Practices, Practices, Practices
How do they relate?

A Meaningful Framework for Supporting Them All!

@TravisLemon
#NCTM100
Our Session Today

- Practices, Practices, Practices, ...
- 8 Effective Teaching Practices
- 8 Standards for Mathematical Practice
- Consider a framework for supporting it all
<table>
<thead>
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Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.
Where do they come from? Who are they for? Which came first? How do they relate? What to do about it?

**Standards for Mathematical Practice**

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Given a specific lens of practices
Identify from the other set of practices 3 that are most critical.
Rank them in order.

**8 x 8: Teacher by Student Practices**

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What patterns have emerged? What do you notice? What do you wonder? How do you feel?

- Some rows/columns are not marked or not marked much at all.
  - Practices that are natural for you
  - Practices not on your radar that need more attention
What patterns have emerged? What do you notice? What do you wonder? How do you feel?

- Wondering what correlations there might be and which ETPs might foster certain SMPs
What patterns have emerged? What do you notice? What do you wonder? How do you feel?

Feeling-

- excited about the possibilities
- intrigued by the potential connections
- overwhelmed by the amount of things to attend to and consider
How can we realize all of the practices?

- Progression
- Coherence
- Frameworks
Calls for Coherence and Progressions

NCTM Curriculum Principle:
A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.
(PSSM, 2000)
Calls for Coherence and Progression

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.

The most important observation we make about these five strands is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal.

(Adding it Up, 2001)
Calls for Coherence and Progression

“The Common Core State Standards in mathematics were built on progressions...informed both by research on children’s cognitive development and by the logical structure of mathematics.”

ProgressionDocument Introduction
http://ime.math.arizona.edu/progressions/
“Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject…

(CCSSMMPublisher’s criteria, pg. 3)
“Fragmenting the Standards into individual standards, or individual bits of standards, erases all these relationships and produces a sum of parts that is decidedly less than the whole.”

Phil Daro, Jason Zimba, Bill McCallum
Calls for Coherence and Progression
Effective Teaching Practices

- Establish mathematics goals to focus learning.
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- Pose purposeful questions.
- Building Procedural Fluency from Conceptual understanding
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Effective Teaching Practices

- Establish mathematics goals to focus learning.
- **Implement tasks that promote reasoning and problem solving.** "Student learning is greatest in classrooms where tasks consistently encourage high-level student thinking and reasoning and least in classrooms where tasks are routinely procedural in nature."
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- Elicit and use evidence of student thinking.
Effective Teaching Practices

Fig. 10.1. A framework for mathematics teaching that highlights the relationships between and among the eight effective teaching practices

(NCTM, 2017)
Effective Teaching Practices

Figure 1: A framework for mathematics teaching that highlights the relationships among effective teaching practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Facilitate meaningful mathematical discourse
  - Pose purposeful questions
  - Elicit and use evidence of student thinking
  - Use and connect mathematical representations
  - Support productive struggle in learning mathematics

Taking Action 9-12 (NCTM, 2017)
Enhancing Classroom Practice

“Tasks ... gain more traction when used within sequences of tasks that develop students’ understanding of larger mathematical ideas or processes.”

Progressions and Coherence Facilitate engagement in all of the Practices!

Considering progressions increases my hope for improvement.
There is a need for a framework

- Chazan and Ball (1999), argue that educators are often left “with no framework for the kinds of specific, constructive pedagogical moves that teachers might make.”
- Stein et al. (2008) refer to a first generation of instructional reform from which “many teachers got the impression that in order for discussion to be focused on student thinking, they must avoid providing any substantive guidance at all,” and they refer to a second generation of instructional reform “that re-asserts the critical role of the teacher in guiding mathematical discussions.”
There are some excellent frameworks

- Levels of Cognitive Demand
There are some excellent frameworks

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<td>- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory.</td>
<td>- Focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.</td>
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<td>- Cannot be solved by using procedures, because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.</td>
<td>- Suggest, explicitly or implicitly, pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.</td>
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<td>- Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.</td>
<td>- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.</td>
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<td>- Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced.</td>
<td>- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.</td>
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<td>- Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task.</td>
<td>- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.</td>
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<td>- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done or how to do it.</td>
<td>- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.</td>
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<td>- Have no connection to the concepts or meaning that underlies the procedure being used.</td>
<td>- Demand self-monitoring or self-regulation of one’s own cognitive processes.</td>
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<td>- Are focused on producing correct answers instead of on developing mathematical understanding.</td>
<td>- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.</td>
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<td>- Require no explanations or explanations that focus solely on describing the procedure that was used.</td>
<td>- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.</td>
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Purple Book (NCTM, 2000, 2009)
There are some excellent frameworks

- Levels of Cognitive Demand
- Launch, Explore, Summarize models of instruction.
There are some excellent frameworks

- Levels of Cognitive Demand
- Launch, Explore, Summarize models of instruction.
- 5 Practices for Orchestrating Productive Mathematics Discussions
There are some excellent frameworks

(NCTM, 2011, 2018)
Putting it into Practice!

- The 5 Practices in Practice: Successfully Orchestrating Mathematics Discussions in Your Elementary Classroom
- The 5 Practices in Practice: Successfully Orchestrating Mathematics Discussions in Your Middle School Classroom
- The 5 Practices in Practice: Successfully Orchestrating Mathematics Discussions in Your High School Classroom
There are some excellent frameworks

- Levels of Cognitive Demand
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- 5 Practices for Orchestrating Productive Mathematics Discussions
- And more...
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- These are Great!
There are some excellent frameworks

- Levels of Cognitive Demand
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- 5 Practices for Orchestrating Productive Mathematics Discussions
- And more...

- These are Great!
- All have supported improvements in pedagogy
  However, focus at the task level!
Calls for Coherence and Progressions

- The CCSSM publisher’s criteria, Principles to Actions, Catalyzing Change and others, wouldn’t urge us to do more if we had arrived.
- We need a sustained, persistent press for student thinking, development of conceptual understanding and procedural fluency, supported productive struggle that occurs on a daily basis.
- The effort to implement a task needs to lead to the implementation of a progression of tasks and a curriculum that is coherent, rigorous and focused.
Calls for Coherence and Progressions

**NCTM Curriculum Principle:**

A curriculum is *more than a collection of activities*: it must be *coherent, focused on important mathematics*, and well *articulated across the grades*.

(PSSM, 2000)
8 and 8
8 for teachers related to teaching
8 for students related to doing mathematics

Standards for Mathematical Practice
- Make sense of problems and persevere in solving them.
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- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

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Seems sensible to have a framework that connects the 8 with the 8.

And at the same time promotes progression!
Comprehensive Mathematics Instruction Framework (CMI)
CMI Framework Supporting Practice

When it comes to all of the practices we have to consider, what advantage is there to a framework containing cycles?
Comprehensive Mathematics Instruction Framework (CMI)
Supporting Task Implementation

Discuss
- Connect strategies, ideas & Representations to achieve goals

Launch
- Determine Goals & Select a task
- Anticipate Student Responses
- Plan questions

Explore
- Sequence work to produce Meaningful discussion
- Select student work to be used to accomplish goals

Monitor Student work & Thinking

5 Practices for Orchestrating Discussions
Supporting Task Implementation

Discuss
- Facilitate Meaningful Mathematics Discourse
- Establish Mathematical Goals to Focus Learning

Explore
- Use and Connect Mathematical Representations
- Elicit and Use Evidence of Student Thinking
- Support Productive Struggle in Learning Mathematics
- Pose Purposeful Questions

Launch
- Implement Tasks That Promote Reasoning and Problem Solving
Comprehensive Mathematics Instruction Framework (CMI)

BYU-Cites partnership
Supporting Coherence & Progressions

- **Practice Understanding** tasks surface student thinking
- **Develop Understanding** tasks examine and extend
- **Solidify Understanding** tasks build fluency
- **Practice Understanding** tasks
Develop Understanding

- Low threshold, high ceiling (easy entry, but extendable for all learners)
- Contextualized (problematic story context, diagrams, symbols)
- Multiple pathways to solutions or multiple solutions
- Surface student thinking (misconceptions and correct thinking)
- Purposeful selection of the vocabulary, numbers, etc. to reveal rather than obscure the mathematics
- Introduce a number of representations
- Constructing viable arguments and critiquing the reasoning of others
Solidify Understanding

- Task context, scaffolding questions and constraints focus students’ attention on:
  - looking for patterns and making use of structure
  - looking for repeated reasoning and expressing regularities as generalized methods
  - attending to precision in language and use of symbols
  - constructing viable arguments and critiquing the reasoning of others
  - using representations and tools strategically for the purpose of developing deeper levels of understanding of mathematical ideas, strategies, and/or representations
Practice Understanding

- Practice tasks focused on refining understanding
  - Task allows student to use reasoning habits to **contextualize** (symbolic to real-world) and **decontextualize** (real-world to symbolic) problems and situations.
  - Tasks involve sufficient complexity to refine mathematical thinking beyond rote memorization.
  - The task requires a **high level of cognitive demand** because students are required to draw upon multiple concepts and procedures, make use of structure and recognize complex relationships among facts, definitions, rules, formulas and/or models.
Practice Understanding

- Practice tasks focused on acquiring fluency
  - Task involves either reproducing previously learned facts, definitions, rules, formulas or models; OR drawing upon previously learned facts, definitions, rules, formulas or models; OR committing facts, definitions, rules, formulas or models to memory
  - An appropriate vehicle of practice is selected (e.g., routines, games, worksheets, etc.) which allows for reproducing, drawing upon, or committing to memory previously examined mathematics
  - Task focuses on a broad definition of fluency: accuracy, efficiency, flexibility, automaticity
6.1 Photocopy Faux Pas

A Develop Understanding Task

Burnell has a new job at a copy center helping people use the photocopy machines. Burnell thinks he knows everything about making photocopies, and so he didn’t complete his assignment to read the training manual.

Mr. and Mrs. Donahue are making a scrapbook for Mr. Donahue’s grandfather’s 75th birthday party, and they want to enlarge a sketch of their grandfather which was drawn when he was in World War II. They have purchased some very expensive scrapbook paper, and they would like this image to be centered on the page. Because they are unfamiliar with the process of enlarging an image, they have come to Burnell for help.

“We would like to make a copy of this image that is twice as big, and centered in the middle of this very expensive scrapbook paper,” Mrs. Donahue says. “Can you help us with that?”

“Certainly,” says Burnell. “Glad to be of service.”

Burnell taped the original image in the middle of a white piece of paper, placed it on the glass of the photocopy machine, inserted the expensive scrapbook paper into the paper tray, and set the enlargement feature at 200%.

In a moment, this image was produced.

“You’ve ruined our expensive paper,” cried Mrs. Donahue.

“Much of the image is off the paper instead of being centered.”

“And this image is more than twice as big,” Mr. Donahue complained. “One fourth of grandpa’s picture is taking up as much space as the original.”
6.2 Triangle Dilations

**A Solidify Understanding Task**

1. Given $\triangle ABC$, use point $M$ as the center of a dilation to locate the vertices of a triangle that has side lengths that are three times longer than the sides of $\triangle ABC$.

2. Now use point $N$ as the center of a dilation to locate the vertices of a triangle that has side lengths that are one-half the length of the sides of $\triangle ABC$. 
6.4 Cut by a Transversal

**A Solidify Understanding Task**

Draw two intersecting transversals on a sheet of lined paper, as in the following diagram. Label the point of intersection of the transversals $A$. Select any two of the horizontal lines to form the third side of two different triangles.

1. What convinces you that the two triangles formed by the transversals and the horizontal lines are similar?

2. Label the vertices of the triangles. Write some proportionality statements about the sides of the triangles and then verify the proportionality statements by measuring the sides of the triangles.

3. Select a third horizontal line segment to form a third triangle that is similar to the other two. Write some additional proportionality statements and verify them with measurements.
6.5 Measured Reasoning

A Practice Understanding Task

Find the measures of all missing sides and angles by using geometric reasoning, not rulers and protractors. If you think a measurement is impossible to find, identify what information you are missing.

Lines \(p, q, r,\) and \(s\) are all parallel.

\[
\begin{align*}
\text{\(r\)} & \quad 33.7^\circ \\
\text{\(g\)} & \quad 21.6 \\
\text{\(e\)} & \quad 53^\circ \\
\text{\(s\)} & \quad 9 \\
\text{\(t\)} & \quad 18 \\
\text{\(u\)} & \quad 12 \\
\text{\(v\)} & \quad 12 \\
\end{align*}
\]
Supporting Coherence & Progressions

Attending to Precision

Practice Understanding
- Look for and express regularity in repeated reasoning
- Construct viable arguments and critique the reasoning of others
- Look for and make use of structure

Develop Understanding
- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Use appropriate tools strategically
- Model with Mathematics

Learning Cycle

Where do they come from?  
Who are they for?  
Which came first?  
How do they relate?  
What to do about it?

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Given a specific lens of practices
Identify from the other set of practices 3 that are most critical.
Rank them in order.

8 x 8: Teacher by Student Practices

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