Museum

Entrance = £6 x 30 = £180
miles = 30 miles x £6 = £180

£186 each

Space Science Show

Entrance = £10 x 30 = £300
miles = 10 miles x £6 = £ 60

£70 each

1. I think they should all go to the Space science museum as it would be less pay.

2. Each Person would have to pay £70 each to go to the space science show.

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Analyze in Small Groups

We think Robyn made the most fair choice since she was thinking of how much each place cost. She multiplied the admission price by how many kids were going. Then she found the bus price. She subtracted the money the school would pay and then divided.

Olly used the votes to get his answer. He used the first and second choices and gave them point values. Even though most people's first choice was the zoo, when he looked at the tallys it was really the Space Show that won. He got the same price as Robyn did too.

Emily, Robyn, and Olly all chose Space Science Show. We think this is the most fair choice.

The zoo got the most first votes. But Olly was the only person to look at the second choices too. We think the Zoo would have been the most fair but don't think Chloe knew what she was doing.

Robyn and Olly did the money work the same. They both got Space Science. Robyn didn't do anything with the votes but she wrote that the cheapest was the Space Science.

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Thank you!

- Thank you for your time.
- jennifer.w.perego@gmail.com