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Practices, Practices, Practices
How do they relate?

A Meaningful Framework for Supporting Them All!

@TravisLemon
#NCTM100

Our Session Today

- Practices, Practices, Practices, …
- 8 Effective Teaching Practices
- 8 Standards for Mathematical Practice
- Consider a framework for supporting it all

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Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Where do they come from?
Who are they for?
Which came first?
How do they relate?
What to do about it?

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Given a specific lens of practices
Identify from the other set of practices 3 that are most critical. Rank them in order.

8 x 8: Teacher by Student Practices

	Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
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Lens of Student Practices

8 x 8: Teacher by Student Practices

		Standards for Mathematical Practice (Students Engaged)							
		Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
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Given a specific lens of practices Identify from the other set of practices 3 that are most critical. Rank them in order.

Lens of Teacher Practices

8 x 8: Teacher by Student Practices

		Standards for Mathematical Practice (Students Engaged)							
EFFECTIVE TEACHING PRACTICES		Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
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	Elicit and use evidence of student thinking.								

What patterns have emerged? What do you notice? What do you wonder? How do you feel?

8 x 8: Teacher by Student Practices

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Establish mathematics goals to focus learning								
Implement tasks that promote reasoning and problem solving.	1	3	1	3	1			1
Use and connect mathematical representations.		1		1	2	3	1	2
Facilitate meaningful mathematical discourse.	2		2					
Pose purposeful questions.			3			2		
Build procedural fluency from conceptual understanding.	3			2	3			3
Support productive struggle in learning mathematics.		2					2	
Elicit and use evidence of student thinking.						1	3	

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What patterns have emerged? What do you notice? What do you wonder? How do you feel?

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EFFECTIVE TEACHING PRACTICES	Standards for Mathematical Practice (Students Engaged)							
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Establish mathematics goals to focus learning	1			3				2
Implement tasks that promote reasoning and problem solving.	1		2		3			
Use and connect mathematical representations.				1	2		3	
Facilitate meaningful mathematical discourse.	2		1					3
Pose purposeful questions.	2		3				1	
Build procedural fluency from conceptual understanding.						3	2	1
Support productive struggle in learning mathematics.	1		2					
Elicit and use evidence of student thinking.		2				3		1

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OPENUPRESOURCES.ORG

What patterns have emerged? What do you notice?
What do you wonder? How do you feel?

- Some rows/columns are not marked or not marked much at all.
  - Practices that are natural for you
  - Practices not on your radar that need more attention

What patterns have emerged? What do you notice?
What do you wonder? How do you feel?

- Wondering what correlations there might be and which ETPs might foster certain SMPs

What patterns have emerged? What do you notice?
What do you wonder? How do you feel?

Feeling-
- excited about the possibilities
- intrigued by the potential connections
- overwhelmed by the amount of things to attend to and consider

How can we realize all of the practices?

- Progression
- Coherence
- Frameworks

Calls for Coherence and Progressions

NCTM Curriculum Principle:

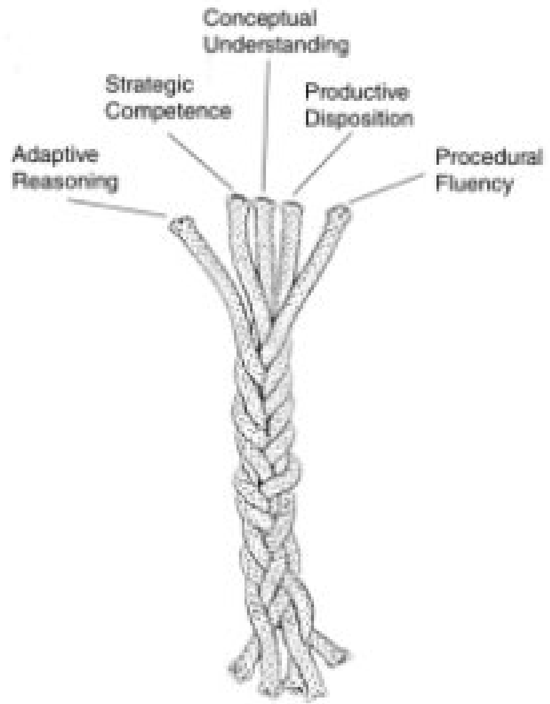
A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

(PSSM,2000)

Calls for Coherence and Progression

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

The most important observation we make about these five strands is that they are interwoven and interdependent. This observation has implications for how students acquire mathematical proficiency, how teachers develop that proficiency in their students, and how teachers are educated to achieve that goal.



Intertwined Strands of Proficiency

(Adding it Up, 2001)

Calls for Coherence and Progression

"The Common Core State Standards in mathematics were built on progressions…informed both by research on children's cognitive development and by the logical structure of mathematics."

Progression Document Introduction
http://ime.math.arizona.edu/progressions/

Calls for Coherence and Progression

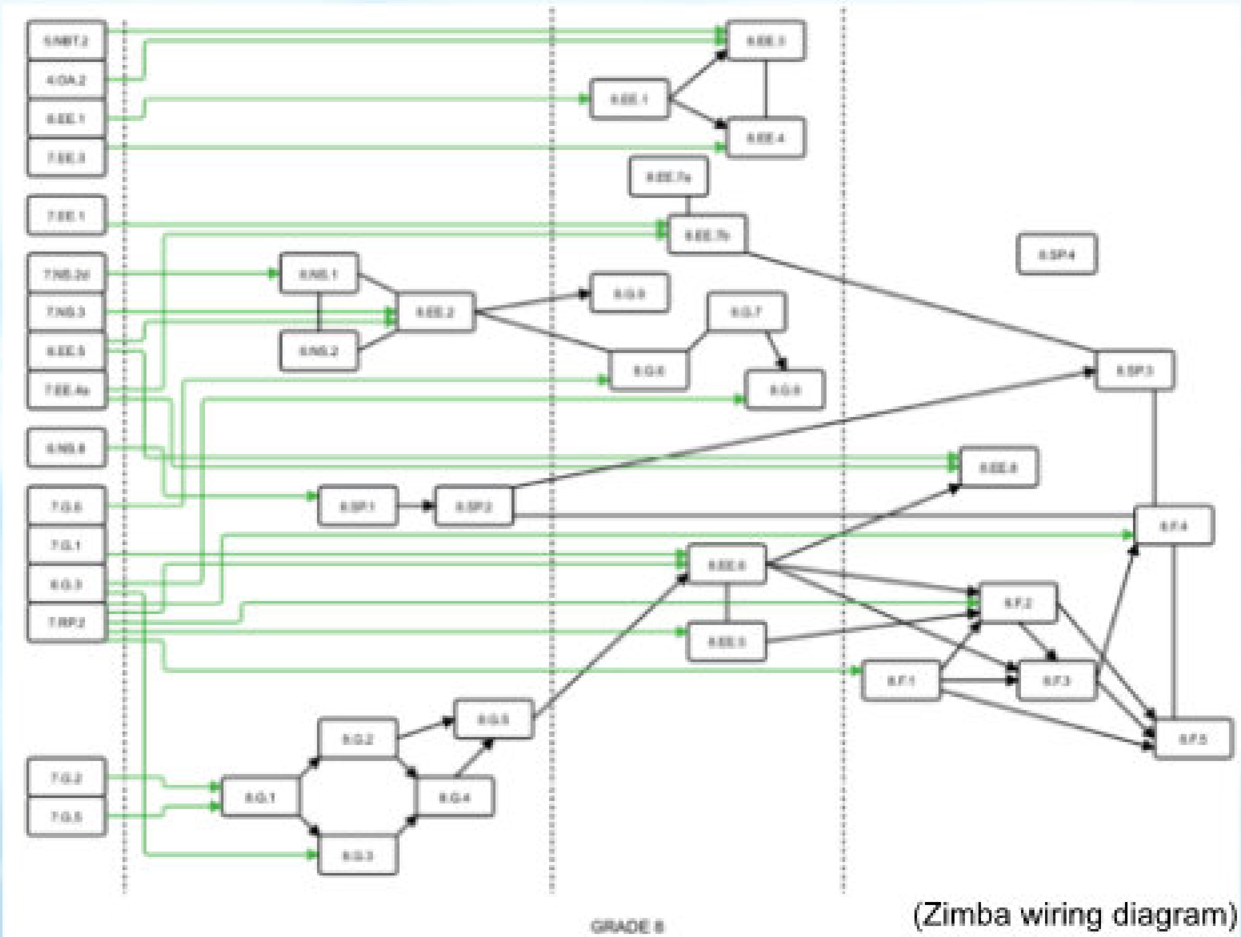
"Coherence is about making math make sense. Mathematics is not a list of disconnected tricks or mnemonics. It is an elegant subject…

(CCSSM Publisher's criteria, pg. 3)

"Fragmenting the Standards into individual standards, or individual bits of standards, erases all these relationships and produces a sum of parts that is decidedly less than the whole."

Phil Daro, Jason Zimba, Bill McCallum

Calls for Coherence and Progression



(Zimba wiring diagram)

Effective Teaching Practices

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Effective Teaching Practices

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving. " Student learning is greatest in classrooms where tasks consistently encourage high-level student thinking and reasoning and least in classrooms where tasks are routinely procedural in nature."
- Use and connect mathematical representations.
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- **Support productive struggle in learning mathematics.** Effective teaching of mathematics *consistently provides* students, individually and collectively, with opportunities and supports to engage in productive struggle as they *grapple with mathematical ideas and relationships*.
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Effective Teaching Practices

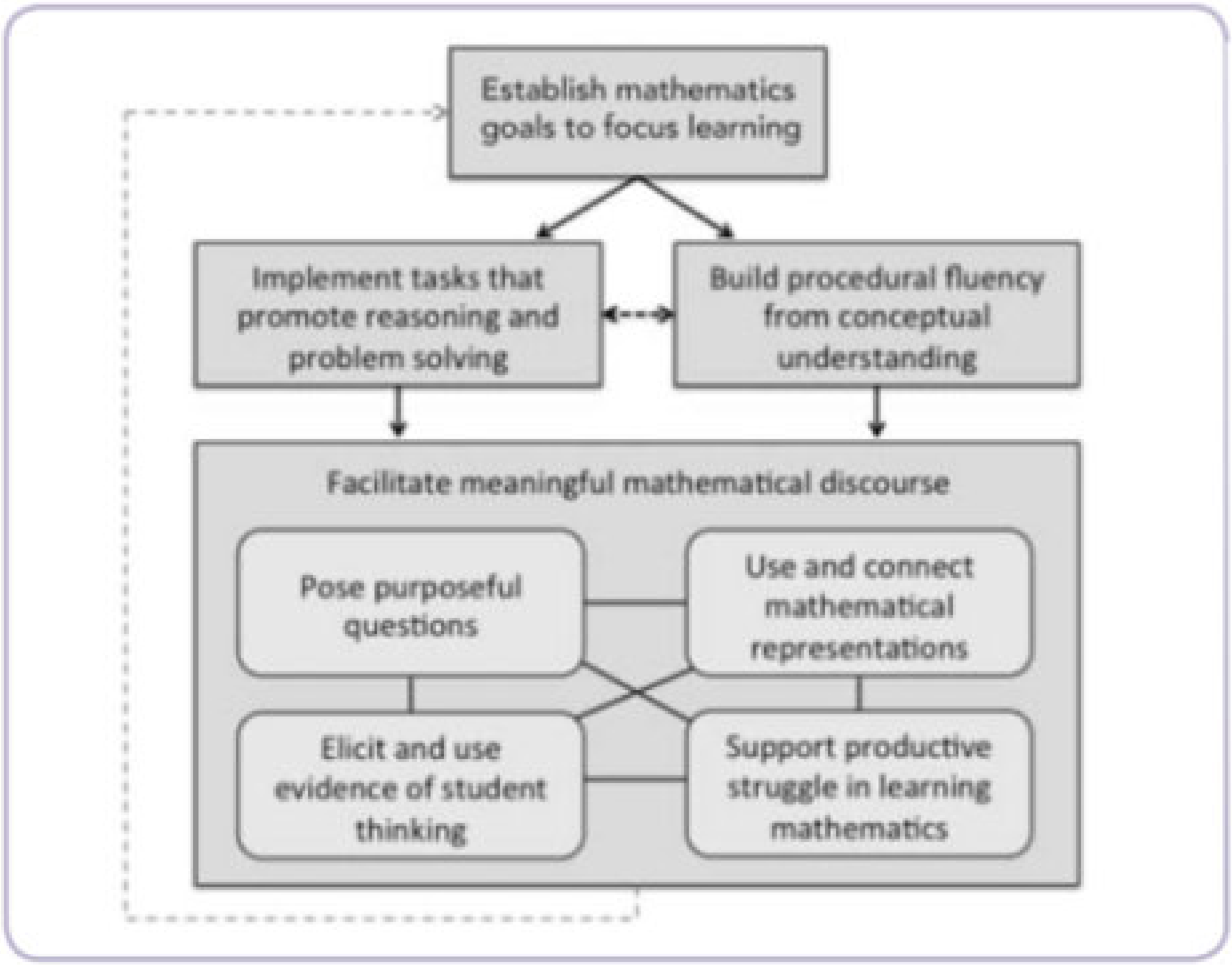

Fig. 10.1. A framework for mathematics teaching that highlights the relationships between and among the eight effective teaching practices

Taking Action 9-12 (NCTM, 2017)

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Effective Teaching Practices

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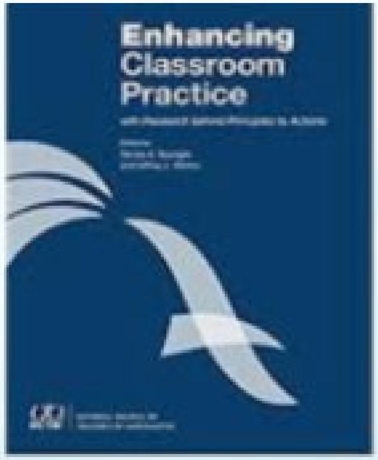
...t effective teaching practices

Taking Action 9-12 (NCTM, 2017)

Enhancing Classroom Practice

“ Tasks … gain more traction when used within sequences of tasks that develop students’ understanding of larger mathematical ideas or processes.”



Boston, Madler & Cutone, “Implementing Tasks That Promote Reasoning and Problem Solving.” Enhancing Classroom Practice with Research Behind Principles to Actions. Reston: NCTM, 2017. 13-26)

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Progressions and Coherence Facilitate engagement in all of the Practices!

Considering progressions increases my hope for improvement.

There is a need for a framework

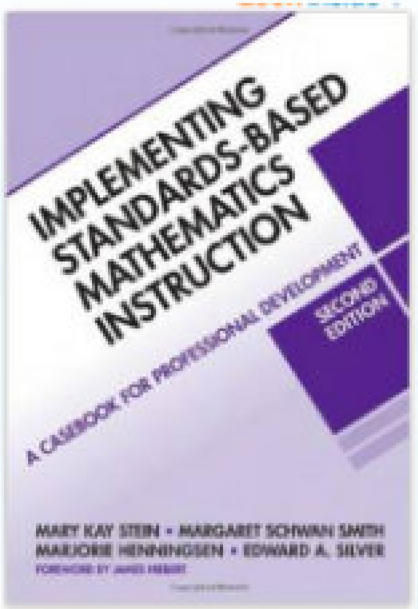
- Chazan and Ball (1999), argue that **educators are often left "with no framework for the kinds of specific, constructive pedagogical moves that teachers might make."**
- Stein et al. (2008) refer to a ***first generation*** of instructional reform from which "many teachers got the impression that in order for discussions to be focused on student thinking, they must avoid providing any substantive guidance at all," and they refer to a ***second generation*** of instructional reform "that re-asserts the critical role of the teacher in guiding mathematical discussions."

There are some excellent frameworks

- Levels of Cognitive Demand

There are some excellent frameworks

Lower-level demands	Higher-level demands
Memorization • Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory • Cannot be solved by using procedures, because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced	Procedures with connections • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • Suggest, explicitly or implicitly, pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts • Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.
Procedures without connections • Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done or how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used • Are focused on producing correct answers instead of on developing mathematical understanding • Require no explanations or explanations that focus solely on describing the procedure that was used	Doing mathematics • Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example • Require students to explore and understand the nature of mathematical concepts, processes, or relationships • Demand self-monitoring or self-regulation of one's own cognitive processes • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions • Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required



Purple Book
(NCTM, 2000, 2009)

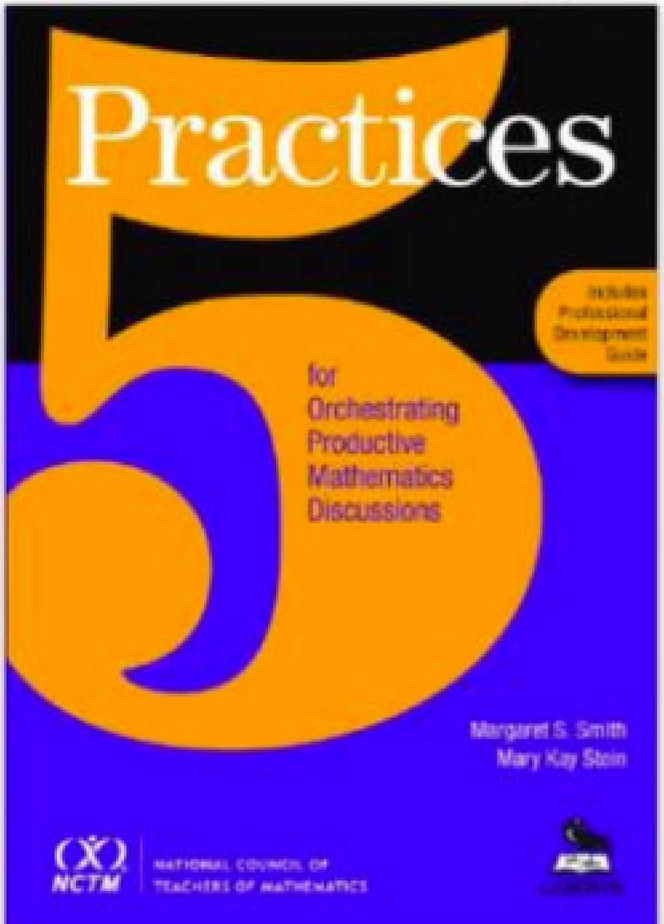
There are some excellent frameworks

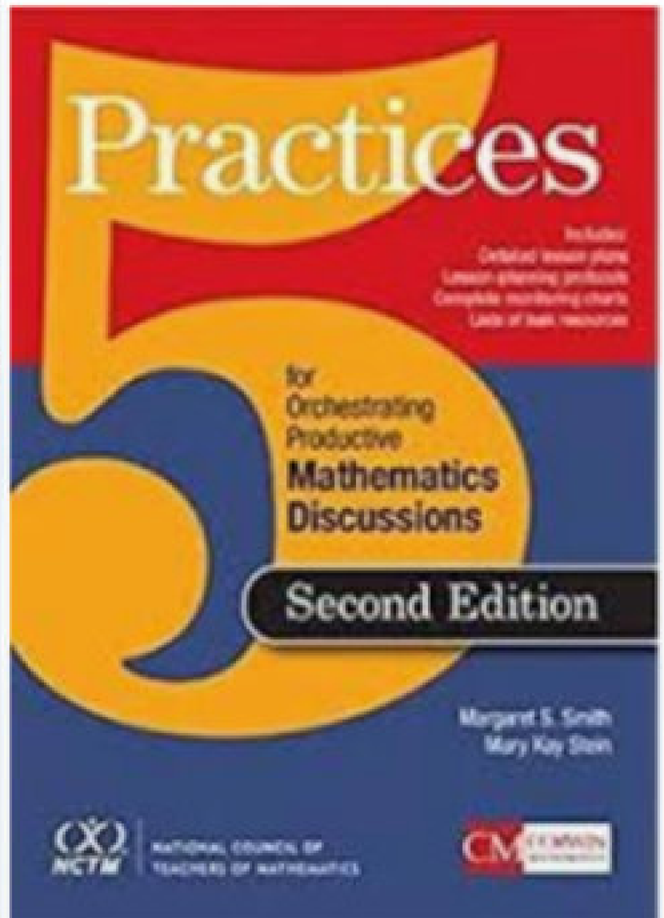
- Levels of Cognitive Demand
- Launch, Explore, Summarize models of instruction.

There are some excellent frameworks

- Levels of Cognitive Demand
- Launch, Explore, Summarize models of instruction.
- 5 Practices for Orchestrating Productive Mathematics Discussions

There are some excellent frameworks





(NCTM, 2011, 2018)

Putting it into Practice!

There are some excellent frameworks

- Levels of Cognitive Demand
- Launch, Explore, Summarize models of instruction.
- 5 Practices for Orchestrating Productive Mathematics Discussions
- And more…

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- These are Great!

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- Levels of Cognitive Demand
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- 5 Practices for Orchestrating Productive Mathematics Discussions
- And more…

- These are Great!
- All have supported improvements in pedagogy

However, focus at the task level!

Calls for Coherence and Progressions

- The CCSSM publisher's criteria, Principles to Actions, Catalyzing Change and others, wouldn't urge us to do more if we had arrived.
- We need a sustained, persistent press for student thinking, development of conceptual understanding and procedural fluency, supported productive struggle that occurs on a <u>daily basis</u>.
- The effort to implement a ***task*** needs to lead to the implementation of a ***progression of tasks*** and a ***curriculum*** that is coherent, rigorous and focused.

Calls for Coherence and Progressions

***NCTM Curriculum Principle*:**

A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

(PSSM,2000)

8 and 8
8 for teachers related to teaching
8 for students related to doing mathematics

Standards for Mathematical Practice

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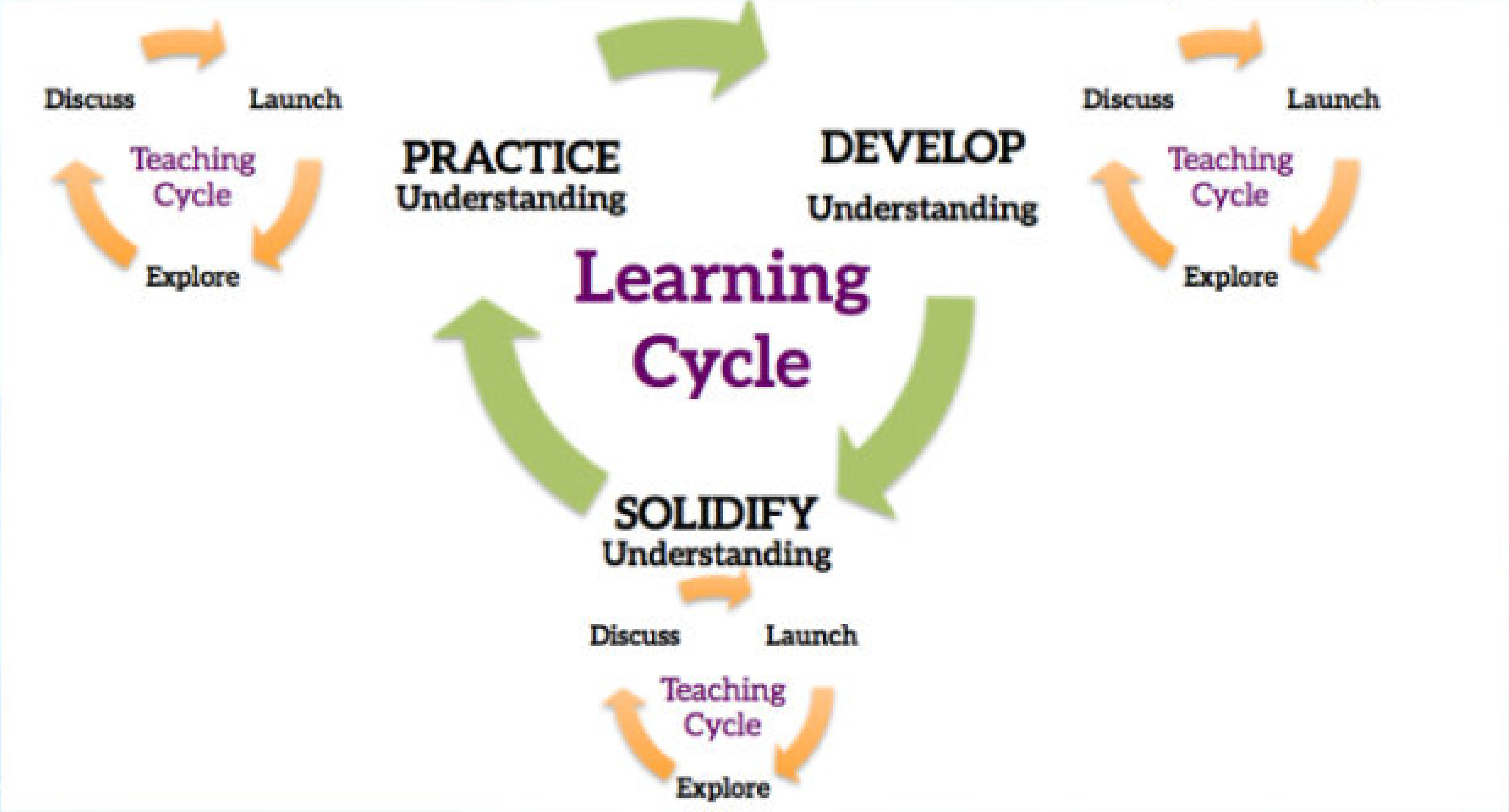
Seems sensible to have a framework that connects the 8 with the 8.

And at the same time promotes progression!

Comprehensive Mathematics Instruction Framework (CMI)



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CMI Framework Supporting Practice

When it comes to all of the practices we have to consider, what advantage is there to a framework containing cycles?

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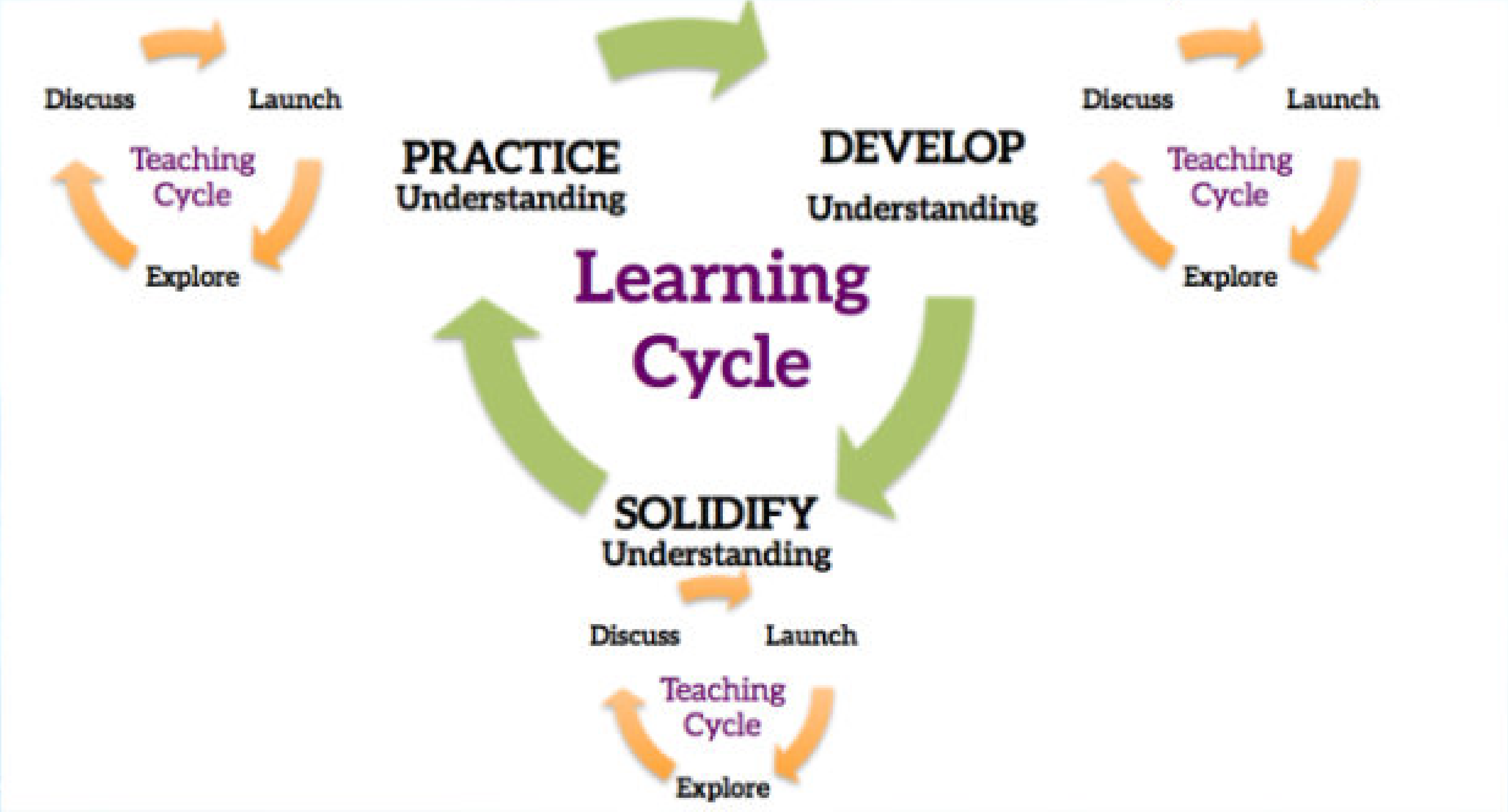
Comprehensive Mathematics Instruction Framework (CMI)



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Supporting Task Implementation

Determine Goals &
Select a task

Anticipate Student Responses
Plan questions

Launch

Monitor Student work &
Thinking

Explore

Select student work to be used
to accomplish goals

Sequence work to produce
Meaningful discussion

Discuss

Connect strategies, ideas &
Representations to achieve goals

5 Practices for Orchestrating Discussions

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Supporting Task Implementation

Facilitate Meaningful
Mathematics Discourse

Establish Mathematical Goals
to Focus Learning

Discuss

Launch

Implement Tasks That Promote
Reasoning and Problem Solving

Use and Connect Mathematical
Representations

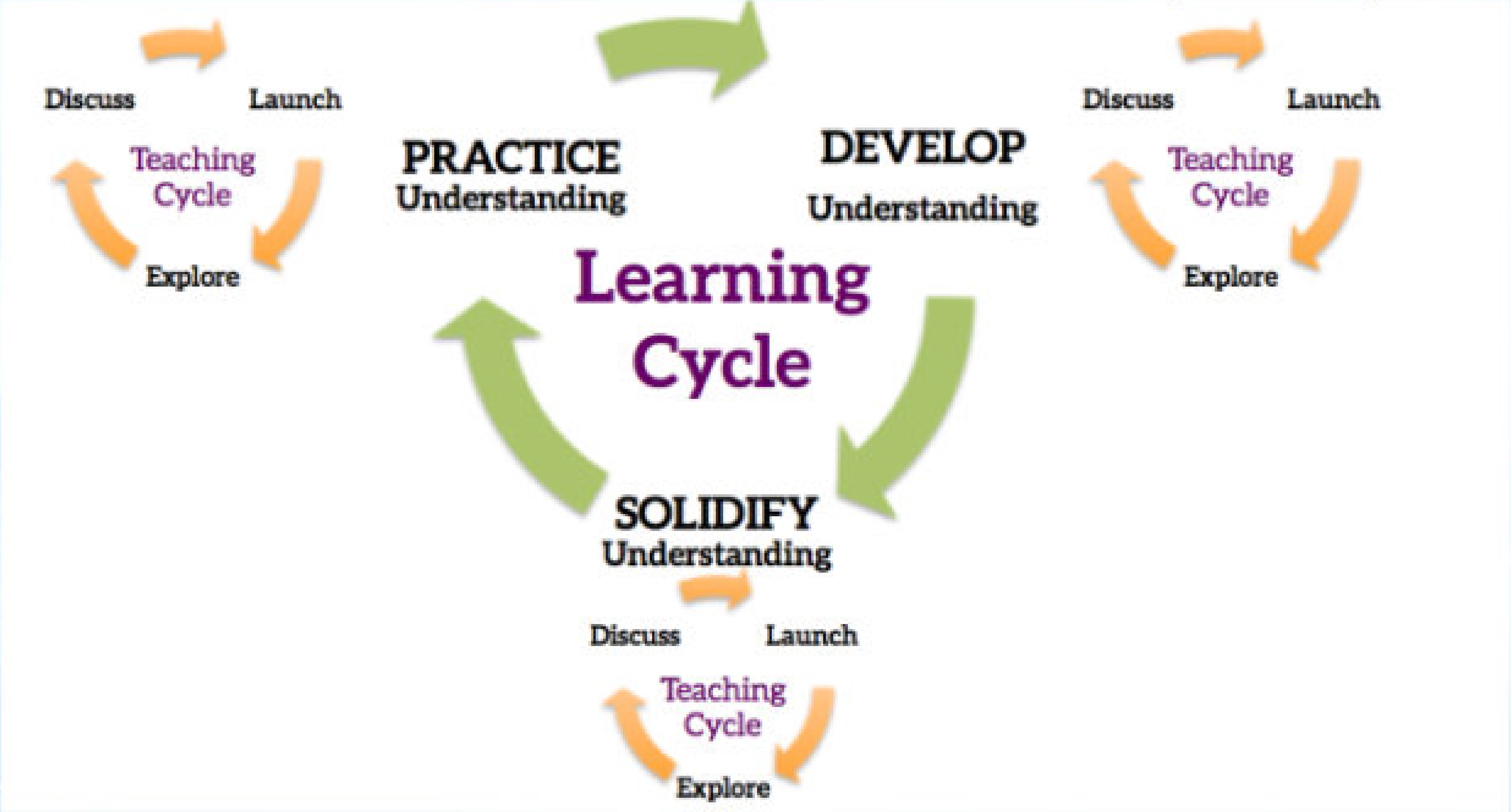
Elicit and Use Evidence of
Student Thinking

Explore

Pose Purposeful Questions

Support Productive Struggle
In Learning Mathematics

Comprehensive Mathematics Instruction Framework (CMI)



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Supporting Coherence & Progressions

- ***Develop Understanding*** tasks surface student thinking
- ***Solidify Understanding*** tasks examine and extend
- ***Practice Understanding*** tasks build fluency

Develop Understanding

- Low threshold, high ceiling (easyentry, but extendable for all learners)
- Contextualized (problematic story context, diagrams, symbols)
- Multiple pathways to solutions or multiple solutions
- Surfacestudent thinking (misconceptionsandcorrect thinking)
- Purposeful selection of the vocabulary, numbers, etc. to reveal rather than obscurethe mathematics
- Introduce a number of representations
- constructing viable arguments and critiquing the reasoning of others

Solidify Understanding

– Task context, scaffolding questions and constraints **focus students' attention** on:
  - looking for patterns and making use of structure
  - looking for repeated reasoning and expressing regularities as generalized methods
  - attending to precision in language and use of symbols
  - constructing viable arguments and critiquing the reasoning of others
  - using representations and tools strategically

  for the purpose of developing deeper levels of understanding of mathematical ideas, strategies, and/or representations

Practice Understanding

- Practice tasks focused on refining understanding
  - Task allows student to use reasoning habits to **contextualize** (symbolic to real-world) and **decontextualize** (real-world to symbolic) problems and situations.
  - Tasks involve sufficient complexity to refine mathematical thinking **beyond rote memorization**
  - The task requires a **high level of cognitive demand** because students are required to draw upon multiple concepts and procedures, make use of structure and recognize complex relationships among facts, definitions, rules, formulas and/or models

Practice Understanding

- Practice tasks focused on acquiring fluency
  - Task involves either reproducing previously learned facts, definitions, rules, formulas or models; OR drawing upon previously learned facts, definitions, rules, formulas or models; OR committing facts, definitions, rules, formulas or models to memory
  - An appropriate vehicle of practice is selected (e.g., routines, games, worksheets, etc.) which allows for reproducing, drawing upon, or committing to memory previously examined mathematics
  - Task focuses on a **broad definition of fluency**: accuracy, efficiency, flexibility, automaticity

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6.1 Photocopy Faux Pas

A Develop Understanding Task

Burnell has a new job at a copy center helping people use the photocopy machines. Burnell thinks he knows everything about making photocopies, and so he didn't complete his assignment to read the training manual.

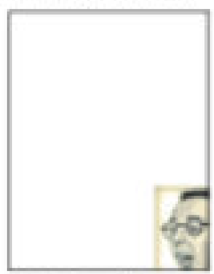
Mr. and Mrs. Donahue are making a scrapbook for Mr. Donahue's grandfather's 75th birthday party, and they want to enlarge a sketch of their grandfather which was drawn when he was in World War II. They have purchased some very expensive scrapbook paper, and they would like this image to be centered on the page. Because they are unfamiliar with the process of enlarging an image, they have come to Burnell for help.

"We would like to make a copy of this image that is twice as big, and centered in the middle of this very expensive scrapbook paper," Mrs. Donahue says. "Can you help us with that?"

"Certainly," says Burnell. "Glad to be of service."

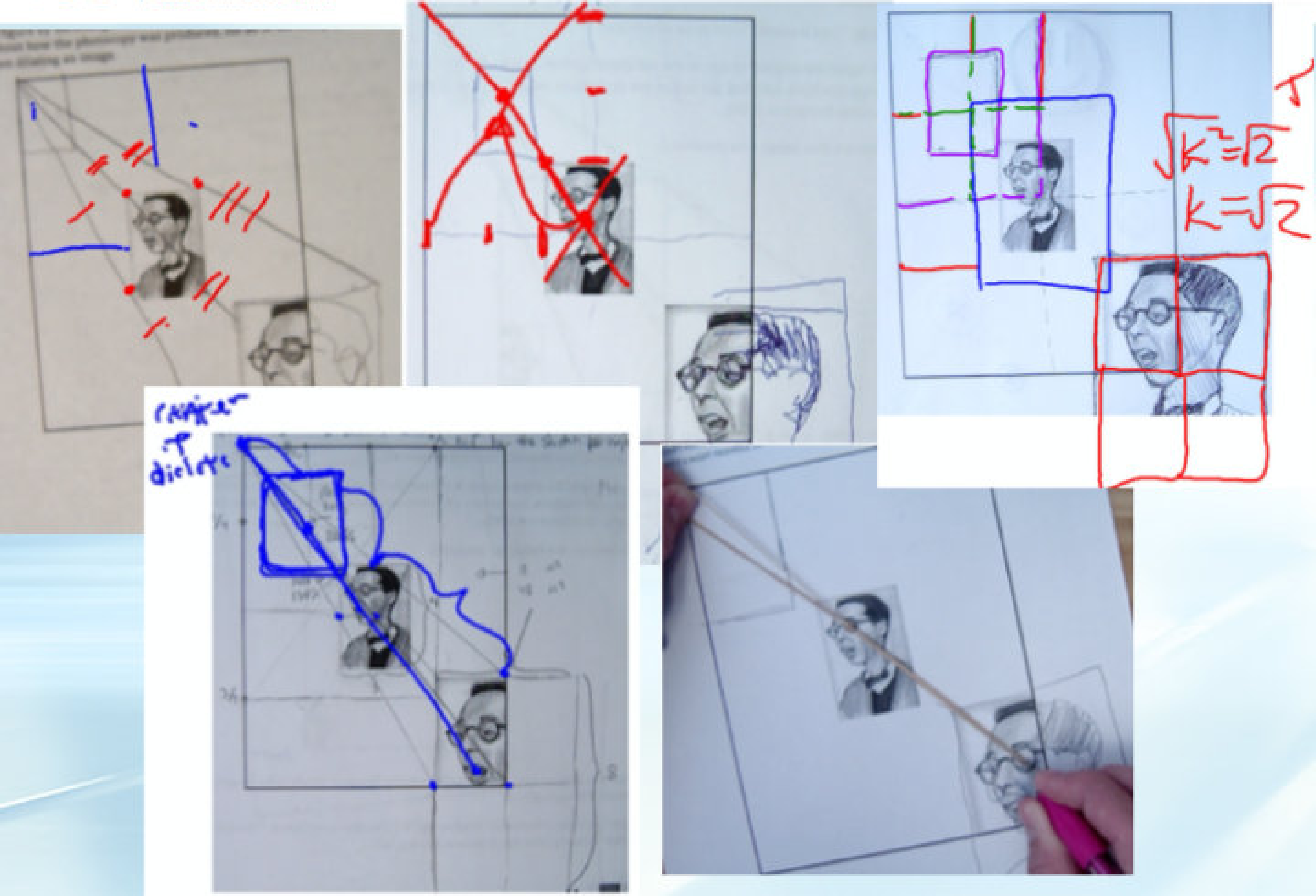
Burnell taped the original image in the middle of a white piece of paper, placed it on the glass of the photocopy machine, inserted the expensive scrapbook paper into the paper tray, and set the enlargement feature at 200%.

In a moment, this image was produced.



"You've ruined our expensive paper," cried Mrs. Donahue. "Much of the image is off the paper instead of being centered."

"And this image is more than twice as big," Mr. Donahue complained. "One fourth of grandpa's picture is taking up as much space as the original."


$\sqrt{k^2}=\sqrt{2}$
$k=\sqrt{2}$

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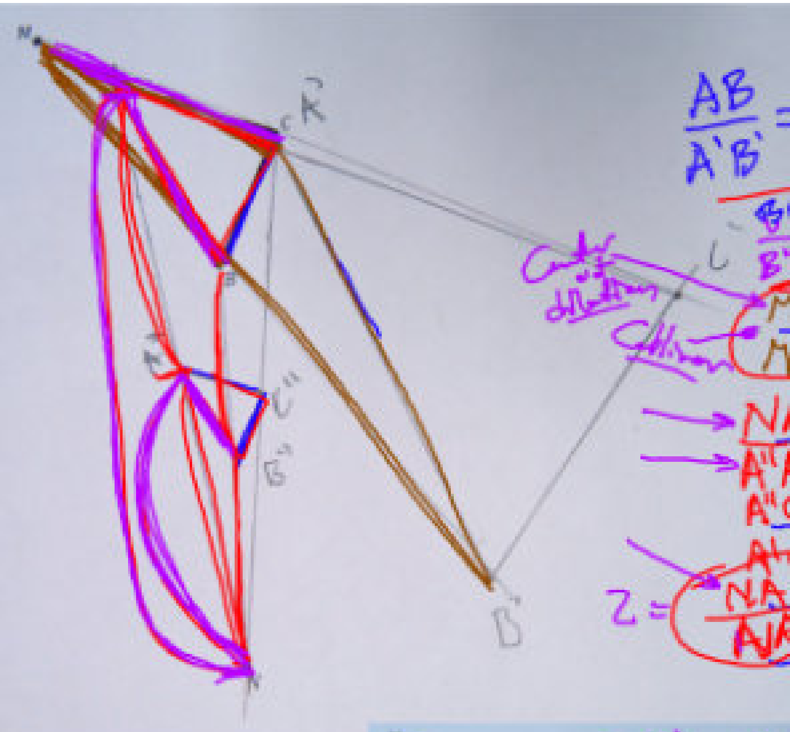
6.2 Triangle Dilations

A Solidify Understanding Task

1. Given ΔABC, use point M as the center of a dilation to locate the vertices of a triangle that has side lengths that are three times longer than the sides of ΔABC.

2. Now use point N as the center of a dilation to locate the vertices of a triangle that has side lengths that are one-half the length of the sides of ΔABC.

mathematics vision project



$$\frac{AB}{A'B'} = \frac{BC}{B'C'}$$ Triangle

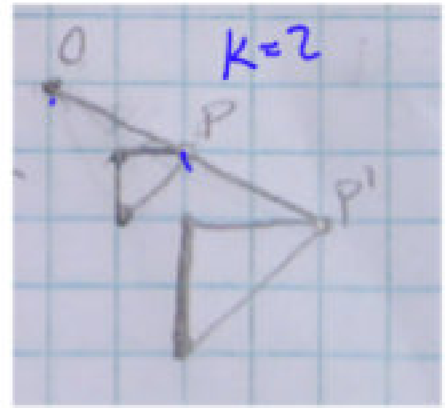
$$\frac{BC}{B''C''} = \frac{AB}{A''B''}$$

$$\frac{MA}{MA'} = \frac{MB}{MB'}$$ Center Dilation

$$\frac{NA''}{A''A} = \frac{NB''}{B''B}$$

$$\frac{A''C''}{A'C'} = \frac{B''C''}{B'C'}$$

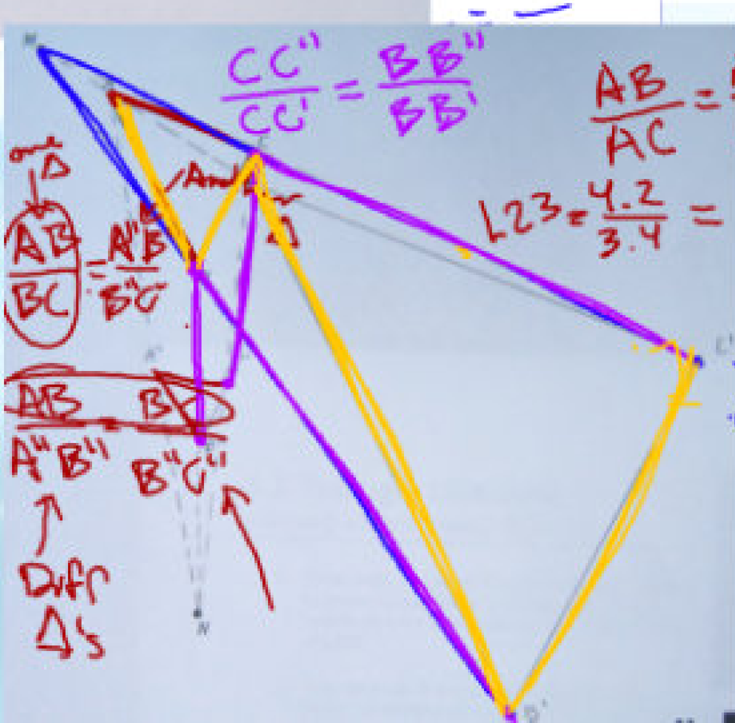
$$2 = \frac{NA}{NA''} = \frac{AB}{A''B''} = 2$$



O P' P

K = less than 1.

P' closer to O



$$\frac{CC''}{CC'} = \frac{BB''}{BB'}$$

$$\frac{AB}{AC} = \frac{A''B''}{A''C''}$$

$$1.23 = \frac{4.2}{3.4} = \frac{2.1}{1.7} = 1.23$$

$$\frac{MB}{MB'} = \frac{MC}{MC'}$$

one Δ

$$\frac{AB}{BC} = \frac{A''B''}{B''C''}$$

Another Δ

$$\frac{AB}{A''B''} \quad \frac{BC}{B''C''}$$

Diff Δ's

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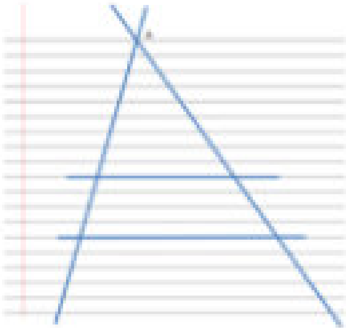
6. 4 Cut by a Transversal

A Solidify Understanding Task

Draw two intersecting transversals on a sheet of lined paper, as in the following diagram. Label the point of intersection of the transversals *A*. Select any two of the horizontal lines to form the third side of two different triangles.

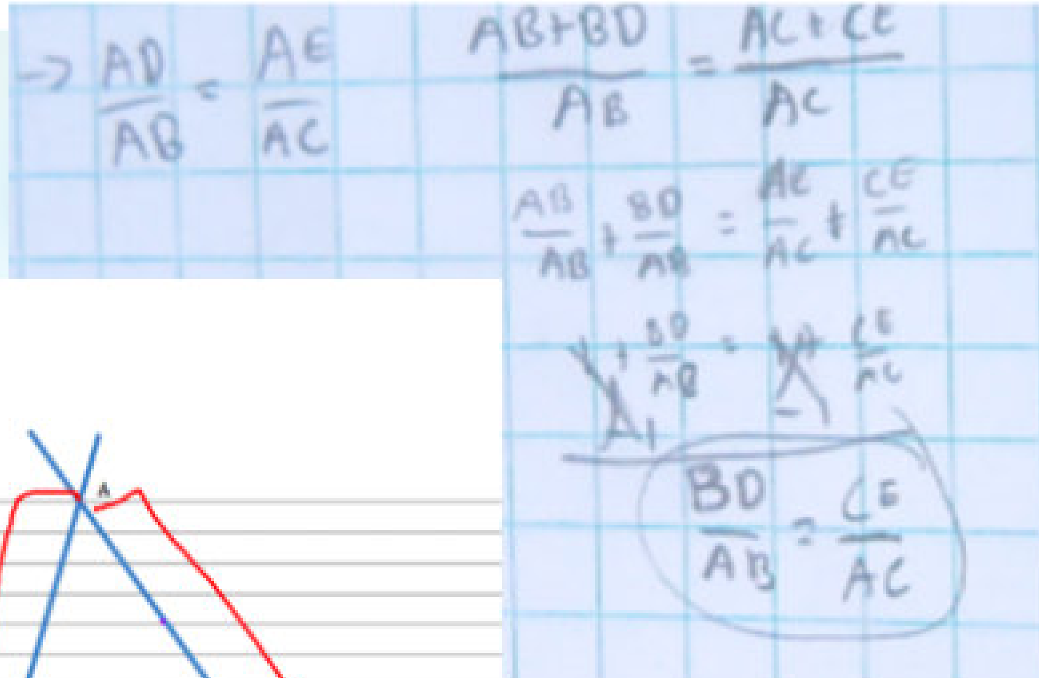


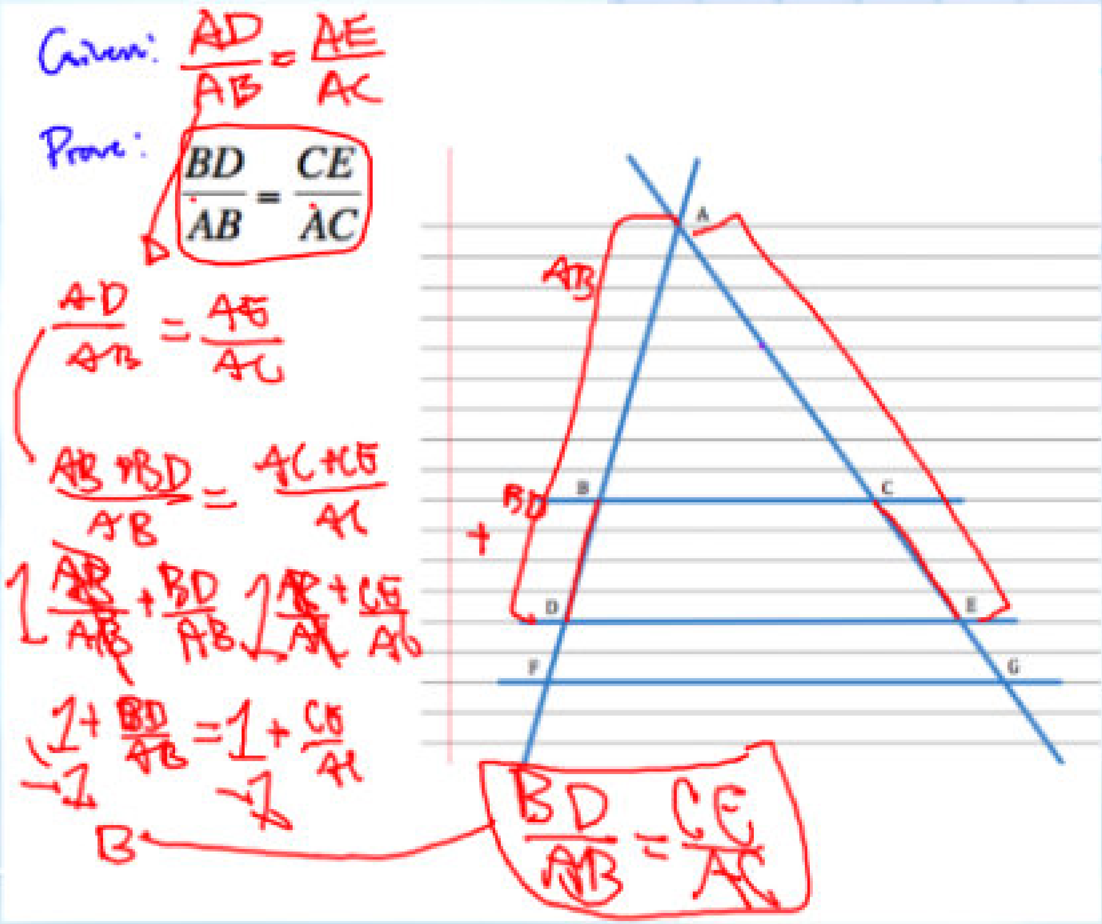
1. What convinces you that the two triangles formed by the transversals and the horizontal lines are similar?

2. Label the vertices of the triangles. Write some proportionality statements about the sides of the triangles and then verify the proportionality statements by measuring the sides of the triangles.

3. Select a third horizontal line segment to form a third triangle that is similar to the other two. Write some additional proportionality statements and verify them with measurements.

mathematics vision project


$\rightarrow \frac{AD}{AB} = \frac{AE}{AC}$
$\frac{AB+BD}{AB} = \frac{AC+CE}{AC}$
$\frac{AB}{AB} + \frac{BD}{AB} = \frac{AC}{AC} + \frac{CE}{AC}$
$\frac{BD}{AB} = \frac{CE}{AC}$

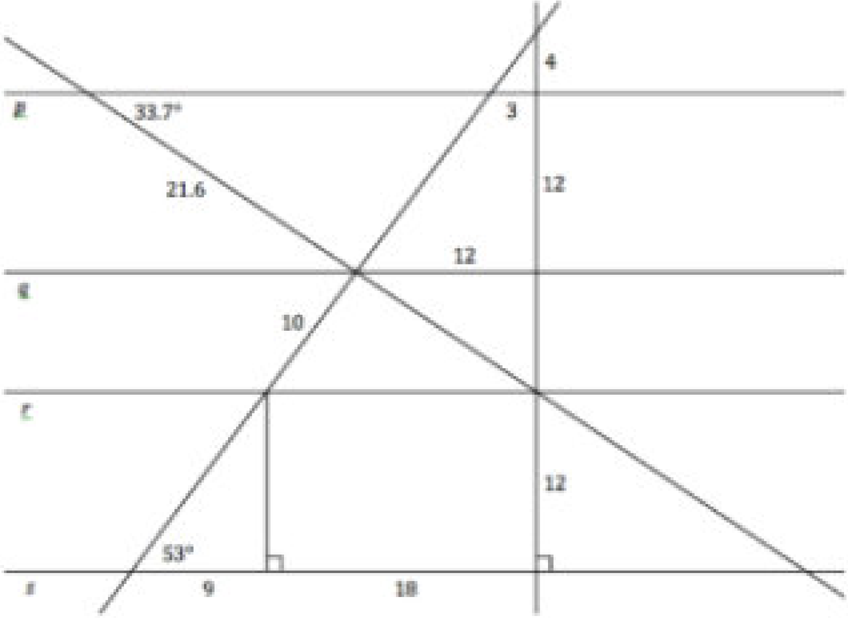

Given: $\frac{AD}{AB} = \frac{AE}{AC}$
Prove: $\frac{BD}{AB} = \frac{CE}{AC}$
$\frac{AD}{AB} = \frac{AE}{AC}$
$\frac{AB+BD}{AB} = \frac{AC+CE}{AC}$
$1\frac{AB}{AB} + \frac{BD}{AB}$ $1\frac{AC}{AC} + \frac{CE}{AC}$
$1 + \frac{BD}{AB} = 1 + \frac{CE}{AC}$
-1 -1
$\frac{BD}{AB} = \frac{CE}{AC}$
A
B
C
D
E
F
G
AB
BD
+

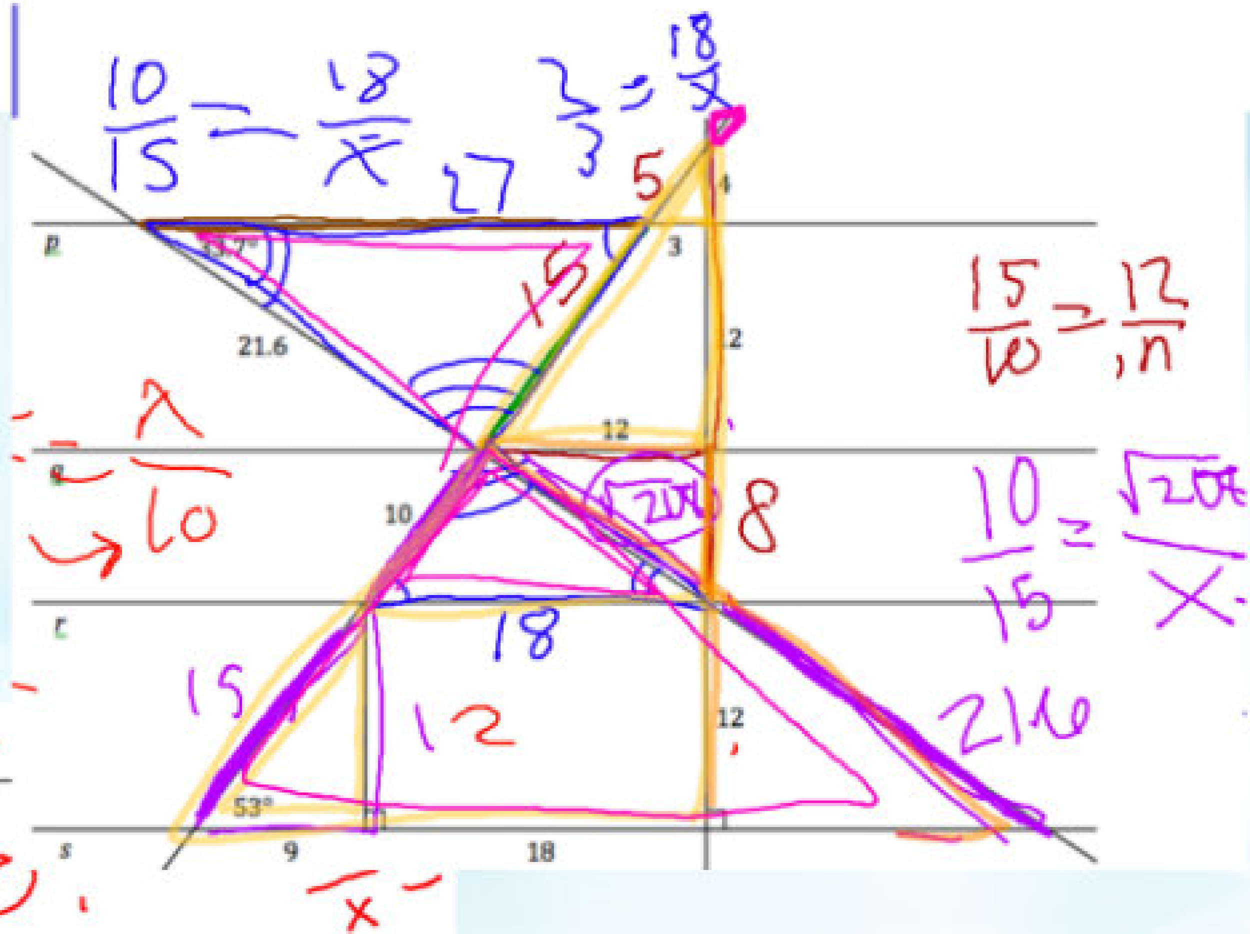
6.5 Measured Reasoning

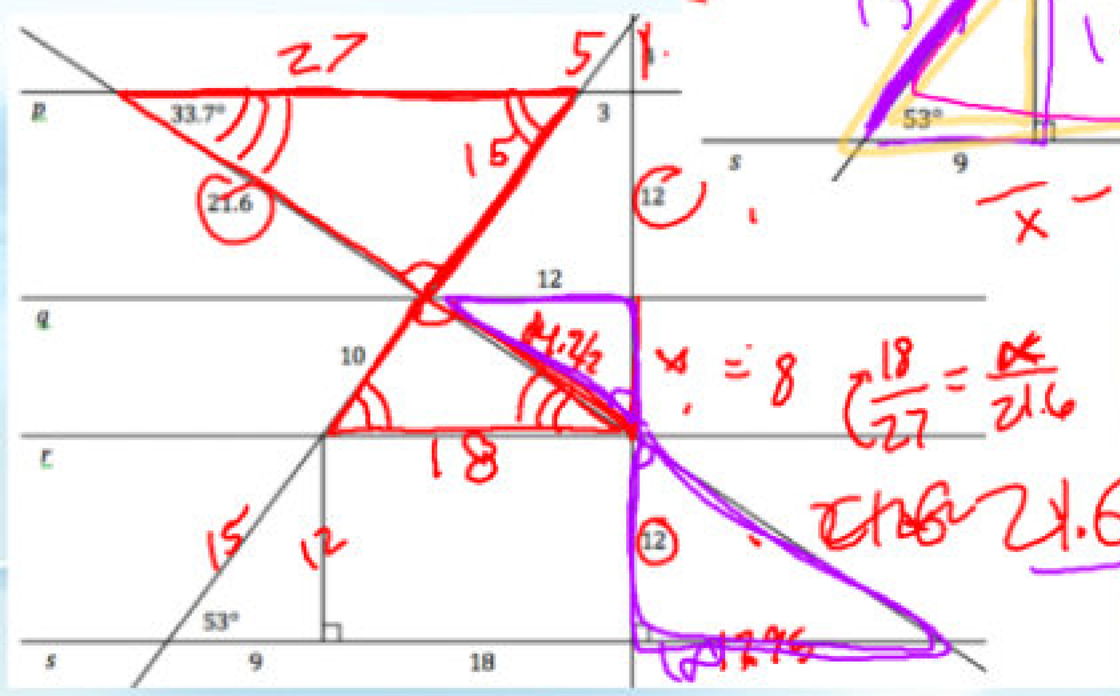
A Practice Understanding Task

Find the measures of all missing sides and angles by using geometric reasoning, not rulers and protractors. If you think a measurement is impossible to find, identify what information you are missing.

Lines *p*, *q*, *r*, and *s* are all parallel.




p
q
r
s
33.7°
21.6
3
12
10
18
53°
9


p
q
r
s
33.7°
21.6
3
12
10
18
53°
9

Supporting Coherence & Progressions

Attending to Precision

PRACTICE
Understanding

DEVELOP
Understanding

Look for and express regularity
In repeated reasoning

Learning
Cycle

Make Sense of problems and
Persevere in solving them

Construct viable arguments and
Critique the reasoning of others

Reason Abstractly and
Quantitatively

SOLIDIFY
Understanding

Look for and make
Use of structure

Use Appropriate tools strategically

Model with Mathematics

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Where do they come from?
Who are they for?
Which came first?
How do they relate?
What to do about it?

Standards for Mathematical Practice

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

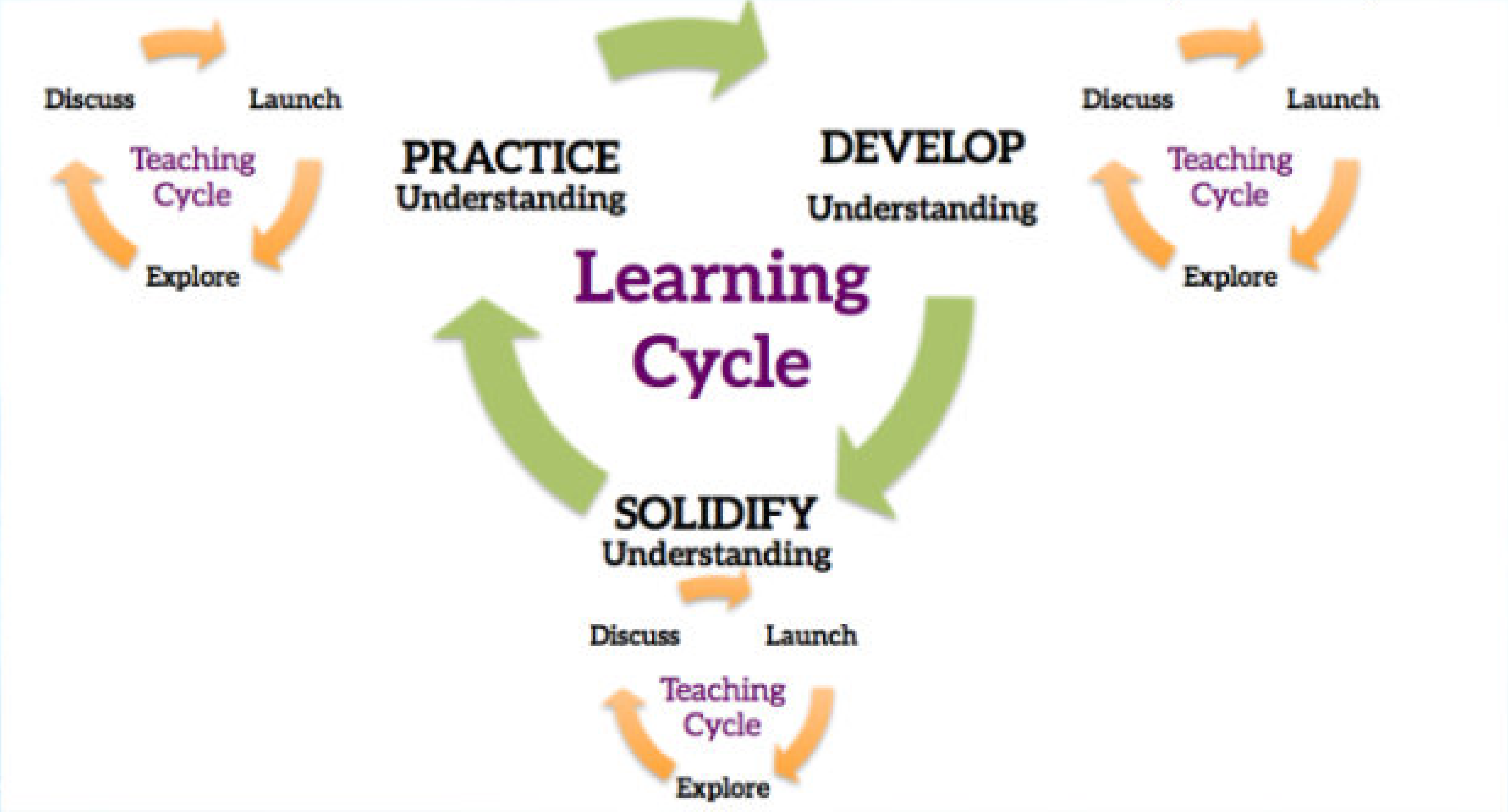
Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Given a specific lens of practices
Identify from the other set of practices 3 that are most critical. Rank them in order.

8 x 8: Teacher by Student Practices

	Make sense of problems and persevere in solving them	Reason abstractly and quantitatively	Construct viable arguments and critique the reasoning of others	Model with mathematics	Use appropriate tools strategically	Attend to precision	Look for and make use of structure	Look for and express regularity in repeated reasoning
Establish mathematics goals to focus learning								
Implement tasks that promote reasoning and problem solving.								
Use and connect mathematical representations.								
Facilitate meaningful mathematical discourse.								
Pose purposeful questions.								
Build procedural fluency from conceptual understanding.								
Support productive struggle in learning mathematics.								
Elicit and use evidence of student thinking.								

Comprehensive Mathematics Instruction Framework (CMI)



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