Supporting Students’ Productive Struggle

Overarching Questions

• What is productive struggle and why does it matter?
• How does engaging in productive struggle enable us to support students?

Supporting Students’ Productive Struggle

Part 1: Common Experiences and Why it Matters?
Driving Questions

- What is productive struggle?
- Why is productive struggle important to students' mathematics learning?
- How do we support productive struggle for each and every student?

What is Productive Struggle?

- In the breakout rooms:
  - Discuss what productive struggle means to you
  - Discuss what productive struggle means to your students
  - Be Ready to Share out

Defining Productive Struggle

The phenomenon of struggle….refers to the intellectual effort students expend to make sense of mathematical concepts that are challenging but fall within the students’ reasonable capabilities.

Explore the task

• Consider the pair of polygons.
• Explain whether or not the polygons are similar.
• If you finish quickly, explain a different way.

What Does it Look Like?

• Read the vignette. Consider the following:
• Write down one or two ideas that you notice
• Write down one or two wonders about students’ experiences in these two experiences
• How are the learning experiences different? The same?
Why Does it Matter?

- Share your ideas with a small group.
- Determine which teacher has created an environment where students can explore, think critically, and grapple with mathematics.
- What are the characteristics of this environment?

Creating an Environment

Help students realize struggle will take time.
They may not be successful – especially in the early stages of solving a problem.
Help students identify what they can do when they are stuck.
Foster a growth mindset.

Student Expectations (Roy, Bush, Hodges, & Safi, 2017)

- participate, even those who do not raise their hands;
- explain and justify their thinking;
- restate a classmate’s reasoning;
- make sense of another classmate’s reasoning; &
- ask a question if they are not sure that they understand.
Facets of Productive Struggle
(NCTM, 2014)

Wrap-Up

• Revisit your definition of productive struggle; what would you add to it?

Food for Thought:
• How do learning goals factor into productive struggle?

Supporting Students’ Productive Struggle

Part 2: Laying the Foundation
Driving Questions

• What does productive struggle look like in a classroom?
• How do I establish mathematical goals?
• How does valuing students’ identities empower students to engage in productive struggle?

Effective Mathematics Teaching Practices (NCTM 2014)

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem-solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

Setting Goals

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Three Goals

• What is the same similar among the goals?

• What is different among the three goals?

• Which goal(s) seems more likely to enable students to solve a wider range of problems?

• How would you assess student success with each goal?
Connection to Standards

7.RP.A.2 Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships
   c. Represent proportional relationships by equations

Fig. 2.1. A seventh-grade proportional reasoning standard from the Common Core State Standards (CCSS) for Mathematics (NGA Center and CCSSO 2010)

Establishing Effective Goals

• Goals help you select a task that focuses on the important mathematics.
• Goals help you make instructional decisions during the lesson (and about assessment).
• Goals are more than a standard or a procedure, but also include information about the conceptual understandings students will gain.
• Goals connect to broader learning trajectories.

Revisiting Earlier Task

Consider the pair of polygons. Explain whether or not the polygons are similar. If you finish quickly, explain a different way.

https://www.illustrativemathematics.org/content-standards/7/G/A/1/tasks/1201
Selecting a Task – Our Goals

• Students will recognize that similar shapes have the same angle measures.
• Students will recognize corresponding segments of similar figures have equal proportional lengths (this common ratio is called the scale factor).
• Students will recognize scale factors and proportions are two ways to solve problems involving similar figures.

Debrief: The Task

How does this task meet our goals?
Why or why not?

What are some considerations or reflections following our debrief?

The task

Does this task meet our goals?
Why or why not?
• Students will recognize that similar shapes have the same angle measures.
• Students will recognize corresponding segments of similar figures have equal proportional lengths – this common ratio is called the scale factor.
• Students will recognize scale factors and proportions are two ways to solve problems involving similar figures.
Wrap – Up

• Revisit your definition of productive struggle; what about learning goals would you add to your definition?

Food for Thought:
• What do you do when you have a task in your current curricular materials that does not adequately allow for productive struggle?

Supporting Students’ Productive Struggle

Part 3: Provoking and Supporting

Driving Questions

• How do we create an environment for nurturing productive struggle?
• What are the characteristics of an effective task?
• How do we adapt tasks to connect and extend student understanding beyond the learning goal?
• In what ways does intentionality in teaching practices support equitable instruction?
Addressing Student Struggle (NCTM 2014)

“Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students. As a result, they jump in to ‘rescue’ students by breaking down the task and guiding students step by step through the difficulties.

Although well intentioned, such ‘rescuing’ undermines the efforts of students, lowers the cognitive demand of the task, and deprives students of opportunities to engage fully in making sense of the mathematics (Reinhart 2000; Stein et al. 2009).”

What does this look like in ...

• We are using a task called Amazing Amanda to see how these pieces fit together.

Amazing Amanda
Mathematical Learning Goals

• Students can subdivide polygons into non-overlapping triangles to make generalizations.
• Students can apply previous learned mathematics to determine the sum of the interior angles of any polygon;
• Students will write equations to describe the relationship between the number of sides and the sum of the measures of the interior angles of any polygon.
Amanda claims to have an amazing talent. “Draw any polygon. Don’t show it to me. Just tell me the number of sides it has and I can tell you the sum of its interior angles.”

Is Amanda’s claim legitimate? Does she really have an amazing gift, or is it possible for anyone to do the same thing?
Read the Mini-Dialogues
Amazing Amanda Task

Looking at the Mini-Dialogues

With your group members, discuss your answers to the questions posed for the mini-dialogues.

- When did the teacher maintain the cognitive demand of the task?
- How did the teacher support (or not support) productive struggle?

Mini-Discussion Reflections

Group Members:

- When did the teacher maintain the cognitive demand of the task?
- How did the teacher support (or not support) productive struggle?
Research

There is no decision that teachers make that has a greater impact on students’ opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

Lappan & Briars, 1995

Explore 3 of the 4 tasks

- Joe’s Ice Cream
- Fictional Stairs
- Marching Graphs
- Petoskey Problem

Mathematical Tasks: A Critical Focus of Instruction

- Implement tasks that promote reasoning and problem solving
- Facilitate meaningful mathematical discourse
- Establish mathematics goals to focus learning
- Select, design, and use appropriate technological tools
- Support and extend student thinking and reasoning
The level and kind of thinking in which students engage determines what they will learn.

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Olivier, & Human, 1997

Mathematical Task Analysis Guide

Based on the Mathematics Task Analysis Guide, what are the characteristics of high level tasks that promote productive struggle?
Revisiting the Four Tasks

Identify the tasks using the TAG. Be prepared to share your reasoning.

- Joe’s Ice Cream
- Fictional Stairs
- Matching Graphs
- Petoskey Problem

Mathematical Tasks: A Critical Starting Point for Instruction

All tasks are not created equal—different tasks require different levels and kinds of student thinking.

Recommendations for Quality Tasks

- Aligns with relevant mathematics content standard(s)
- Encourages the use of multiple representations
- Provides opportunities for students to develop and demonstrate the mathematical practices
- Involves students in an inquiry-oriented or exploratory approach


**Recommendations for Quality Tasks**

- Allows entry to the mathematics at a low threshold (all students can begin the task) but also has a high ceiling (some students can extend the activity to higher-level activities)
- Connects to previous knowledge to new learning
- Allows for multiple solution approaches and strategies
- Engages students in explaining the meaning of the result
- May include a relevant and interesting context


**Compare the Tasks**

- What goals would you have for each of these tasks?
- What does one task offer that the other may not?

**Let’s Look at Some Tasks**

- We are comparing ...
  - Original vs. Modified Tasks

All of these tasks are adapted from the Institute for Learning unless otherwise noted.
Task 1: Original Task

78 is 30% of what number?

Task 1: Modified Task

Autumn Leaves

• In the first week of November, Trevor the Tree lost 65% of his leaves.
• In the second week of November, Trevor lost 45% of all the leaves he had at the beginning of the week.
• In the third week of November, Trevor lost 55% of the leaves he had at the beginning of the week. Trevor had 212 leaves after three weeks of November.
• How many leaves did Trevor have at the beginning of November?
• Use a model and words to explain your thinking.

Task adapted from United IF's Solve by Tim Erickson, 1996.

Task 1: Debrief

• How is the cognitive demand different between the two tasks?
• How will students engage in the task differently?
• How may the classroom discourse differ for the two tasks?
• How does the modified task support productive struggle?
Task 2: Original Task

- Answer the following:

\[
\frac{1}{2} + \frac{2}{3} \\
\frac{3}{7} \times \frac{1}{8} \\
\frac{4}{5} \div \frac{1}{2}
\]

Task 2: Modified Task

Chocolate Milk Task

Jenny was mixing herself a glass of chocolate milk. “You certainly have enough syrup in the glass,” remarked Kevin, who then found a glass of milk of his own to drink.

“Only a third of a glass of syrup,” said Jenny. “And you’re certainly taking your share.”

“I only have one-fourth of a glass,” estimated Kevin.

“But Kevin, your glass holds twice as much!”

“Tell you what,” said Kevin, after they both had mixed milk and syrup in their glasses. “Let’s combine our drinks in a larger pitcher, and then split the whole amount.”

While Jenny is trying to decide whether or not this arrangement is to her advantage, can you say what part of the combined mixture would be syrup? Use diagrams, pictures, numbers, or words to show your solution strategy.


Task 2: Debrief

- How is the cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?
Task 3: Original Task

Find the value of the unknown in each of the proportions shown below.

\[
\begin{array}{l}
\frac{5}{2} = \frac{y}{10} \\
\frac{a}{24} = \frac{7}{8} \\
\frac{m}{8} = \frac{3}{12} \\
\frac{30}{6} = \frac{b}{7}
\end{array}
\]


Task 3: Modified Task

The Candy Jar Task

A candy jar contains 5 Jolly Ranchers and 13 jawbreakers. Suppose that you had a new candy jar with the same ratio of Jolly Ranchers to jawbreakers, but it contains 100 Jolly Ranchers.

How many jawbreakers would you have?

Explain how you know.


Task 3: Debrief

• How is the cognitive demand different between the two tasks?
• How will students engage in the task differently?
• How may the classroom discourse differ for the two tasks?
• How does the modified task support productive struggle?
Task 4: Original Task

For each of the following, name the slope and \(y\)-intercept.

- a. \(y = 2x + 5\)
- b. \(y = -x + 1\)
- c. \(y = -\frac{1}{3}x + 2\)
- d. 
- e. 
- f. 

Task 4: Modified Task

Joe’s On the Beach Ice Cream

At Joe’s On the Beach, single-scoop ice cream cones sell for $2.99 and ice cream cakes sell for $24.99. Rosa buys an ice cream cake for her party. She also decides to buy a single-scoop cone for each of her friends.

1. Write a function that can be used to determine the cost (\(y\)) of a cake and any number of cones (\(x\)) that Rosa buys. Explain the meaning of the terms in your function.

2. Sketch a graph that models the problem situation. Explain how you know your graph models the problem situation.

3. How does the total cost increase with the number of cones bought? How does this appear in the function and the graph?

Task 4: Debrief

- How is cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?
Sisters Aya and Jun keep track of the amount they have saved in the bank using the tables below.

### Aya’s Savings

<table>
<thead>
<tr>
<th>Month</th>
<th>Amount in Bank in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10.00</td>
</tr>
<tr>
<td>2</td>
<td>$12.50</td>
</tr>
<tr>
<td>3</td>
<td>$15.00</td>
</tr>
<tr>
<td>4</td>
<td>$17.50</td>
</tr>
</tbody>
</table>

### Jun’s Savings

<table>
<thead>
<tr>
<th>Month</th>
<th>Amount in Bank in Dollars</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$19.00</td>
</tr>
<tr>
<td>4</td>
<td>$23.00</td>
</tr>
<tr>
<td>6</td>
<td>$27.00</td>
</tr>
<tr>
<td>8</td>
<td>$31.00</td>
</tr>
</tbody>
</table>

### Task 5: Original Task

**Bank Accounts**

a. Write an equation to describe the amount of money in dollars that Aya has in the bank after any number of months.

b. Graph Aya’s equation.

c. Write an equation to describe the amount of money in dollars that Jun has in the bank after any number of months.

d. Graph Jun’s equation on the same axes as Aya’s graph.

e. What is the coordinate of the point of intersection of the two graphs?

f. After how many months will the sisters have the same amount of money in the bank?

### Task 5: Original Task

**Bank Accounts (continued)**

a. Write an equation to describe the amount of money in dollars that Aya has in the bank after any number of months.

b. Graph Aya’s equation.

c. Write an equation to describe the amount of money in dollars that Jun has in the bank after any number of months.

d. Graph Jun’s equation on the same axes as Aya’s graph.

e. What is the coordinate of the point of intersection of the two graphs?

f. After how many months will the sisters have the same amount of money in the bank?

### Task 5: Modified Task

**Bank Accounts**

a. Explain how you can use tables and graphs to solve the problem below:

   **After how many months will the sisters have the same amount of money in the bank?**

b. Write an equation to describe the amount of money in dollars that each sister has in the bank after any number of months.

c. Explain how you can use the equations you wrote to solve the problem.

d. Will each of the solution strategies (tables, graphs, and equations) produce the same solution? Explain why or why not.
Task 5: Debrief

- How is cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?

Task 6: Original Task

Find the slopes of the lines containing the following points.

- a. (-3, 0) and (4, 7)
- b. (0, 5) and (-3, 2)
- c. (0, 0) and (-4, 2)
- d. (-1, 9) and (3, 7)
- e. (6, -1/2) and (4.1/2)
- f. (1/8, 3/4) and (1/4, -3/8)

Task 6: Modified Task

Slope Task

For each pair of points, find a third point that is on the same line. Explain how you used the information given to find the third point.

- a. (-3, 0) and (4, 7)
- b. (0, 5) and (-3, 2)
- c. (-1, 9) and (3, 7)
- d. (6, -1/2) and (4, -1/2)
Task 6: Debrief

- How is the cognitive demand different between the two tasks?
- How will students engage in the task differently?
- How may the classroom discourse differ for the two tasks?
- How does the modified task support productive struggle?

Consider...

What can you do if you want students to develop the capacity to think, reason, and problem-solve, but your textbook doesn’t have many high-level cognitively demanding tasks?

How can these tasks be modified?

- Simplify: $4(3+5y)$
- Multiply $\frac{2}{3} \times \frac{3}{5}$
- Find the area of a rectangle that measures 4 cm by 6 cm.
- Solve:
  \[
  \begin{cases}
  -2x + 3 = 2x - 1 \\
  -2x + 3 = \frac{1}{2}(4x + 6)
  \end{cases}
  \]
Things to keep in mind about modifying tasks:

- Right environment – trust, risk taking, collaborative
- Prior knowledge – do they have foundational understanding so they can enter/access
- Right access points – can all students at different levels enter the task
- Right tools/representations
- Right partners/groups – partners that have complementary attributes

Strategies for Modifying Textbook Tasks

- Ask students to create real-world stories for “naked number” problems.
- Include a prompt that asks students to represent the information another way (with a picture, in a table, a graph, an equation, within a context).
- Include a prompt that requires students to make a generalization.
- Use a task “out of sequence” before students have memorized a rule or have practiced a procedure that can be routinely applied.
- Eliminate components of the task that provide too much scaffolding.

Debriefing Modifying Tasks

What is the role of modifying problems found in textbooks in supporting productive struggle?
Supporting Students’ Productive Struggle

Part 4: Apply it – Live it!

Revisiting Session Focus

• How do we create an environment for nurturing productive struggle?
• What are the characteristics of an effective task?
• How do we adapt tasks to connect and extend student understanding beyond the learning goal?
• In what ways does intentionality in teaching practices support equitable instruction?

My Next Steps

• What big ideas about productive struggle resonate with me?
• What do others in similar professional roles think about productive struggle?
• What actions will I take to enhance productive struggle in my classroom, building, or district?
• Why is it important to address productive struggle coherently as part of a systemic learning culture?
Collaborative Next Steps

Discuss:

• What challenges might you face with productive struggle?
• How will you share your message about what you've learned?
• What other tools/resources do you need?
• What actions might you take next year?

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