Supporting Students' Productive Struggle

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Overarching Questions

- What is productive struggle and why does it matter?
- How does engaging in productive struggle enable us to support students?

Supporting Students' Productive Struggle

Part 1: Common Experiences and Why it Matters?

Driving Questions

- What is productive struggle?
- Why is productive struggle important to students' mathematics learning?
- How do we support productive struggle for each and every student?

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Group Work

What is Productive Struggle?

- In the breakout rooms:
  - Discuss what productive struggle means to *you*
  - Discuss what productive struggle means to *your students*
  - Be Ready to Share out

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Defining Productive Struggle

The phenomenon of struggle….refers to the intellectual effort students expend to make sense of mathematical concepts that are challenging but fall within the students' reasonable capabilities.

Warshauer, H. K. (2015). Strategies to support productive struggle. *Mathematics Teaching in the Middle School*, *20*(7), 390–393.

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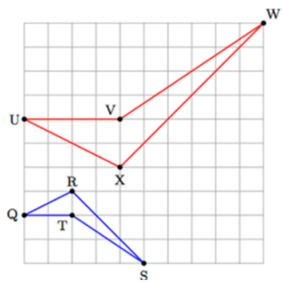
Explore the task

- Consider the pair of polygons.
- Explain whether or not the polygons are similar.
- If you finish quickly, explain a different way.



https://www.illustrativemathematics.org/content-standards/7/G/A/1/tasks/1201

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What Does it Look Like?

- Read the vignette. Consider the following:

- Write down one or two ideas that you notice
- Write down one or two wonders about students' experiences in these two experiences

- How are the learning experiences different? The same?

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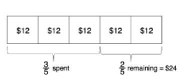
Ms. Flahive and Ms. Ramirez teach fifth grade and plan their lessons collaboratively. Their current instructional unit focuses on fractions. They have selected the Shopping Trip task shown below because they think it will be accessible to their students yet provoke some struggle and challenge, since a solution pathway is not straightforward. The mathematics goal for students is to draw on and apply their understanding of how to build non-unit fractions from unit fractions and to use visual representations to solve a multi-step word problem:

> **Shopping Trip Task**
>
> Joseph went to the mall with his friends to spend the money that he had received for his birthday. When he got home, he had $24 remaining. He had spent 3/5 of his birthday money at the mall on video games and food. How much money did he spend? How much money had he received for his birthday?

When Ms. Flahive and Ms. Ramirez present the problem in their classrooms, both teachers see students struggling to get started. Some students in both classrooms immediately raise their hands, saying, "I don't get it," or "I don't know what to do."

Ms. Flahive is very directive in her response to her students. She tells them to draw a rectangle and shows them how to divide it into fifths to represent what Joseph had spent and what he had left. She then guides her students step by step until they have labeled each one-fifth of the rectangle as worth $12, as shown below. Finally, she tells the students to use the information in the diagram to figure out the answers to the questions.



Ms. Ramirez approaches her students' struggles very differently. After she sees them struggling, she has them stop working on the problem and asks all the students to write down two things that they know about the problem and one thing that they wish they knew because it would help them make progress in solving the problem. Then Ms. Ramirez initiates a short class discussion in which several ideas are offered for what to do next. Suggestions include drawing a tape diagram or number line showing fifths, or just picking a number, such as $50 and proceeding through trial and error. Ms. Ramirez encourages the students to consider the various ideas that have been shared as they continue working on the task.

Fig. 21. Two teachers' responses to students' struggles to solve a multi-step word problem involving fractions

Group Work

Why Does it Matter?

- Share your ideas with a small group.
- Determine which teacher has created an environment where students can explore, think critically, and grapple with mathematics.
- What are the characteristics of this environment?

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Creating an Environment

Help students realize struggle will take time.

They may not be successful – especially in the early stages of solving a problem.

Help students identify what they can do when they are stuck.

Foster a growth mindset.

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Student Expectations (Roy, Bush, Hodges, & Safi, 2017)

- participate, even those who do not raise their hands;
- explain and justify their thinking;
- restate a classmate's reasoning;
- make sense of another classmate's reasoning; &
- ask a question if they are not sure that they understand.

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Facets of Productive Struggle
(NCTM, 2014)

Expectations for students	Teacher actions to support students	Classroom-based indicators of success
Most tasks that promote reasoning and problem solving take time to solve, and frustration may occur, but perseverance in the face of initial difficulty is important.	Use tasks that promote reasoning and problem solving; explicitly encourage students to persevere; find ways to support students without removing all the challenges in a task.	Students are engaged in the tasks and do not give up. The teacher supports students when they are "stuck" but does so in a way that keeps the thinking and reasoning at a high level.
Correct solutions are important, but so is being able to explain and discuss how one thought about and solved particular tasks.	Ask students to explain and justify how they solved a task. Value the quality of the explanation as much as the final solution.	Students explain how they solved a task and provide mathematical justifications for their reasoning.
Everyone has a responsibility and an obligation to make sense of mathematics by asking questions of peers and the teacher when he or she does not understand.	Give students the opportunity to discuss and determine the validity and appropriateness of strategies and solutions.	Students question and critique the reasoning of their peers and reflect on their own understanding.
Diagrams, sketches, and hands-on materials are important tools to use in making sense of tasks.	Give students access to tools that will support their thinking processes.	Students are able to use tools to solve tasks that they cannot solve without them.
Communicating about one's thinking during a task makes it possible for others to help that person make progress on the task.	Ask students to explain their thinking and pose questions that are based on students' reasoning, rather than on the way that the teacher is thinking about the task.	Students explain their thinking about a task to their peers and the teacher. The teacher asks probing questions based on the students' thinking.

Fig. 20. Redefining student and teacher success. Adapted from Smith (2000, p. 382).

Group Work

Wrap – Up

- Revisit your definition of productive struggle; what would you add to it?

Food for Thought:

- How do learning goals factor into productive struggle?

Supporting Students' Productive Struggle

Part 2: Laying the Foundation

Driving Questions

- What does productive struggle look like in a classroom?
- How do I establish mathematical goals?
- How does valuing students' identities empower students to engage in productive struggle?

Effective Mathematics Teaching Practices (NCTM 2014)

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem-solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Setting Goals

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Principles to Action: Ensuring Mathematical Success for All (NCTM 2014, p.12)

Setting Goals

Analyzing Teaching and Learning 2.1

Comparing Goal Statements

Review goal statements A, B, and C (shown below), written for a lesson on proportional relationships, and consider the following:

- How are they the same, and how are they different?
- How might the differences matter?

Goal A: Students will learn the procedure (cross multiplication) for finding the missing value in a proportional situation.

Goal B: Students will be able to (SWBAT) use cross multiplication to find the missing value in problems in which the quantities being compared are in a proportional relationship.

Goal C: Students will recognize that a proportion consists of two ratios that are equivalent to each other (e.g., $\frac{a}{b}=\frac{ax}{bx}$) and that missing values in the proportion can be found by determining the scale factor x that relates the two ratios or by determining the unit rate—the relationship (multiplicative) between a and b and recognizing that ax and bx must have the same relationship as a and b.

Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8; *Smith, M.S., Steele, M., Raith, L., NCTM, 2017.*

Three Goals

- What is the same similar among the goals?
- What is different among the three goals?

Analyzing Teaching and Learning 2.1

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Three Goals

- Which goal(s) seems more likely to enable students to solve a wider range of problems?
- How would you assess student success with each goal?

Analyzing Teaching and Learning 2.1

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Connection to Standards

7.RP.A.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships

c. Represent proportional relationships by equations

Fig. 2.1. A seventh-grade proportional reasoning standard from the Common Core State Standards (CCSS) for Mathematics (NGA Center and CCSSO 2010)

Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8; Smith, M.S., Steele, M., Raith, L., NCTM, 2017.

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Establishing Effective Goals

- Goals help you select a task that focuses on the important mathematics.
- Goals help you make instructional decisions during the lesson (and about assessment).
- Goals are more than a standard or a procedure, but also include information about the conceptual understandings students will gain.
- Goals connect to broader learning trajectories.

Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8; Smith, M.S., Steele, M., Raith, L., NCTM, 2017.

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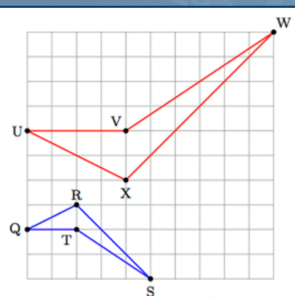
Revisiting Earlier Task

Consider the pair of polygons. Explain whether or not the polygons are similar.

If you finish quickly, explain a different way.



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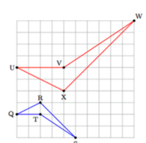
Selecting a Task – Our Goals

- Students will recognize that similar shapes have the same angle measures.
- Students will recognize corresponding segments of similar figures have equal proportional lengths (this common ratio is called the scale factor).
- Students will recognize scale factors and proportions are two ways to solve problems involving similar figures.

Debrief: The Task

How does this task meet our goals?

Why or why not?



What are some considerations or reflections following our debrief?

The task

Does this task meet our goals?

Why or why not?

- Students will recognize that similar shapes have the same angle measures.
- Students will recognize corresponding segments of similar figures have equal proportional lengths – this common ratio is called the scale factor.
- Students will recognize scale factors and proportions are two ways to solve problems involving similar figures.

Wrap – Up

- Revisit your definition of productive struggle; what about learning goals would you add to your definition?

Food for Thought:

- What do you do when you have a task in your current curricular materials that does not adequately allow for productive struggle?

Supporting Students' Productive Struggle

Part 3: Provoking and Supporting

Driving Questions

- How do we create an environment for nurturing productive struggle?
- What are the characteristics of an effective task?
- How do we adapt tasks to connect and extend student understanding beyond the learning goal?
- In what ways does intentionality in teaching practices support equitable instruction?

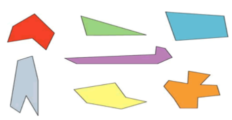
Addressing Student Struggle (NCTM 2014)

"Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students. As a result, they jump in to 'rescue' students by breaking down the task and guiding students step by step through the difficulties.

Although well intentioned, such 'rescuing' undermines the efforts of students, lowers the cognitive demand of the task, and deprives students of opportunities to engage fully in making sense of the mathematics (Reinhart 2000; Stein et al. 2009)."

What does this look like in …

- We are using a task called Amazing Amanda to see how these pieces fit together.

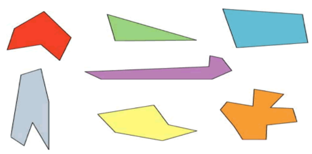


Amazing Amanda Mathematical Learning Goals

- Students can subdivide polygons into non-overlapping triangles to make generalizations.
- Students can apply previous learned mathematics to determine the sum of the interior angles of any polygon;
- Students will write equations to describe the relationship between the number of sides and the sum of the measures of the interior angles of any polygon.

Amanda claims to have an amazing talent. "Draw any polygon. Don't show it to me. Just tell me the number of sides it has and I can tell you the sum of its interior angles."

Is Amanda's claim legitimate? Does she really have an amazing gift, or is it possible for anyone to do the same thing?



Putting It All Together

- Read the mini-dialogues
- Discuss the nature of each student's struggle.
- Identify what the teacher does to help students move beyond the impasse they had reached.
- Determine whether or not the teacher supported students' productive struggle.

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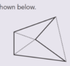
Read the Mini-Dialogues Amazing Amanda Task

Dialogue 1

A student made the drawing shown below.



T: What did you do here?

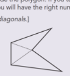
S: I drew a polygon with 5 sides.

T: Then what?

S: I divided it into triangles. And I got 4 triangles. But I don't think it is right because when I asked around, no one else had 4.

T: Your triangles can't go outside the polygon. If you take your picture and just get rid of one of your diagonals, you will have the right number of triangles.

[*Student erases one of the diagonals.*]



T: That's right. So how many triangles do you have now?

S: 3.

T: Okay. So now you just need to multiply 3 by 180 and you will be set. So now try another one using this method.

Dialogue 2

A student made the table shown below.

# of Sides	Degrees of Interior Angles
3	180
4	360
5	540
6	720
7	900
8	1,080

T: Tell me how you constructed your table.

S: I decided to try all of the polygons from 3 to 8. I knew that the 3-sided polygon—a triangle—had angles that summed to 180 degrees because we did that last week. Then I drew polygons with more sides on scrap paper. I subdivided each polygon into non-overlapping triangles. Then I counted the number of triangles in each polygon and multiplied by 180.

T: Why did you multiply by 180?

S: Because the angles of each triangle sum up to 180 so to find the sum of all the angles in a polygon you need to multiply the number of triangles in the polygon by 180.

T: So how does this help you determine the relationship between the number of sides of the polygon and the sum of measures of the interior angles?

S: I am not sure. I know that you multiply the number of triangles in the polygon by 180 like I said, so I guess I need to figure out how many triangles there are in each polygon. Maybe I will add a column to the table to keep track of this.

T: That sounds like a good plan. I will check back in with you later.

Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12; Boston, M., Dillon, F., Smith, M.S., Smith, S., NCTM, 2017.

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Read the Mini-Dialogues
Amazing Amanda Task

Dialogue 3
A student made the drawing shown below.

T: So tell me about your drawing?
S: I made a 5-sided polygon and subdivided into 5 non-overlapping triangles.
T: And then what?
S: Well, since each triangle has angles that sum to 180 degrees, I multiplied 180 by 5 and got 900. [*Student sounds unsure of herself.*]
T: So what is the problem?
S: I think it is too big. I took out my protractor and did a rough measure of the angles and I got closer to 500.
T: Nice way to check if your answer is reasonable. So let's take a closer look at your diagram. Can you show me where the angles of the triangles are?
[*Student points to the angles in each triangle.*]
T: So are all the angles you just pointed to included in the interior angles of the polygon?
S: No. All these [*points to the angles formed around the center point*] are not included in the interior angles. Oh, so somehow I need to figure out how not to count these.
T: I will leave you to figure out what you know about the angles around a point and how this can help you solve your problem. I will check in with you later.

Dialogue 4
A student can't get started.
T: What have you figured out so far?
S: Nothing. I am not sure what to do.
T: The first thing I want you to do is to draw a polygon with 4 sides.
[*Student draws a square.*]
T: Now you need to divide it into triangles, starting at one of the vertices.
[*Student divides the square into two triangles by drawing the diagonal.*]
T: Okay. So you have two triangles. What is the sum of the angles of a triangle equal to?
S: 180?
T: So if you have two triangles, what would the sum of the angles be?
S: 360?
T: Yes! So the angles of a 4-sided polygon sum up to 360 degrees. Now try a five-sided polygon and use the same method of breaking it up into triangles that we just did.

Dialogue 5
A student can't get started.
T: What have you figured out so far?
S: Nothing. I am not sure what to do.
T: Go back through your notes and review the work you did when we proved that the sum of the angles of a triangle sum to 180.

Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12; Boston, M., Dillon, F., Smith, M.S., Smith, S., NCTM, 2017.

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Group Work

Looking at the Mini-Dialogues

With your group members, discuss your answers to the questions posed for the mini-dialogues.

- When did the teacher maintain the cognitive demand of the task?
- How did the teacher support (or not support) productive struggle?

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Group Work

Mini-Dialogue Reflections

Group Members:

- When did the teacher maintain the cognitive demand of the task?
- How did the teacher support (or not support) productive struggle?

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Research

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars, 1995

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Explore 3 of the 4 tasks

- Joe's Ice Cream

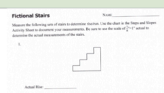
Joe's on the Beach Ice Cream

At Joe's on the Beach, single-scoop ice cream cones sell for $2.99 and ice cream cakes sell for $24.99. Rosa buys an ice cream cake for her party. She also decides to buy a single-scoop cone for each of her friends.

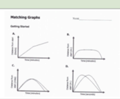
1. Write an equation that can be used to determine the cost (y) of a cake and any number of cones (x) that Rosa buys. Explain the meaning of the terms in your equation.

- Fictional Stairs

Fictional Stairs



- Matching Graphs



- Petoskey Problem

The population of Petoskey, Michigan, was 6,076 in 1990 and was growing at the rate of 3.7% per year. The city planners want to know what the population will be in the year 2025. Write and evaluate an expression to estimate this population. (Source: Holt Algebra 2 [Schultz et al. 2004, p. 415])

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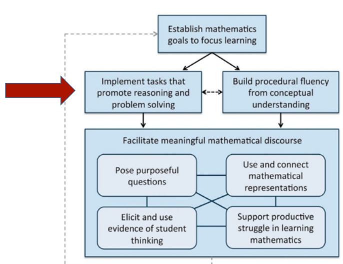
Mathematical Tasks:
A Critical Focus of Instruction



Mathematics Teaching Framework

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Mathematical Tasks:
A Critical Focus of Instruction

The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997

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Mathematical Task Analysis Guide

Lower-Level Demands	Higher-Level Demands
Memorization Tasks • Involve either reproducing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous—such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	*Procedures with Connections Tasks* • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	*Doing Mathematics Tasks* • Require complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). • Require students to explore and understand the nature of mathematical concepts, processes, or relationships. • Demand self-monitoring or self-regulation of one's own cognitive processes. • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development* (p. 6). New York, NY: Teachers College Press.

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Reflection

Based on the Mathematics Task Analysis Guide, what are the characteristics of high level tasks that promote productive struggle?

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Revisiting the Four Tasks

Identify the tasks using the TAG. Be prepared to share your reasoning.

- Joe's Ice Cream
- Fictional Stairs
- Matching Graphs
- Petoskey Problem

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Mathematical Tasks: A Critical Starting Point for Instruction

All tasks are not created equal–*different tasks require different levels and kinds of student thinking.*

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development*, p. 3. New York: Teachers College Press.

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Recommendations for Quality Tasks

- Aligns with relevant mathematics content standard(s)
- Encourages the use of multiple representations
- Provides opportunities for students to develop and demonstrate the mathematical practices
- Involves students in an inquiry-oriented or exploratory approach

NCTM (2013). Putting Essential Understandings of Multiplication and Division into Practice. Reston, VA: Author.

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Recommendations for Quality Tasks

- Allows entry to the mathematics at a low threshold (all students can begin the task) but also has a high ceiling (some students can extend the activity to higher-level activities)
- Connects to previous knowledge to new learning
- Allows for multiple solution approaches and strategies
- Engages students in explaining the meaning of the result
- May include a relevant and interesting context

NCTM (2013). Putting Essential Understandings of Multiplication and Division into Practice. Reston, VA: Author.

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Compare the Tasks

- What goals would you have for each of these tasks?
- What does one task offer that the other may not?

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Let's Look at Some Tasks

- We are comparing ...
  - Original vs. Modified Tasks

All of these tasks are adapted from the Institute for Learning unless otherwise noted.

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Task 1: Original Task

78 is 30% of what number?

Task 1: Modified Task Autumn Leaves

- In the first week of November, Trevor the Tree lost 65% of his leaves.
- In the second week of November, Trevor lost 45% of all the leaves he had at the beginning of the week.
- In the third week of November, Trevor lost 55% of the leaves he had at the beginning of the week. Trevor had 212 leaves after three weeks of November.
- How many leaves did Trevor have at the beginning of November?
- *Use a model and words to explain your thinking.*

Task adapted from *United We Solve* by Tim Erickson, 1996.

Task 1: Debrief

- How is the cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?

Task 2: Original Task

- Answer the following:

$$1/2 + 2/3$$

$$3/7 \times 1/8$$

$$4/5 \div 1/2$$

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Task 2: Modified Task
Chocolate Milk Task

Jenny was mixing herself a glass of chocolate milk. "You certainly have enough syrup in the glass," remarked Kevin, who then found a glass of milk of his own to drink.

"Only a third of a glass of syrup," said Jenny. "And you're certainly taking your share."

"I only have one-fourth of a glass," estimated Kevin.

"But Kevin, your glass holds twice as much!"

"Tell you what," said Kevin, after they both had mixed milk and syrup in their glasses. "Let's combine our drinks in a larger pitcher, and then split the whole amount."

While Jenny is trying to decide whether or not this arrangement is to her advantage, can you say *what part of the combined mixture would be syrup*? Use diagrams, pictures, numbers, or words to show your solution strategy.

Newton, K.J. (2010). The sweetest chocolate milk. Mathematics Teaching in the Middle School, 16(3), 149-153.

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Task 2: Debrief

- How is the cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
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Task 3: Original Task

Find the value of the unknown in each of the proportions shown below.

$\frac{5}{2} = \frac{y}{10}$ $\frac{a}{24} = \frac{7}{8}$ $\frac{n}{8} = \frac{3}{12}$ $\frac{30}{6} = \frac{b}{7}$

Adapted from Smith et al. 2005. Found in *Smith, M.S., Steele, M. D., & Raith, M. L. (2017). Taking Action: Implementing Effective Mathematics Teaching Practices. Reston, VA: NCTM*

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Task 3: Modified Task The Candy Jar Task

A candy jar contains 5 Jolly Ranchers and 13 jawbreakers. Suppose that you had a new candy jar with the same ratio of Jolly Ranchers to jawbreakers, but it contains 100 Jolly Ranchers.

How many jawbreakers would you have?

Explain how you know.

Adapted from Smith et al. 2005. Found in *Smith, M.S., Steele, M. D., & Raith, M. L. (2017). Taking Action: Implementing Effective Mathematics Teaching Practices. Reston, VA: NCTM*

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Task 3: Debrief

- How is the cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?

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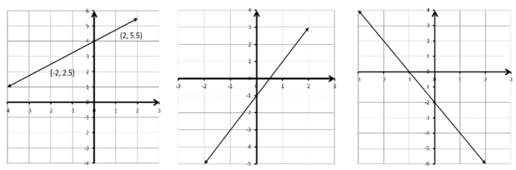
Task 4: Original Task

For each of the following, name the slope and *y*-intercept.

a. $y = 2x + 5$ b. $y = -x + 1$ c. $y = -1/3x + 2$

d. e. f.



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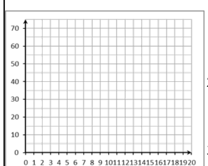
Task 4: Modified Task Joe's On the Beach Ice Cream

At Joe's On the Beach, single-scoop ice cream cones sell for $2.99 and ice cream cakes sell for $24.99. Rosa buys an ice cream cake for her party. She also decides to buy a single-scoop cone for each of her friends.



1. Write a function that can be used to determine the cost (*y*) of a cake and any number of cones (*x*) that Rosa buys. Explain the meaning of the terms in your function.
2. Sketch a graph that models the problem situation. Explain how you know your graph models the problem situation.
3. How does the total cost increase with the number of cones bought? How does this appear in the function and the graph?

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Task 4: Debrief

- How is cognitive demand different between the two tasks?
- How will students engage in the task differently?
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Task 5: Original Task
Bank Accounts

Sisters Aya and Jun keep track of the amount they have saved in the bank using the tables below.

Aya's Savings	
Month	Amount in Bank in Dollars
1	$10.00
2	$12.50
3	$15.00
4	$17.50

Jun's Savings	
Month	Amount in Bank in Dollars
2	$19.00
4	$23.00
6	$27.00
8	$31.00

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Task 5: Original Task
Bank Accounts (continued)

a. Write an equation to describe the amount of money in dollars that Aya has in the bank after any number of months.
b. Graph Aya's equation.
c. Write an equation to describe the amount of money in dollars that Jun has in the bank after any number of months.
d. Graph Jun's equation on the same axes as Aya's graph.
e. What is the coordinate of the point of intersection of the two graphs?
f. After how many months will the sisters have the same amount of money in the bank?

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Task 5: Modified Task
Bank Accounts

a. Explain how you can use tables and graphs to solve the problem below:

   After how many months will the sisters have the same amount of money in the bank?

b. Write an equation to describe the amount of money in dollars that each sister has in the bank after any number of months.
c. Explain how you can use the equations you wrote to solve the problem.
d. Will each of the solution strategies (tables, graphs, and equations) produce the same solution? Explain why or why not.

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Task 5: Debrief

- How is cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?

Task 6: Original Task

Find the slopes of the lines containing the following points.

a. (-3, 0) and (4, 7)
b. (0, 5) and (-3, 2)
c. (0, 0) and (-4, 2)
d. (-1, 9) and (3, 7)
e. (6, - 1/2) and (4, -1/2)
f. (1/8, 3/4) and (1/4 , -3/8)

Task 6: Modified Task
Slope Task

For each pair of points, find a third point that is on the same line. Explain how you used the information given to find the third point.

a. (-3, 0) and (4, 7)
b. (0, 5) and (-3, 2)
c. (-1, 9) and (3, 7)
d. (6, -1/2) and (4, -1/2)

Task 6: Debrief

- How is the cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?

Consider…

What can you do if you want students to develop the capacity to think, reason, and problem-solve, but your textbook doesn't have many high-level cognitively demanding tasks?

How can these tasks be modified?

- Simplify: $4(3+5y)$
- Multiply $2/3 \times 3/5$
- Find the area of a rectangle that measures 4 cm by 6 cm.
- Solve:
  - $-2x + 3 = 2x - 1$
  - $-2x + 3 = \frac{1}{2}(4x + 6)$

Things to keep in mind about modifying tasks….

- Right environment – trust, risk taking, collaborative
- Prior knowledge – do they have foundational understanding so they can enter/access
- Right access points – can all students at different levels enter the task
- Right tools/representations
- Right partners/groups– partners that have complementary attributes

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Strategies for Modifying Textbook Tasks

- Ask students to create real-world stories for "naked number" problems.
- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, within a context).
- Include a prompt that requires students to make a generalization.
- Use a task "out of sequence" before students have memorized a rule or have practiced a procedure that can be routinely applied.
- Eliminate components of the task that provide too much scaffolding.

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Debriefing Modifying Tasks

What is the role of modifying problems found in textbooks in supporting productive struggle?

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Supporting Students' Productive Struggle

Part 4: Apply it – Live it!

Revisiting Session Focus

- How do we create an environment for nurturing productive struggle?
- What are the characteristics of an effective task?
- How do we adapt tasks to connect and extend student understanding beyond the learning goal?
- In what ways does intentionality in teaching practices support equitable instruction?

My Next Steps

- What big ideas about productive struggle resonate with me?
- What do others in similar professional roles think about productive struggle?
- What actions will I take to enhance productive struggle in my classroom, building, or district?
- Why is it important to address productive struggle coherently as part of a systemic learning culture?

Collaborative Next Steps

Discuss:

- What challenges might you face with productive struggle?
- How will you share your message about what you've learned?
- What other tools/resources do you need?
- What actions might you take next year?

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