NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

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Rich Tasks Require Rich Implementation For Rich Learning to Result

Rich Tasks { Deserve / Depend on / Demand

Rich Implementation

For Rich Learning

To Result

Black Lives Matter

"The pedagogy typically faced by African American students is very directive, controlling and debilitating." Strutchens

"… reduction of the relational work of teaching to carceral pedagogies preoccupied with the control of children's bodies over the development of their intellect and agency." Horn

"An autocratic 'pedagogy of poverty' forces compliance without involving students in learning and self-discipline." Haberman

For over 25 years, research has documented the critical role of rich tasks in generating high-quality opportunities for students to learn math, develop conceptual understanding & procedural fluency, and build efficacy & agency.

QUASAR

Memorization

- Recall previously learned facts /rules
- Non–ambiguous

Procedures w /Connections

- Focus on math concepts /ideas
- Usually represented in multiple ways

Procedures w /o Connections

- Algorithmic
- Procedure defined by prior instruction

Doing Mathematics

- Require complex nonalgorithmic thinking
- Require exploration & self–monitoring

Complex Instruction

Groupworthy Tasks

- Tasks are designed specifically to maximize engagement of all group members

Positive interdependence

- Tasks require a range of perspectives and contributions to be "solved"

Low threshold /high ceiling

- Everyone is able to begin but no one should "max out" on their mathematical sense-making

Array of mathematical smartnesses

- Not just one way to be mathematically competent

NSF Funded Curricula

Connected Mathematics Project (CMP)

- Simultaneous development of conceptual & procedural knowledge
- Multiple access points

College Prep Mathematics (CPM)

- Spiral through material adding complexity on successive cycles
- Build group roles into task structure

Interactive Mathematics Program (IMP)

- Emphasis on communicating mathematical thinking
- Large unit tasks motivate student learning

Core–PLUS Mathematics

- Embed statistical reasoning into all units
- Emphasis on mathematical modeling

BUT WAIT …

Culture eats strategy

Expectation for student interactions

Ask;

Don't

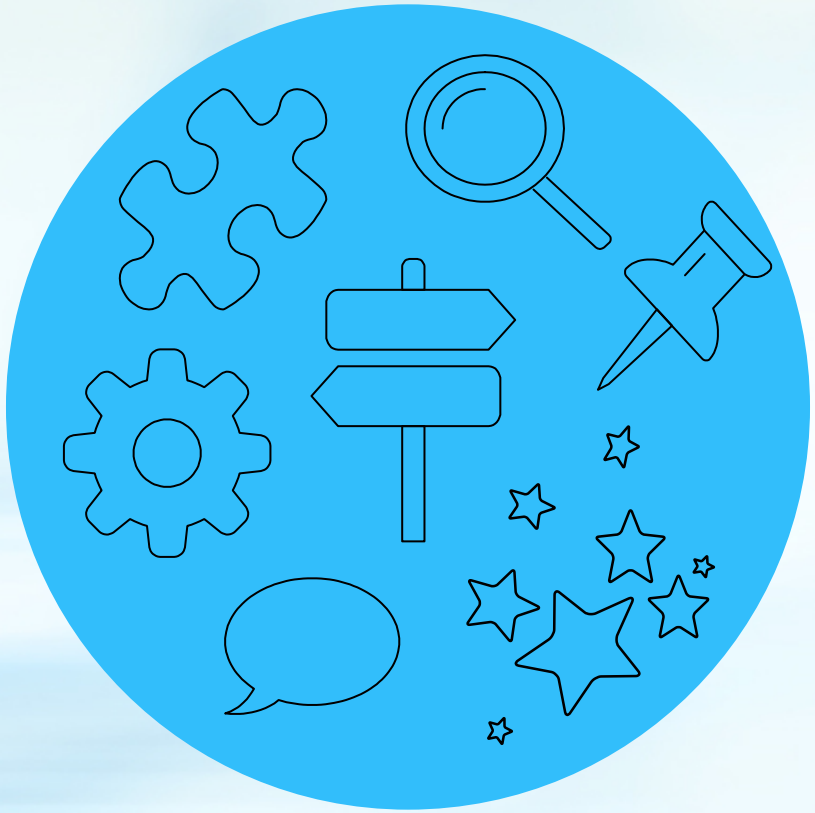
Tell!

Expectation for student work

Convince yourself

Convince a friend

Convince a skeptic

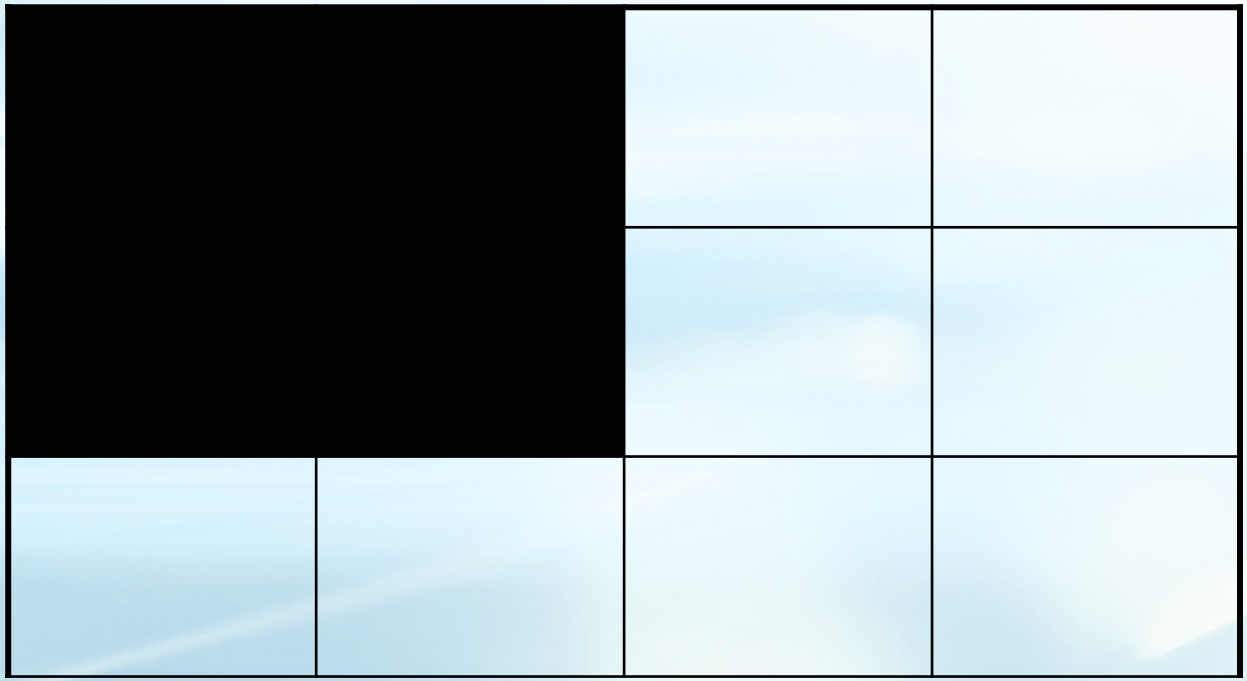


What makes a task rich?

Richness lies in the potential for each and every student to engage in meaningful sense-making about the mathematical content, to make that sense-making visible for their own reflection, to communicate it to others, and develop mathematical curiosity

A tale of two tasks …

What fraction of the overall rectangle is shaded?



a.) 1/6

b.) 1/5

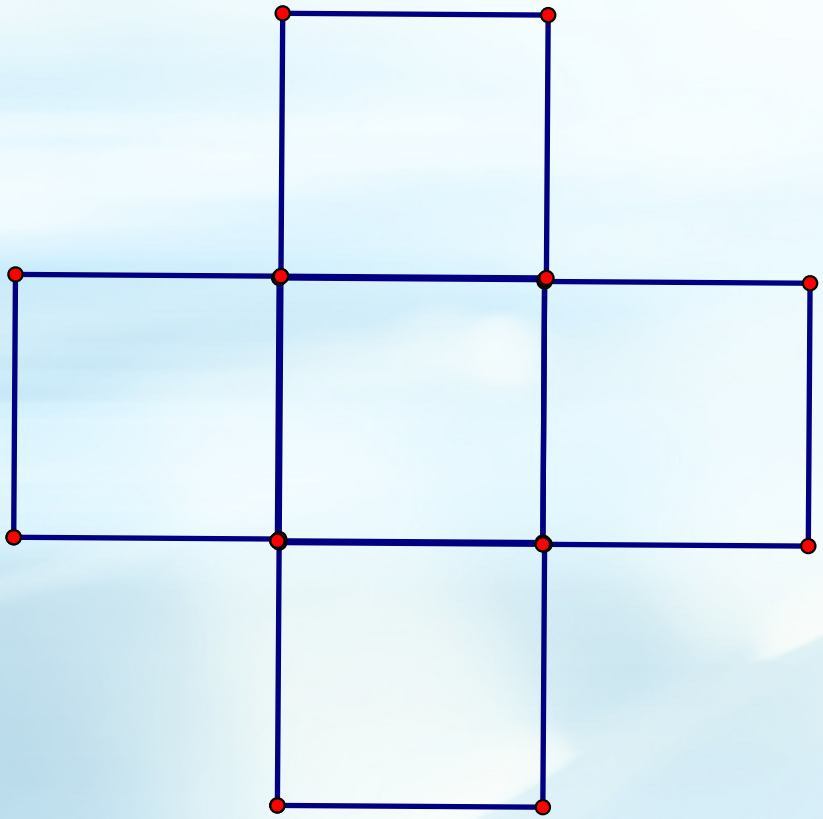
c.) 1/4

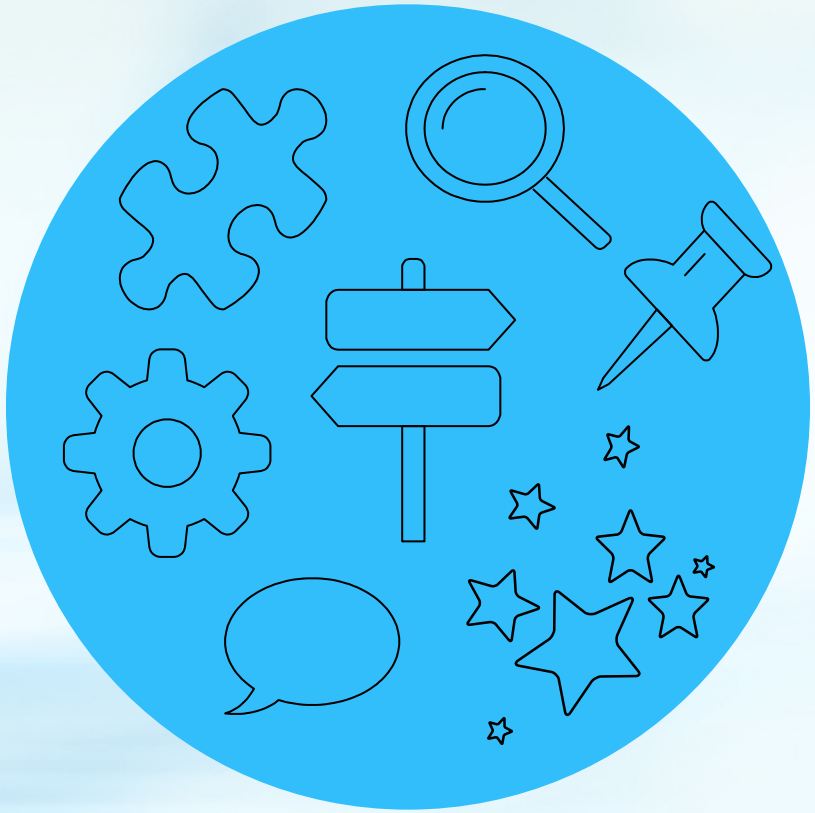
d.) 1/3

e.) 1/2

A tale of two tasks …

Shade ¼ of the area of the overall figure.





What makes implementation rich?

Richness lies in preserving & expanding the potential for each & every student to grapple with & make sense of meaningful math, in leveraging the artifacts of that sense-making, and in validating & expanding their math agency & identity

A tale of two directions …

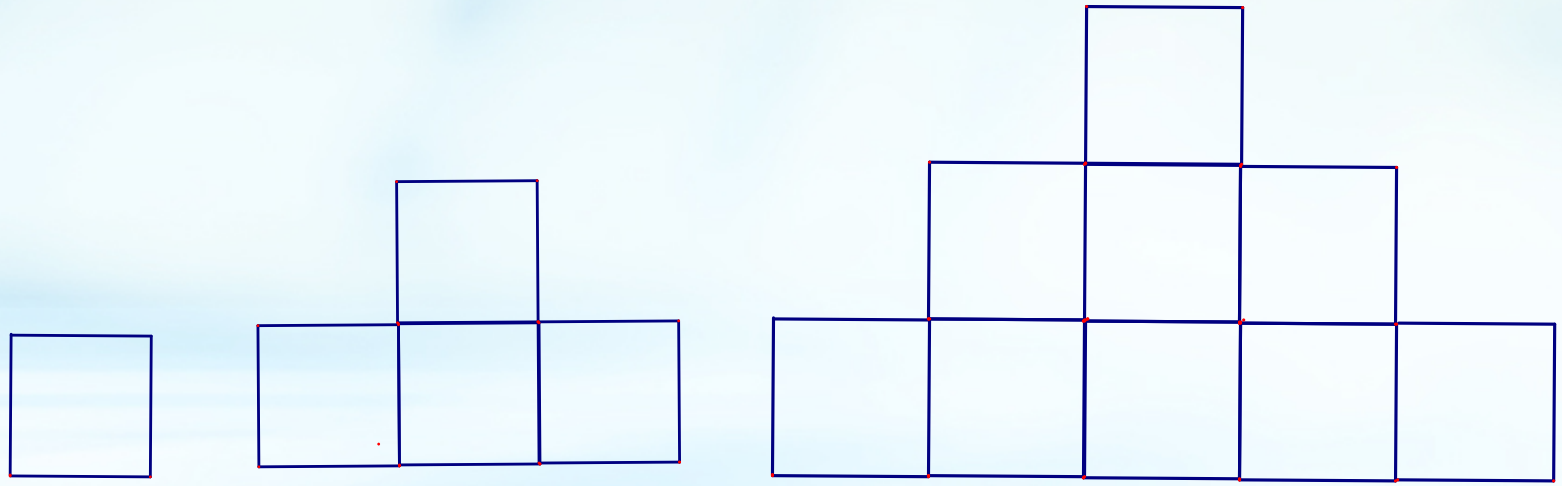


figure 1 figure 2 figure 3

Make a table of values that records the number of unit squares in each figure.
How many unit squares will be in the 10th figure? How many in the 100th figure?
Use your table to determine an equation.

A tale of two directions …

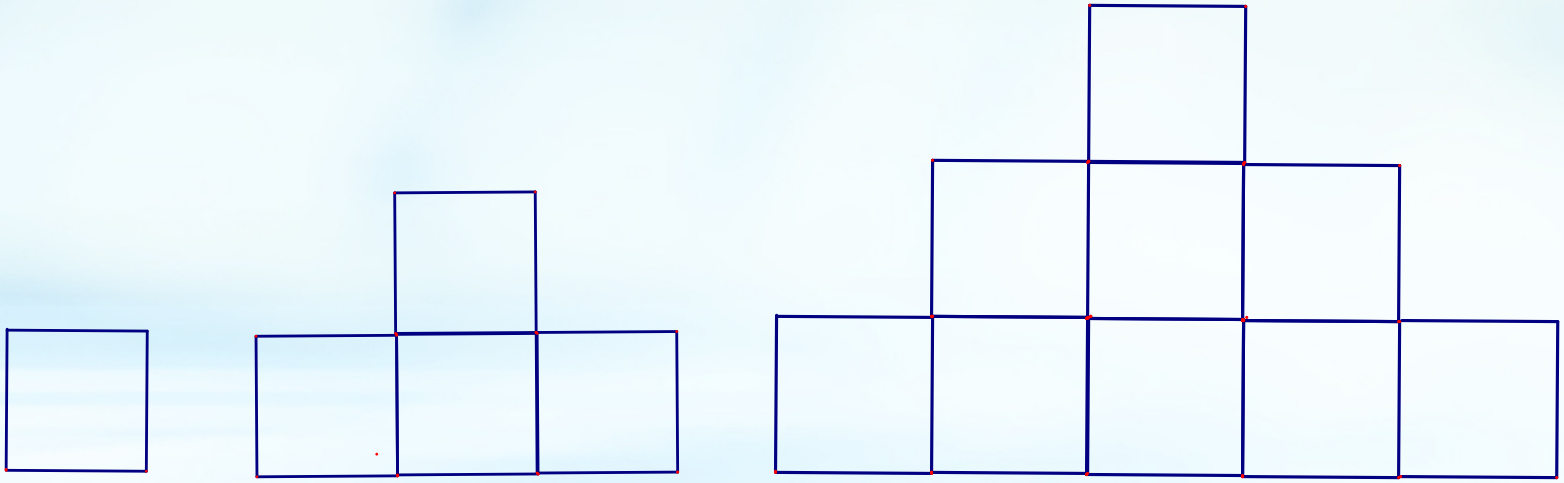


figure 1 figure 2 figure 3

As the figure number changes, _____ changes.

Structure

Too little structure often leads to uneven engagement

Just right…

Too much structure often leads to reduced opportunity to learn

"Just right" structure

Invites everyone

Gives sufficient guidance so no one is left without a place to productively get started

Privileges no one

Frames the work so no one individual student could accomplish all aspects of the task as effectively or as deeply if working in isolation

Purposely ambiguous

Leaves room for a range of student interpretations & perhaps even some confusion, but with intent

When no structure is actually a structure

Notice and Wonder

When the answer actually isn't the goal

Give

the

Answer

The richest implementation makes the mathematical meaning memorable …

Rich instruction creates touchstone events for students

Intentional discourse to debrief tasks attaches meaning

Public use of student work and a record of the ensuing discussion increases access for all

Possible rich implementation pacing

Day one

Engage all students in a groupworthy task, while circulating to eavesdrop, attend to status issues & assign competence

Day two

Debrief by posting all groupwork, begin with notice & wonder, and support students to develop their own mathematical story vs teacher's predetermined narrative

Day three & beyond

Add residue from conversation to the display of groupwork, then intentionally refer to it including pointing out particular aspects throughout the ensuing unit of study

No matter the structure, routine, practice or protocol, our focus must always be on **the "why" rather than the "how"**

ALL

~~Rich~~ Tasks

Deserve
Depend on
Demand

Rich Implementation

For Rich Learning

to Result

Every single instructional day does not need to be based in a profoundly deep, rich, open, groupworthy task.

There are places in a balanced curriculum better occupied by what might be called "everyday tasks".

Even very intentionally low threshold tasks require prior capacity for students to engage.

That capacity is often only built through more everyday tasks designed specifically to support incremental growth.

"Typical" everyday assignments employed with atypically rich implementation can maximize their capacity to：

1.) provide opportunities to learn

2.) provide opportunities to make sense of students' learning

Sorting vs Solving

Sorting – Given 16 different systems of equations, arrange them into groups. Create at least two different set of groupings based on shared characteristics of the systems.

$5x + y = 9$ $10x - 7y = -18$	$3x - 2y = 2$ $y = -10 - x$	$6x-2y=7$ $3x-y=5$	$-14 = -20y - 7$ $10y + 4 = 2x$
$y = x^2 + 4x + 3$ $y = 2x + 6$	$2x-3y=6$ $6x-18=9y$	$8x + y = -1$ $-3x + y = -5$	$x - y = 11$ $2x + y = 19$
$x^2 + y^2-4=0$ $2y^2+x+2=0$	$x^2 + y^2=25$ $x - y=5$	$-4x -2y = -12$ $4x + 8y = -24$	$3 + 2x - y = 0$ $-3 -7y = 10x$
$x^2+y^2-16x+39=0$ $x^2- y^2-9=0$	$-7x + y = -19$ $-2x + 3y = -1$	$x=3y-5$ $y=2x+4$	$2x-y=3$ $y-3=3x$

Solve each system by substitution.

1) $4x + 3y = -8$
$-8x + y = -12$

2) $4x - 2y = 8$
$y = -2$

3) $14x - 2y = 46$
$-7x + y = -23$

4) $5x + y = 8$
$-3x + 2y = -10$

Solve each system by elimination.

5) $10x - 8y = 4$
$-5x + 3y = -9$

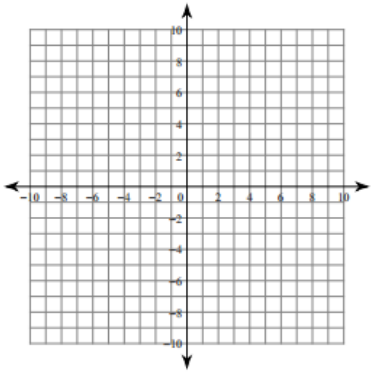
6) $-15x + 9y = 27$
$-5x - y = 17$

7) $-7x - 8y = -23$
$4x + 4y = 12$

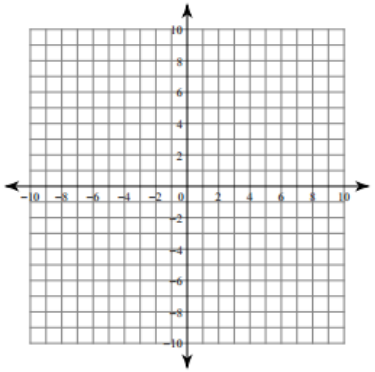
8) $-3x - 10y = -4$
$x - 5y = 18$

Solve each system by graphing.

9) $y = \frac{5}{7}x + 4$
$y = -\frac{1}{7}x - 2$



10) $x = 7$
$y = -x + 9$



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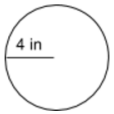
Analyzing vs Answering

Area, Perimeter and Circumference Review Sheet

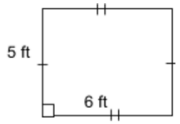
1. Suzanne is making a frame for an 8-inch by 10-inch photo. How much wood does she need for the frame?
2. The diameter of a quarter is 24 mm. What is the quarter's circumference?
3. The floor of our classroom is 50 feet by 75 feet. What is the area of our floor?
4. Each tire on your bicycle has a diameter of 26 inches. About how far will you travel when the tires make on complete revolution?

Find the area and perimeter (or circumference) of each figure.

5.


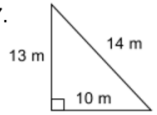
C = _____

A = _____

6.


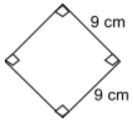
P = _____

A = _____

7.


P = _____

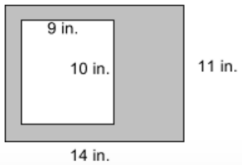
A = _____

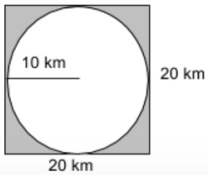
8.


P = _____

A = _____

9. Find the area of each shaded region.

a)


b)


$$\frac{x}{x^2-1}+\frac{4}{x-1} \qquad \frac{x}{x^2-1}+\frac{x}{x+1} \qquad \frac{x^2}{x+1}+\frac{1}{x+3}$$

$$\frac{x}{x^2+4x+3}+\frac{4}{x+1} \qquad \frac{3x}{x^2+7x+10}+\frac{x}{x+5} \qquad \frac{x^2}{x^2-5x+6}+\frac{1}{x-3}$$

$$\frac{x}{x^2+4x+3}-\frac{4}{x+1} \qquad \frac{3x}{x^2+7x+10}-\frac{x}{x+5} \qquad \frac{x^2}{x^2-5x+6}-\frac{1}{x-3}$$

$$\frac{x}{x^2-1}*\frac{4}{x-1} \qquad \frac{x}{x^2-1}*\frac{x}{x+1} \qquad \frac{x}{x^2+1}*\frac{1}{x+3}$$

$$\frac{x}{x^2+4x+3}\div\frac{4}{x+1} \qquad \frac{3x}{x^2+7x+10}\div\frac{x}{x+5} \qquad \frac{x^2}{x^2-5x+6}\div\frac{1}{x-3}$$

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Contemplating vs Calculating

$x - 11 = -21$
$+11 \quad +11$
$x = -10$

$\frac{x}{-3} = 7$
$*-3 \quad *-3$
$x = -21$

$4x + 8 = -12$
$-8 \quad -8$
$\frac{4x}{4} = \frac{-20}{4}$
$x = -5$

$\frac{x}{2} - 9 = -13$
$+4 \quad +9$
$\frac{x}{2} = -4$
$*2 \quad *2$
$x = -8$

Each of these equations has been simplified incorrectly. For each, explain what the person was confused about the led to the mistake.
Bonus: Give another problem that might help them to see & understand their misconception.

1.) $y = -2x(x + 4)$

$y = -2x^2 + 8x$

2.) $-3x + x(x - 6) = y$

$-3x^2 + 18x + x^2 - 6x = y$
$-2x^2 + 12x = y$

3.) $y = (x - 3)(x + 3)$

$y = x - 3x + 3$
$y = -2x + 3$

4.) $y = (x - 3)(x + 2)$

$y = x^2 - 6$

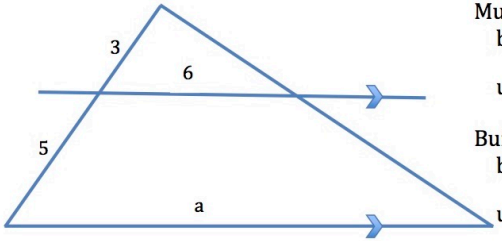
5.) $(3x + 2)(x - 5) = y$

$3x^2 - 15x + 2x - 10 = y$
$3x^2 - 13x^2 - 10 = y$
$-10x^2 - 10 = y$

6.) $y = (x + 3)^2$

$y = x^2 + 9$

Muffy & Buffy have been investigating triangles that have been intersected by a line parallel to one of the sides. They are using proportional reasoning to determine the length of the side marked as *"a"*.



Muffy says that "*a*" would equal 10 because she wrote a proportion using $\frac{3}{6} = \frac{5}{a}$ and solved for "*a*".

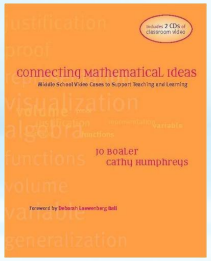
Buffy says that "*a*" would equal 16 because she wrote a proportion using $\frac{3}{6} = \frac{8}{a}$ and solved for "*a*".

Do you agree with Muffy ? or with Buffy? Or do you disagree with them both?
Be prepared to share your thinking.

Remember that there is no single right answer about which task to use, or how to implement it.

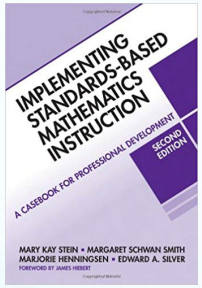
The right answer is whatever best supports your students in making progress towards the identified mathematical learning goal.

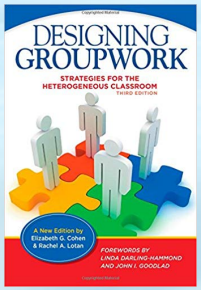
If you want to read more



Connecting Mathematical Ideas
by Boaler & Humphreys

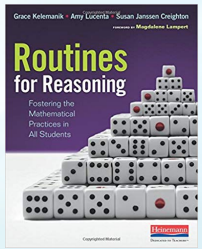
Implementing Standards Based Mathematics Instruction
by Stein, Smith, Henningsen & Silver





Designing Groupwork
by Cohen & Lutan

Routines for Reasoning
by Kelemanik, Lucenta & Creighton



Thank you, but before you leave …

Questions?

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