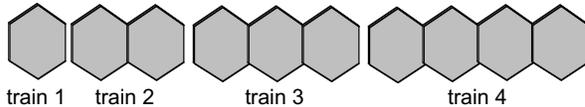


## Exploring Linear Functions: *The Case of Barbara Peterson*<sup>1</sup>

Ms. Peterson wanted her eighth grade students to understand three key ideas about linear functions: 1) variables can be used to represent two quantities that change in the relationship to each other; 2) that there are different but equivalent ways of writing an explicit rule; and 3) that connections can be made between different representational forms – tables, graphs, equations, words, and pictures. She selected the Hexagon Pattern Train Task because it aligned with her goals for the lesson and with state standards, it had multiply entry points and solution paths, and it would challenge her students to think and reason.

Trains 1, 2, 3, and 4 are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



1. Compute the perimeter for each of the first four trains;
2. Draw the fifth train and compute the perimeter of the train;
3. Determine the perimeter of the 10th train without constructing it;
4. Write a description that could be used to compute the perimeter of any train in the pattern; and
5. Determine which train has a perimeter of 110.

Ms. Peterson told students that questions 1-3 were to help them get started if they needed it, question 5 was for students who finished quickly, but that question 4 would be the focus of the whole class discussion. She reminded students that they could use any of materials (hexagon shapes, copies of the hexagon trains, colored pencils, calculators) that were available on their tables.

As students began working in their groups, Ms. Peterson walked around the room stopping at different groups to listen in on their conversations and to ask questions as needed. When groups struggled to get started, she asked: "What would train 5 look like? How much bigger is the perimeter of train five than the perimeter of train 4? What is different as you move from one train to the next? How does the pattern seem to be growing?" She also encouraged students to use the hexagon shapes to build new trains and to make observations about what changes and what remains the same as the trains get bigger. When groups determined a correct algebraic representation she pressed them to explain what each part of the equation meant, to relate each part of the equation to the picture, and to explain how they knew it would always work.

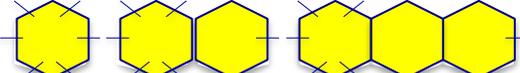
Ms. Peterson spent more time with Groups 2 and 5. Group 2 had identified the growth rate as +4 but was not able to find the perimeter of a train without knowing the perimeter of the train that preceded it. She asked: "Why are you adding 4? Where is the 4 in the picture? Can you predict the number of 4's that you are adding each time?" Group 5 had reasoned that since a hexagon has 6 sides that every time you add one more hexagon you add 6 more sides. She asked: "When you use your rule to find the perimeter in train 4, do you get the same answer as you did when you counted? Why not? What gets counted when you multiply by 6?"

As she visited the groups, Ms. Peterson made note of the strategies students were using (see reverse side) so she could decide which groups she wanted to have present their work. Each presenting group would be expected to explain what they did and why, how their equation connected to the visual arrangement of hexagons, and to answer questions posed by their peers. Group 3 would go first since their table would be accessible to all groups. Group 1 would go next because the equation they created modeled the verbal description given by Group 3 but was derived from the picture rather than from numbers in a table. Group 5 would then present a different equation, also derived from the picture, and discuss how they came to see that counting all of the sides did not work.

<sup>1</sup>This case, written by Margaret Smith (University of Pittsburgh), was inspired by lessons planned and taught by Timothy Booth, a graduate student at the University of Pittsburgh and other teachers with whom she has worked over the last decade. It is intended to support the Teaching and Learning Guiding Principle in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014).

51 The presentations would conclude with Group 2. Below is an excerpt from the discussion that took  
 52 place around the explanation presented by Group 2.  
 53  
 54 Michael: If you look at Train 4 we saw that there were five sides on each end of the train that counted  
 55 in the perimeter and that the two hexagons in the middle each had 4 sides that counted. So  
 56 we had  $5 + 4 + 4 + 5$  which is 18.  
 57 Ms. P.: So suppose we were talking about the 10th train. Can someone else explain how we could  
 58 use Group 2's method to find the perimeter of the 10<sup>th</sup> train? (*Several students raise their*  
 59 *hands and the teacher calls on Kelsey.*)  
 60 Kelsey: For Train 10 it would  $5 + 5$  for the two ends and then there would be 8 4's in the middle. So  
 61 it would be  $32 + 10$  which is 42.  
 62 Brian: How did you know how many 4's would be needed?  
 63 Kelsey: I pictured it in my head and if I took off the two ends then there would be 8 hexagons left.  
 64 Ms. P.: I am wondering if we could generalize the number of hexagons that are between the two  
 65 ends of any train. Take two minutes and turn and talk to the person sitting next to you. (*After*  
 66 *two minutes the teacher continues.*) Amber? What did you and Sara decide?  
 67 Amber: Well we think that the number of hexagons in the middle is  $h - 2$ . Then you would need to  
 68 multiply by 4 to get the total number of sides that the ones in the middle contribute to the  
 69 perimeter. That is  $4(n-2)$ .  
 70 Ms. P.: What do others think? (*Students are nodding their heads or giving a "thumbs up".*)  
 71 Ms. P.: If we use  $4(n-2)$  like Amber and Jackie are suggesting, will we get the correct perimeter?  
 72 Sara: No. You need to add on the 5 for the first hexagon and the 5 for the last hexagon. So it has  
 73 to be  $P = 4(n-2) + 10$ .  
 74 Ss: YES!

75  
 76 With only five minutes left in the class, Ms. Peterson asked students to picture in their minds what the  
 77 graph of this function they had been working on would look like, to sketch the graph, and explain why  
 78 they think their sketch made sense. She would use this information to see what students understood  
 79 about relationship between representations and to launch class the following day. She also planned  
 80 to talk about how there could be three (or more) equations for the same function.

Group 1's Solution	Group 2's Solution	Group 3's Solution																			
 <p>Each hex has 2 sides on the top and 2 on the bottom that count. So that's 4. Then you need to add the two on the ends.            It is <math>P = 4h + 2</math></p>	 <p>There are five sides on each end of the train. Then every time you add another hexagon there are 4 more sides on the inside - 2 on the top and 2 on the bottom.</p>	<table border="1" data-bbox="992 1129 1289 1371"> <thead> <tr> <th>Train #</th> <th>Perimeter</th> </tr> </thead> <tbody> <tr><td>1</td><td>6</td></tr> <tr><td>2</td><td>10</td></tr> <tr><td>3</td><td>14</td></tr> <tr><td>4</td><td>18</td></tr> <tr><td>5</td><td>22</td></tr> <tr><td>6</td><td>26</td></tr> <tr><td>7</td><td>30</td></tr> <tr><td>8</td><td>34</td></tr> </tbody> </table> <p>The perimeter of each train is 4 more. So if you multiply the number of hexagons by 4 you need to add on 2 to get the right perimeter.</p>		Train #	Perimeter	1	6	2	10	3	14	4	18	5	22	6	26	7	30	8	34
Train #	Perimeter																				
1	6																				
2	10																				
3	14																				
4	18																				
5	22																				
6	26																				
7	30																				
8	34																				
Group 4 Solution		Group 5's Solution																			
 <p>The first hexagon train has six sides. When you go to bigger trains the right side of the first hexagon moves to the end of the train. Each of the other hexagons in a train adds 4 sides to the perimeter.  <math>P = 4(h - 1) + 6</math></p>		 <p>Each hexagon has six sides, but all of them don't count. So you have to get rid of the vertical ones. So that would be <math>2x</math>. But you then have to add two back for the sides on the ends.  <math>P = 6h - 2h + 2</math></p>																			