NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

Teaching Practices that Support Student Understanding and Learning of Mathematics

Margaret (Peg) Smith
Professor Emerita
University of Pittsburgh

Taking Action: Bringing the Effective Teaching Practices to Life in Your Classroom
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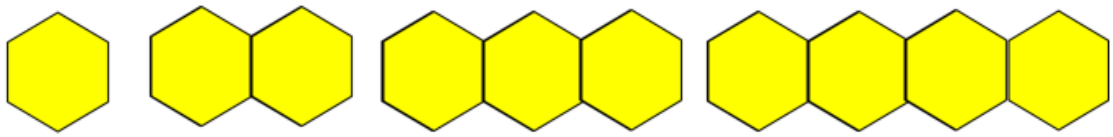
Hexagon Task[1]

Trains 1, 2, 3 and 4 (shown below) are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



Train 1 Train 2 Train 3 Train 4

1. Compute the perimeter for each of the first four trains.
2. Draw the fifth train and compute the perimeter of the train.
3. Determine the perimeter of the 10th train without constructing it.
4. Write a description that could be used to compute the perimeter of any train in the pattern and explain why it works.
5. Determine which train has a perimeter of 110.

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[1] This task was adapted from *Visual Mathematics Course 1, Lessons 16-30* published by the Math Learning Center. Copyright © 1995 by The Math Learning Center, Salem, Oregon.

Patterns

The table of value below describes the perimeter of each figure in the pattern of blue tiles. The perimeter *P* is a function of the number of tiles t.

t	1	2	3	4
P	4	6	8	10

a. Choose a rule to describe the function in the table.
   A. P = t + 3
   B. P = 4t
   C. P = 2t + 2
   D. P = 6t – 2

b. How many tiles are in the figure if the perimeter is 20?

c. Graph the function.

The Task Analysis Guide (TAG)

Levels of Demands

Lower-level demands (memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen 1996; Stein, Lane, and Silver 1996).

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3* (5), 344-350.