100 Problems Involving the Number 100

100 Days of Professional Learning
Webinar – Thursday, 14 May 2020
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Problem 1: Spell It Out

If all the positive integers from 1 to 100 were spelled out, how many letters would be used?

Problem 2: A Funny Name

What name did nine-year-old Milton Sirotta create for the number 10^{100}?

Problem 3. Letter Product

Let A = 1, B = 2, C = 3, …, Z = 26, according to their position in the alphabet. The letter product of a word is the product of the values of the letters within the word. For instance, CAT has a letter product of $3 \cdot 1 \cdot 20 = 60$. How many common English words have a product of 100?

Problem 4: It's Gettin' Kinda Heavy

Which weighs more: $100 worth of quarters, or $100 worth of dimes?

Problem 5. Days are Numbered

How many days is 100 hours?

Problem 6. Walk It Off

How many feet is 100 inches?

Problem 7. Release the Hounds

There are 100 puppies in a shelter to be adopted, and 99% of them are hounds. How many hounds must be removed from the shelter so that 98% of the remaining puppies are hounds?

Problem 8. Digital Throwback

Insert operators (+, –, ×, ÷) between the digits 1–9 below to create a true equation.

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 = 100$$

What is the least number of operators that must be inserted to create a true equation?

Problem 9. Freedom of Expression

Place each digit 0–9 in one of the blanks to make a true equation. Use each digit exactly once.

__ __ – __ __ + __ __ – __ __ + __ __ = 100

Problem 10. It Doesn't Add Up

Insert only addition (+) symbols to make the following equation true.

10 9 8 7 6 5 4 3 2 1 = 100

Problem 11. Give and Take

Insert only addition (+) and subtraction (–) symbols to make the following equation true.

9 8 7 6 5 4 3 2 1 = 100

Problem 12. What's It Gonna Take?

If you solved Problem 11: Give and Take, you know that it's possible to create an expression equal to 100 using the digits 9 through 1 in descending order. What's the least positive integer value of *n* such that the numbers from *n* to 1, arranged in descending order, will allow you to make an expression equal to 100 that involves only addition and subtraction?

Problem 13. Times Square

The multiplication table shown below contains 100 entries. What is the sum of all entries?

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Problem 14. Squares and Square Roots

What's the positive difference between 100^2 and the square root of 100?

Problem 15. What Would Your Computer Think?

What comes next?

1, 10, 11, 100, ___

Problem 16. Desmos Be the Place

What comes next?

2, 5, 10, 20, 50, 100, ___

Problem 17. Even I Know This One

What comes next?

4, 16, 36, 64, 100, ___

Problem 18. Number of Numbers

In the sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, …, what is the 100th term?

Problem 19. What in the World?

In the sequence 0, 1, 2, 2, 3, 3, 4, 4, 4, 4, 5, 5, 6, …, what is the 100th term?

Problem 20. Farey Tales

A Farey sequence F_n is the set of all fractions from 0 to 1 with every possible denominator less than or equal to n. For instance,

$$F_4 = \{0/1, 1/4, 1/3, 1/2, 2/3, 3/4, 1/1\}$$

What is the 100th term of F_{100}?

Problem 21. Gauss and Check

What is $1 + 2 + 3 + \cdots + 100$?

Problem 22. Getting Even

What is $2 + 4 + 6 + \cdots + 100$?

Problem 23. Well, That's Odd

What is $1 + 3 + 5 + \cdots + (2 \times 100 - 1)$?

Problem 24. Square Deal

What is the sum of the first 100 square numbers?

Problem 25. Non-Square Numbers

The square numbers are 1, 4, 9, 16, …, and the non-square numbers are 2, 3, 5, 6, 7, 8, 10, 11, …. What is the 100th non-square number?

Problem 26. In Perfect Harmony

What integer is closest to $1/1 + 1/2 + 1/3 + 1/4 + \cdots + 1/100$?

Problem 27. The Great Divide

How many positive integer divisors does 100 have?

Problem 28. Factor Fiction

What is the least positive integer with exactly 100 positive integer factors?

Problem 29. The Locker Problem

A school with 100 students has 100 lockers numbered 1–100. The first student goes along and opens every locker. The second student closes every even-numbered locker. The third student changes the state (that is, opens it if it's closed, or closes it if it's open) of every locker whose number is a multiple of three. The fourth student changes the state of every locker whose number is a multiple of four. And so on, with the n^{th} student changing the state of every locker whose number is a multiple of n. When all 100 students have finished opening and closing lockers, how many lockers will be open? Which ones?

Problem 30. Zero Hero

How many zeroes are at the end of 100^{100}?

Problem 31. Famous Last Digits

What are the last two digits of 2^{100}?

Problem 32. More Famous Last Digits

What is the units digit of $1^{100} + 2^{100} + 3^{100} + 4^{100} + 5^{100} + 6^{100} + 7^{100} + 8^{100} + 9^{100}$?

Problem 33. At the End, There's Nothing

How many zeroes are at the end of 100! when it is computed?

Problem 34. Can You Digit?

Let $S(n)$ represent the number of digits in integer n. For instance, $S(4) = 1$ and $S(256) = 3$. What is $S(S(100^{100}))$?

Problem 35. Sum Kinda Wonderful

The sum of the digits of a number is 100, and none of the digits are 0. What is the largest possible value of the number?

Problem 36. Product Marketing

The product of the digits of a number is 100, and none of the digits are 1. What is the largest possible value of the number?

Problem 37. Whole Lotta Nothin'

What is the 100th positive integer that contains a 0?

Problem 38. No Zeroes

What is the 100th positive integer that does not contain a 0?

Problem 39. Non-Zero Product

Two positive integers have a product of 100, and neither number contains the digit 0. What are the two numbers?

Problem 40. To Say the Least

If you write the integers 1–100, which digit would appear the least number of times? How many times?

Problem 41. I Got Chills, They're Multiplying

A number with 100 9s (and no other digits) is multiplied by 9. How many 9s are in the resulting product?

Problem 42. Ring, Rang, Rungs

A ladder with 100 rungs leads to the top of a tower. You can climb one or two rungs at a time. How many different ways are there to get to the top?

Problem 43. Leo's Bank Account

Leo made a deposit at the bank every day for a week. On Monday and Tuesday, he deposited a positive whole number of dollars, and every day thereafter he deposited an amount equal to the sum of the deposits from the previous two days. On Sunday, he deposited exactly $100. What were the amounts of his deposits on the other days?

Problem 44. Prime Time

How many pairs of prime numbers have a sum of 100?

Problem 45. In Their Prime

How many sets of three prime numbers have a sum of 100?

Problem 46. Prime Pair

Two prime numbers have a sum of 100. What's the maximum possible product?

Problem 47. Max Product

Two positive integers have a sum of 100. What's the maximum possible product?

Problem 48. To the Max

Some positive integers add up to 100. What's the maximum possible product?

Problem 49. Why, Certainly

For what value of n can you be certain that n consecutive positive integers have a product that is divisible by 100?

Problem 50. Catching Some Zs

For how many positive integer values of k is $100! / 100^k$ an integer?

Problem 51. Don't Put Me on a Shelf

How many Styrofoam cups could fit on a shelf 100 inches tall?

Problem 52. Play Ball!

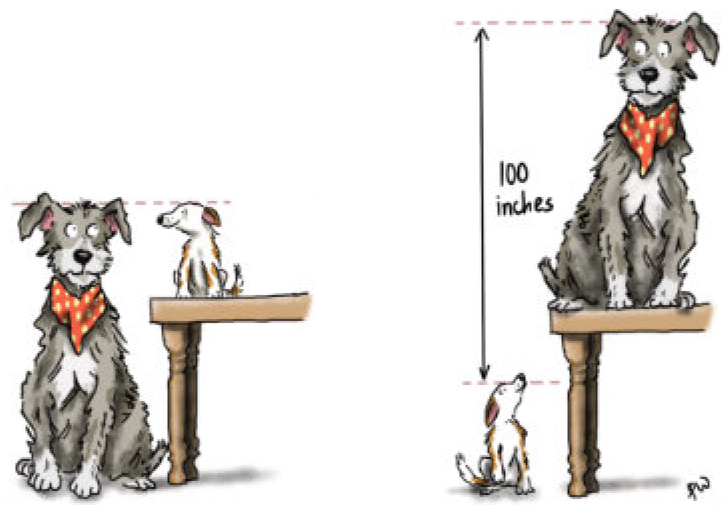
Together, a baseball and a bat cost $110. The bat costs $100 more than the baseball. How much did the bat cost?

Problem 53. The Jug is Half Full

A jug full of water weighs 100 ounces. When the jug is half full of water, it only weighs 60 ounces. What is the weight of the empty jug?

Problem 54. Dog Day Afternoon

When the large dog is on the ground and the small dog is on the table, the tops of their heads are the same distance off the ground. But when the small dog is on the ground and the large dog is on the table, the top of the small dog's head is 100 inches lower than the top of the large dog's head. How tall is the table?



Problem 55. The Barnyard

A farmer buys 100 animals for $100. She buys at least one cow, one pig, and one chicken, but no other type of animal. If a cow costs $10, a pig costs $5, and a chicken costs 50¢, how many of each did she buy?

Problem 56. Sum of Cubes

For what integer values of x, y, and z is $x^3 + y^3 + z^3 = 100$?

Problem 57. Absolute Power

If x and y are both integers, how many different solutions exist for $|x| + |y| = 100$?

Problem 58. Absolutely Dreadful

For the equation below, what is the least possible value of y?

$$y = |||x - 100| + 100| - 100| + 100$$

Problem 59. Don't Be Mean

In a list of 100 numbers, the average value of the first 99 numbers in the list equals the average value of all 100 numbers. What is the 100th number in the list?

Problem 60. Counterintuitive Cards

A dealer is holding a deck with 100 cards, 30 of which are red on both sides, 30 of which are blue on both sides, and 40 of which are red on one side and blue on the other. The dealer tells you that he will select a card at random and show you the color on one side of the card. If you correctly predict the color on the other side, he will give you $1. If not, you must give $1 to the dealer. What strategy should you use?

Problem 61. Tickets, Please

One-hundred passengers board a train with exactly 100 seats. Every person has an assigned seat number, but the first person to board has lost her ticket, so she chooses a seat at random. Every passenger thereafter takes their assigned seat, unless their assigned seat is already occupied, in which case they choose a seat at random. If you are the last passenger to board the train, what's the probability that your assigned seat is unoccupied?

Problem 62. You Say It's Your Birthday

If there are 100 people in a room, what's the probability that two of them have the same birthday?

Problem 63. The Envelope, Please

You are offered a choice of two envelopes. One of them has twice as much money in it as the other. You select one envelope, open it, and find that it has $100 in it. Should you keep the $100, or should you switch to the other envelope?

Problem 64. Wrecked Angle

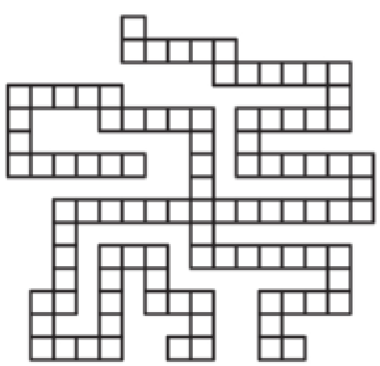
How many different rectangles with integer side lengths and a perimeter of 100 units are possible?

Problem 65. Area of Influence

How many different rectangles with integer side lengths and an area of 100 square units are possible?

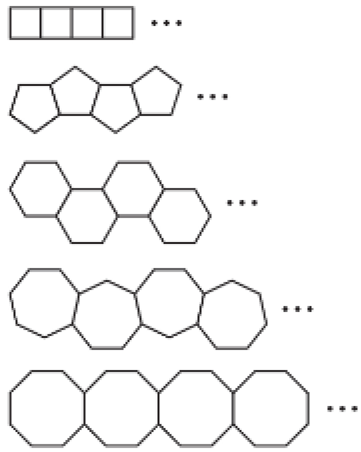
Problem 66. Around the Square

The figure below consists of 100 unit squares, and adjacent squares share a side. What is the perimeter of the figure?



Problem 67. Chains of Fools

Which of the following chains cannot be extended to have a perimeter of exactly 100 units?

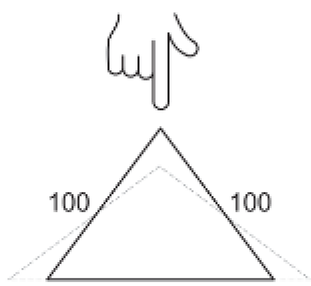


Problem 68. Rock the Octagon

An octagon with side lengths of 9, 10, 11, 12, 13, 14, 15, and 16 units has a perimeter of 100 units. If adjacent sides are perpendicular, what is the maximum possible area of this octagon?

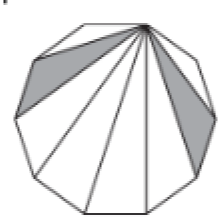
Problem 69. Flattening a Triangle

The triangle shown below lies on a flat surface and is pushed at the top vertex. The length of the two congruent sides is 100 inches and their length does not change when the top vertex is pushed, but the angle between the two congruent sides will increase, and the base will stretch. What is the maximum possible area of the triangle that can be achieved when the top vertex is pushed? And, what is the length of the base when the maximum area is reached?



Problem 70. Pieces of Ten

The regular decagon below has area 100 square units. What is the area of the two shaded regions?



Problem 71. Prism to the Max

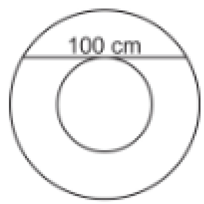
How many different rectangular prisms have integer edge lengths and a volume of 100 cubic units?

Problem 72. Run for Cover

A rectangular prism with integer edge lengths has a volume of 100 cubic centimeters. What is the maximum possible surface area of this solid?

Problem 73. Strike a Chord

Find the area of the annulus if a chord of the outer circle that measures 100 cm is externally tangent to the inner circle.

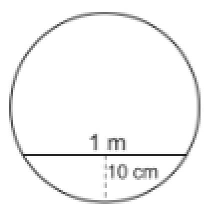


Problem 74. In the Distance

An historic lighthouse is perched on the cliff of a rocky beach. Standing in the lantern room of the lighthouse, you are approximately 100 feet above the surface of the ocean below. How far can you see to the horizon?

Problem 75. Pipe Dream

A meter stick is placed inside a pipe, and the midpoint of the meter stick is 10 centimeters from the pipe wall. What is the diameter of the pipe?

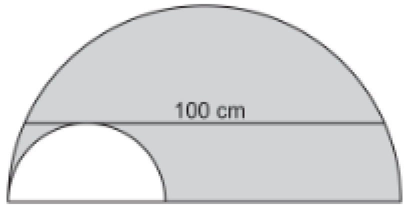


Problem 76. Around the World

A piece of wire is stretched around the Equator. Then, a piece of wire is wrapped around the Earth at a height of 100 meters above the surface. What is the difference between the length of the wire and the circumference of the Earth?

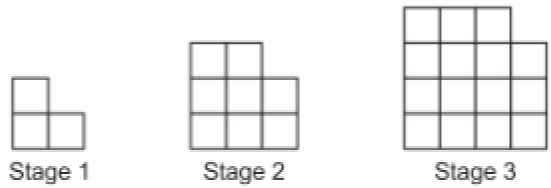
Problem 77. The Setting Sun

In the figure below, the diameter of the small semicircle is coincident with the diameter of the large semicircle. A line segment parallel to the diameter of the large semicircle is tangent to the small semicircle, and its length is 100 centimeters. What is the area of the shaded region?



Problem 78. Growing Pattern

If the pattern below continues, how many squares would be in Stage 100?



Problem 79. Survivor, Part 1

There are 100 flags arranged in a circle. You can remove either one or two flags at a time, and the person to remove the last flag wins. If you go first, how many should you remove on your first turn to guarantee that you win?

Problem 80. Survivor, Part 2

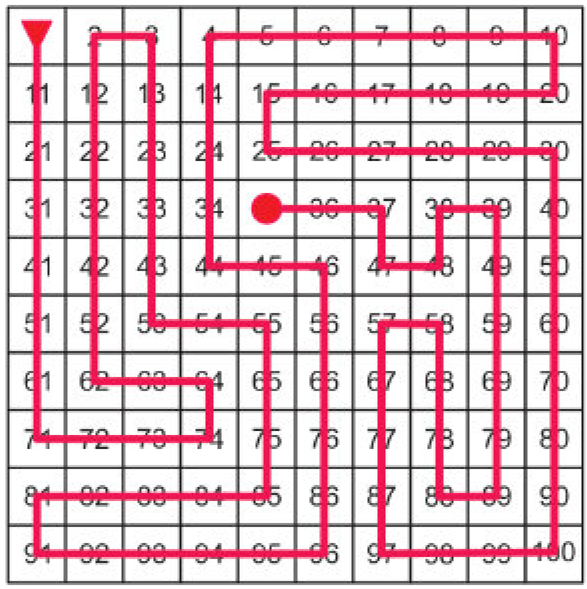
Begin with a stack of 100 coins. Two players take turns removing one, four, or eight coins. The player who removes the last coin wins. If both players follow optimal strategy, which player will win, and how many coins should that player take on the first turn?

Problem 81. Survivor, Part 3

One-hundred people sitting in a circle are given the numbers 1–100. Going clockwise around the circle starting with 1, every second person leaves the circle. As some leave, those remaining form a smaller circle without gaps. They continue to remove people in this manner until only one person remains. Who will be the last one remaining?

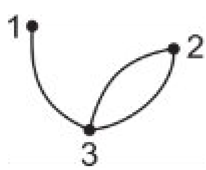
Problem 82. Complete the Circuit

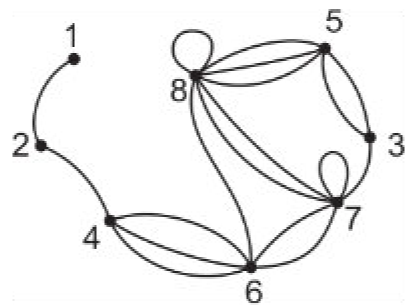
Starting in the top left cell of a 10-by-10 grid of squares, a move can be made to an adjacent cell either vertically or horizontally. A complete circuit consists of starting on the cell numbered 1 and touching every other cell in the grid exactly once. One possible circuit, which ends at 35, is shown below. What is the sum of all numbers on which a circuit could end?



Problem 83. Numbering the Vertices

As shown below, it's possible to create a graph with three vertices so that one vertex has degree one, one vertex has degree two, and one vertex has degree three. (The degree of a vertex is the number of edges connected to that vertex.) It's also possible to create a graph with eight vertices so that each degree one through eight is associated with a different vertex. Is it possible to create a graph with 100 vertices so that the vertices have degrees 1 through 100?





Problem 84. Shake It Off

At a party with 100 people, every person shakes hands with every other person. How many total handshakes occur?

Problem 85. Scope It Out

What is the value of the following expression?

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{99\cdot 100}$$

Problem 86. Erase and Replace

The numbers 1–100 are written on a board. At every stage, two numbers a and b are erased from the board and replaced by $a + b + ab$. For instance, if you erase 14 and 51, you'd replace them with $14 + 51 + 14 \cdot 51 = 779$. This process is repeated until only a single number remains. What are the possible values of the remaining number?

Problem 87. Replace the Hypotenuse

Start with the numbers 1–100 written on a whiteboard. Erase two numbers and replace them with the hypotenuse of a triangle for which those two numbers were the lengths of the two legs. Continue until just one number remains. What number is it?

Problem 88. An Odd Game

The numbers 1–100 are written in ascending order on a board. Players take turns placing either + or – between any two numbers. After all spaces between numbers have been filled with an operator, the result is calculated. If the result is odd, you win. What first move should you make?

Problem 89. Pick 'n Add

Anand and Bela are playing a number game. Each of them picks a number from 1 to 10. (If they pick the same number, they both pick again.) If Anand's number is smaller, then he gets to add his number to Bela's, and the result is his new number. But at that point, Bela's number is now smaller, so she gets to add Anand's number to her number, and the result is her new number. And so on. For example, if Anand chose 2 and Bela chose 5, then Anand's new number would be $2 + 5 = 7$, Bela's new number would be $5 + 7 = 12$, Anand's next number would be $7 + 12 = 19$, Bela's next number would be $12 + 19 = 31$, and so forth.

This continues until one of them reaches 100 or more, and that player wins. What is the best number to choose at the start of this game?

Problem 90. Product of Piles

Marty has 100 tokens and divides them into two piles of, say, 40 and 60 tokens. He then records the product, $40 \times 60 = 2{,}400$. He then divides the pile of 40 into two piles of, say, 17 and 23 and records that product, $17 \times 23 = 391$. He divides the pile of 60 into two piles of, say, 30 and 30 and records the product, $30 \times 30 = 900$. He continues doing this until he has 100 piles of just one token each. Marty then finds the sum of all 99 products that he recorded. What is the greatest possible value for the sum?

Problem 91. Magic Rectangles

A magic rectangle is an $m \times n$ array of the positive integers from 1 to $m \times n$ such that the numbers in each row have a constant sum and the numbers in each column have a constant sum (although the row sum need not equal the column sum). Shown below is a 3 × 5 magic rectangle with the integers 1–15.

6	7	8	9	10
13	3	1	11	12
5	14	15	4	2

How many magic rectangles can be made using the integers 1–100?

Problem 92. Fair and Square

A magic square is a square grid in which each cell contains a distinct positive integer, and the sum of the integers in each row, column, and diagonal is equal. The sum of the numbers in each row, column, and diagonal is known as the magic sum. Create a 4 × 4 magic square in which the magic sum is 100.

Problem 93. Magic Sum

What's the "magic sum" in a 10 × 10 magic square?

Problem 94. One and Done

One-hundred tennis players enter a single-elimination tournament. How many matches are needed to determine a champion?

Problem 95. To the Point

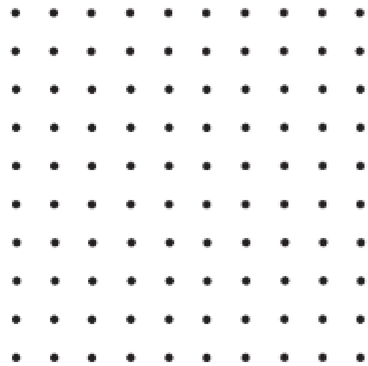
A dartboard contains regions worth 8 and 13 points. Bart scored exactly 100 points. How many times did he hit the region worth 8 points?

Problem 96. Call to Order

All possible permutations of the digits 1, 2, 3, 4, and 5 are written in order from least to greatest. What five-digit number is in the 100th position in the list?

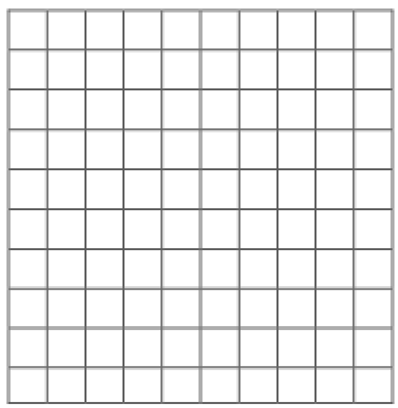
Problem 97. Triangles on a Grid

What's the probability that three randomly chosen points on a 10×10 lattice will form a triangle?



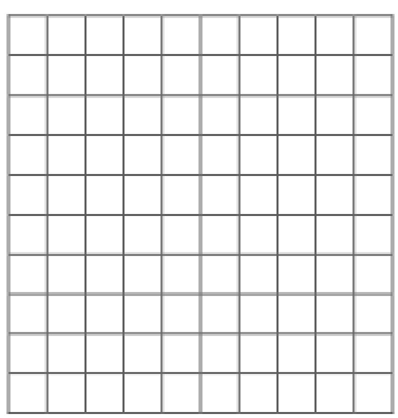
Problem 98. Binary Quandary

The following grid contains 100 squares. If each square is filled with either a 0 or a 1, in how many different ways can the entire grid be filled so that every row contains an even sum?



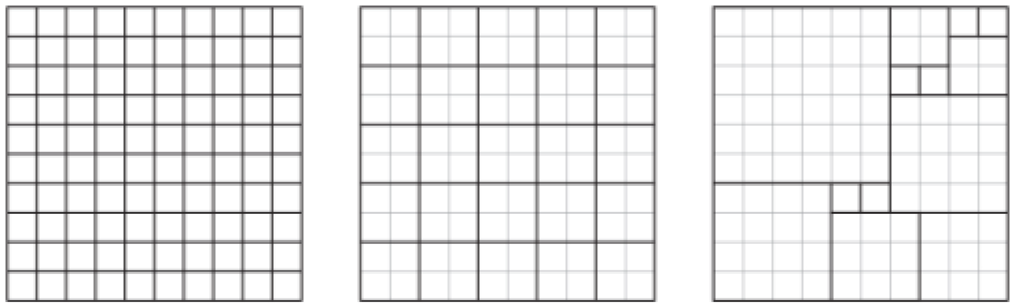
Problem 99. Squares on a Grid

How many different squares of any size are contained in the 10×10 grid shown below?



Problem 100. Covering with Squares

As shown below, a square grid with 100 smaller squares can be covered by 100 squares (each measuring 1 ×1), by 25 squares (each measuring 2 × 2), or by 13 squares (one 6 × 6, two 4 × 4, two 3 × 3, two 2 × 2, and six 1 × 1).



100 Squares 25 Squares 13 Squares

Find all values of n for which it is impossible to cover a 10 ×10 grid with n squares of integer side length.

Problem 101. Falsehoods and Fibs

Which of the following statements are true?

1. Exactly one of the statements in this list is false.
2. Exactly two of the statements in this list are false.
3. Exactly three of the statements in this list are false.
4. Exactly four of the statements in this list are false.
5. Exactly five of the statements in this list are false.

⋮

100. Exactly 100 of the statements in this list are false.