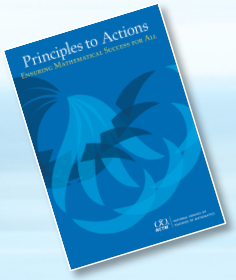
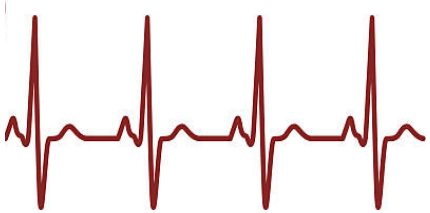
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

Teaching Practices that Support Student Understanding and Learning of Mathematics



Margaret (Peg) Smith
Professor Emerita
University of Pittsburgh

Taking Action: Bringing the **Effective Teaching Practices** to Life in Your Classroom
Webinar Series
November 24, 2020
7:00 – 8:00 pm EST



Taking the Pulse

How familiar are you with the 8 Effective Teaching Practices (ETP's), first introduced in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014)?

Pick the one that best describes you:

a. I have never heard of the ETP's
b. I have heard of the ETP's but have never used them in my teaching
c. I have used the ETP's, but they are not a regular feature of my teaching practice
d. I have used the ETP's routinely in my teaching

Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations**.
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

Effective Teaching Practices

National Council of Teachers of Mathematics (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: Author. [Writing Team: Leinwand, S., Huinker, D., Brahier, D., Berry, R.,. Dillon, F., Larson, M., Leiva, M., Martin, G., & Smith, M.S.]

Effective Teaching Practices

Huinker, DeAnn, and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017. **(November 30, 2020)**

Smith, Margaret, Michael Steele, and Mary Lynn Raith. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics, 2017. **(December 1, 2020)**

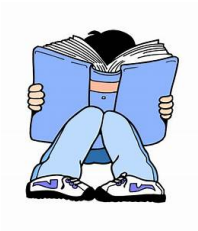
Boston, Melissa, Fredrick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. Reston, VA: National Council of Teachers of Mathematics, 2017. **(December 3, 2020; December 10, 2020)**

Overview

- Read and analyze a short case of a teacher (Ms. Peterson) who is attempting to support her students' learning
- Discuss the eight teaching practices and relate them to the case
- Discuss how to get started in making these practices central to your instruction

The Case of Barbara Peterson

- Read the Case of Ms. Peterson and review the strategies used by her students.



- Make note of what Ms. Peterson did before or during instruction to support her students' learning and understanding of linear relationships.

- Use the chat feature to share one of the teacher actions (or interactions) that you think supported student learning.

This case appears the Principles to Action Professional Learning Toolkit
https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/The-Case-of-Patrica-Rossman-and-the-Hexagon-Task/

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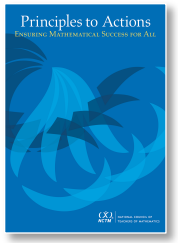

Establish Mathematics Goals To Focus Learning

Learning Goals should:

- Clearly state what it is students are to learn and understand about mathematics as the result of instruction;
- Be situated within learning progressions; and
- Frame the decisions that teachers make during a lesson.

Formulating clear, explicit learning goals sets the stage for everything else.

(Hiebert, Morris, Berk, & Janssen, 2007, p. 57)

Consider Three Different Goals

Which goal do you think would be most useful in planning and teaching a lesson on linear relationships?

a. Students will learn that y = *m*x + b is the slope-intercept form of a linear equation.

b. Students will be able to write linear equations given a visual diagram or problem context.

c. Students will understand that equations can be created to represent two quantities that change in relationship to each other; the rate of change describes **how** one variable quantity (y) changes with respect to another (x) and can be seen in a table as the first difference, in a graph as the slope of the line, and in an equation as the coefficient of the independent (x) variable (the *m* in y = *m*x + b).

- How are the goals the same and how are they different?
- How might the differences matter?

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Consider Three Different Goals

Performance Goals – focus on what students will do

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Learning Goals – focus on what students understand about mathematics

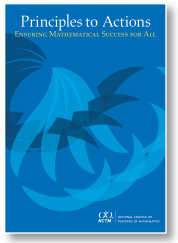
Goals to Focus Learning

Ms. Peterson's goal for students' learning:

1) variables can be used to represent two quantities that change in the relationship to each other;
2) there are different but equivalent ways of writing an explicit rule; and
3) connections can be made between different representational forms – tables, graphs, equations, words, and pictures.

Goals for Learning

- Ms. Peterson was clear about what students would learn (lines 3-6)
- Her goal was grade-level appropriate and consistent with recommendations from CCSS
- She made decisions through out the lesson based on her goal:
  - Asking students questions to help them see the relationship between the train number/number of hexagons and the perimeter (lines 28-31)
  - Inviting groups to present who had written different but equivalent equations (Groups 1, 5, and 2)
  - Highlighting different representations so that connections could be made (Group 3's table, Group 1's equation, and the graph everyone was picturing --lines 77-78)



Implement Tasks that Promote Reasoning and Problem Solving

Mathematical tasks should:

- Provide opportunities for students to engage in exploration or encourage students to use procedures in ways that are connected to concepts and understanding;
- Build on students' current understanding; and
- Have multiple entry points.

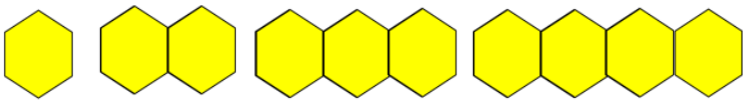
There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics.

(Lappan & Briars, 1995)

Comparing Two Tasks

Task A - Hexagon

Trains 1, 2, 3 and 4 are the the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.

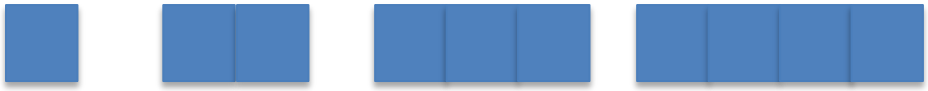


1. Compute the perimeter for each of the first four trains;
2. Draw the fifth train and compute the perimeter of the train;
3. Determine the perimeter of the 25th train without constructing it;
4. Write a description that could be used to compute the perimeter of any train in the pattern and explain why it works; and
5. Determine which train has a perimeter of 110.

Task B - Patterns

The table of values below describes the perimeter of each figure in the pattern of blue tiles. The perimeter *P* is a function of the number of tiles *t*.

t	1	2	3	4
P	4	6	8	10



a. Choose a rule to describe the function in the table.

A. $P = t + 3$ B. $P = 4t$

C. $P = 2t + 2$ D. $P = 6t - 2$

b. How many tiles are in the figure if the perimeter is 20?

c. Graph the function.

Consider Two Different Tasks

Which task would do you think would be *most useful* in building students understanding of mathematics?

1. Task A - Hexagon
2. Task B – Patterns
3. Both tasks
4. Neither task

- How are the tasks the same and how are they different?
- How might the differences matter?

Hexagon

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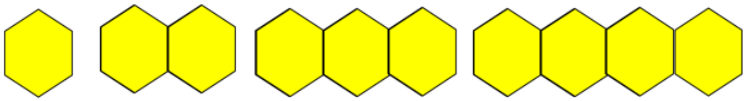

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Comparing Two Tasks

Same

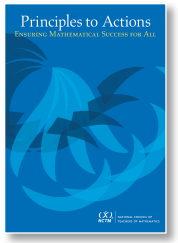
- The tasks involve trains of regular polygons
- When the train number increases by 1 an additional polygon is added to the train
- The number of polygons is the same as the train number
- The tasks ask you to find the number of tiles in a train given the perimeter
- A knowledge of perimeter is needed to solve the tasks

Different

- Multiple ways to enter Hexagon – make table, build trains, inspect diagram
- Multiple ways to solve Hexagon - table, graph, different equations that model the physical arrangement of tiles
- In the Patterns task, the table is done, the equations are provided, very little thinking is needed. Part a is a matching task and b is a plug and chug task. While you are asked to graph in part c, this is nothing more than a plotting points exercise.

Features of the Hexagon Task

- Aligned with goals for lesson
- Significant mathematics content
- <u>High cognitive demand</u>
- Low-threshold, high ceiling
- Different representations
- Requires explanation/justification

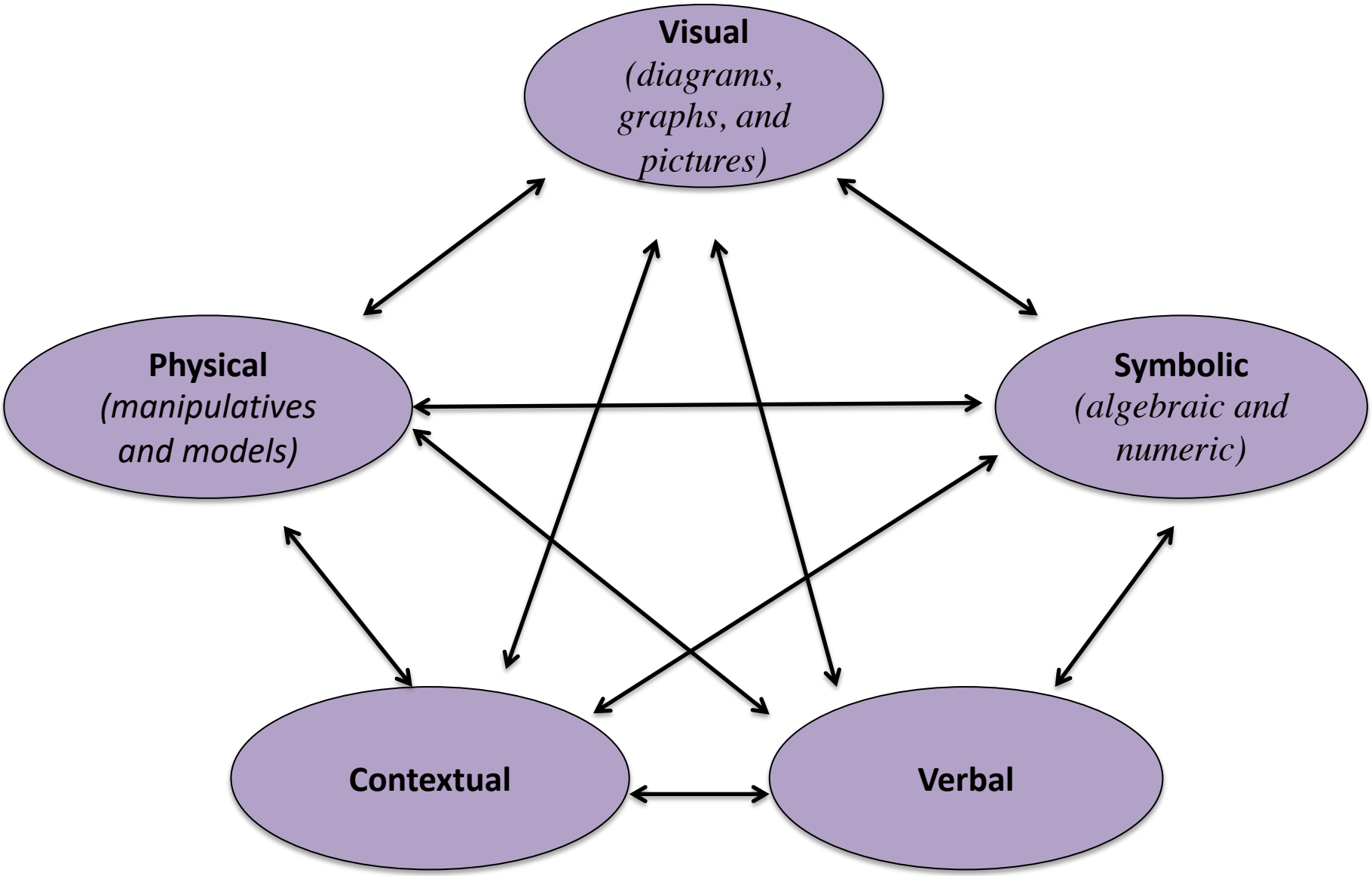


Use and Connect Mathematical Representations

Different Representations should:

- Be introduced, discussed, and connected;
- Focus students' attention on the structure or essential features of mathematical ideas; and
- Support students' ability to justify and explain their reasoning.

Strengthening the ability to move between and among these representations improves the growth of children's concepts. (Lesh, Post & Behr, 1987)



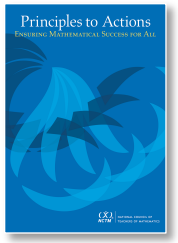
Adapted from Lesh, Post & Behr, 1987

Connecting Representations

- What different representations were used and connected in Ms. Peterson's class?
- How might her students benefit from making these connections?

Connecting Representations

- **Visual** -- the picture of the hexagon trains -- was used repeatedly. Students were asked to make connections between the equations they found (lines 32-34; 46-47) and visual arrangement of tiles.
- **Verbal** -- Group 3's verbal description was connected to Group 1's equation (lines 47-49)
- **Graph** -- Teacher asked students to picture then sketch a graph and explain why it makes sense (lines 76-78). This would provide the opportunity to connect the equation to graph and to the context.
- **Table** -- It is not clear if the teacher explicitly connected group 1's equation P=4h +2 with the table, but she could (and probably should!) do so.



Facilitate Meaningful Mathematical Discourse

Mathematical Discourse should:

- Build on and honor students' thinking;
- Provide students with the opportunity to share ideas, clarify understandings, and develop convincing arguments; and
- Advance the mathematical learning of the whole class.

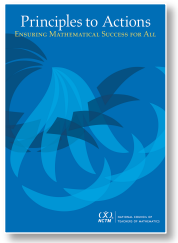
Discussions that focus on cognitively challenging mathematical tasks…are a primary mechanism for promoting conceptual understanding of mathematics (Hatano and Inagaki 1991; Michaels, O'Connor and Resnick 2008). (Smith, Hughes, Engle and Stein 2009, p. 549)

Meaningful Discourse

- What did Ms. Peterson do (before or during the discussion) that may have positioned her to engage her students in a productive discussion?

Meaningful Discourse

- Set a clear goal for learning (lines 3-6)
- Selected a good task (lines 10-18)
- Helped students make connections between different strategies and to the key mathematical ideas in the lesson.
- Asked questions (more about this later….)
- Monitored what her students did during the lesson (lines 25-41) and noted solutions that would be shared (lines 43-52)
- Selected and sequenced solutions in a particular order (46-52)



Pose Purposeful Questions

Effective Questions should:

- Reveal students' current understandings;
- Encourage students to explain, elaborate, or clarify their thinking; and
- Make the mathematics more visible and accessible for student examination and discussion.

Teachers' questions are crucial in helping students make connections and learn important mathematics and science concepts. Teachers need to know how students typically think about particular concepts, how to determine what a particular student or group of students thinks about those ideas, and how to help students deepen their understanding.

(Weiss & Pasley, 2004)

Purposeful Questions

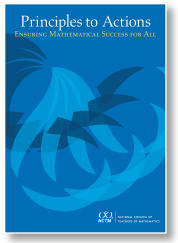
- What do you notice about the questions that Ms. Peterson asked on lines 37-41 and 54-74?

- What purpose did her questions serve?

Purposeful Questions

The questions were very open and helped Ms. Peterson determine what it was the students knew and understood (*elicit student thinking*); they required students to think and explain.

- Made students thinking visible (54-74)
- Engaged students in exploring mathematics (39-41; 64-65)
- Invited others to participate (57-58; 66; 70)
- Helped students make connections (45-46; 76-78)



Build Procedural Fluency from Conceptual Understanding

Procedural Fluency should:

- Build on a foundation of conceptual understanding;
- Result in generalized methods for solving problems; and
- Enable students to flexibly choose among methods to solve contextual and mathematical problems.

Students must be able to do much more than carry out mathematical procedures. They must know which procedure is appropriate and most productive in a given situation, what a procedure accomplishes, and what kind of results to expect. Mechanical execution of procedures without understanding their mathematical basis often leads to bizarre results.

(Martin 2009, p. 165)

$$\frac{x}{4} = \frac{21}{28}$$

$$28x = 84$$

$$\frac{28x}{28} = \frac{84}{28}$$

$$x = 3$$

Solve this system of equations (and check):

$$3y - 2x = 11$$
$$y + 2x = 9$$

1. Solve one of the equations for either "x =" or "y =". This example solves the second equation for "y =".

$$3y - 2x = 11$$
$$y = 9 - 2x$$

2. Replace the "y" value in the first equation by what "y" now equals. Grab the "y" value and plug it into the other equation.

$$3(9 - 2x) - 2x = 11$$

3. Solve this new equation for "x".

$$(27 - 6x) - 2x = 11$$
$$27 - 6x - 2x = 11$$
$$27 - 8x = 11$$
$$-8x = -16$$
$$x = 2$$

4. Place this new "x" value into either of the ORIGINAL equations in order to solve for "y". Pick the easier one to work with!

$$y + 2x = 9 \text{ or}$$
$$y = 9 - 2x$$
$$y = 9 - 2(2)$$
$$y = 9 - 4$$
$$y = 5$$

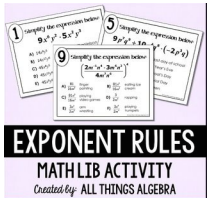
$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$D = \sqrt{(5 - 1)^2 + (2 - 5)^2}$$
$$D = \sqrt{(4)^2 + (-3)^2}$$
$$D = \sqrt{16 + 9}$$
$$D = \sqrt{25}$$
$$D = 5$$

Mindless Use of Procedures

$$(2x-7)(5x+3)$$
$$= (10x^2) + (6x) + (-35x) + (-21)$$

F O I L

$$= 10x^2 + 6x - 35x - 21$$
$$= 10x^2 - 29x - 21$$



$$\text{Mean} = \frac{\text{sum of all values}}{\text{total number of values}}$$

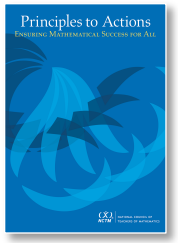
Median = middle value (when the data are arranged in order)

Mode = most common value

Procedural Fluency

What might we expect the students in Ms. Peterson's class to be able to do after they have had the opportunity to develop an understanding of how linear functions model situations, what the slope means in a context, and how slope is represented in a table, graph and equation?

- Be able to create equations that model situations by connecting key aspects of a situation with parameters such as slope and y-intercept. *(As oppose to creating a decontextualize table of values and finding the difference in successive y values when x is incremented by 1. Then plugging it in as the m in y = mx + b.)*
- Identify what slope is given any representational form and explain how the different representations are connected. *(As oppose to only being able to find the slope using two points and the formula and not being able to relate it to anything.)*



Support Productive Struggle in Learning Mathematics

Productive Struggle should:

- Be considered essential to learning mathematics with understanding;
- Develop students' capacity to persevere in the face of challenge; and
- Help students realize that they are capable of doing well in mathematics with effort.

By struggling with important mathematics we mean the opposite of simply being presented information to be memorized or being asked only to practice what has been demonstrated.

(Hiebert and Grouws 2007, pp. 387-88)

Productive Struggle

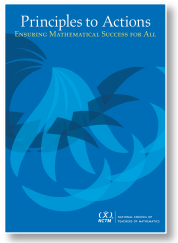
- How did Ms. Peterson support students when they struggled?

Productive Struggle

- When students struggled, she asked questions to help them make progress on the task (27-29) and encouraged them to build and examine a model (29-31).

- When students came up with insufficient or incorrect solutions, she again asked them questions (37-41).

- She supported students' ability to work through the problem without taking over the thinking for them and thereby lowering the demand of the task. In this way she set the message to students that they were capable of figuring it out for themselves. In the end they would have ownership of the work.



Elicit and Use Evidence of Student Thinking

Evidence should:

- Provide a window into students' thinking;
- Help the teacher determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom. (Wiliam 2007, pp. 1054;1091)

Evidence of Student Thinking

- To what extent did Ms. Peterson elicit students' thinking?
- To what extent did (or could) Ms. Peterson use the evidence to inform her instruction?

Evidence of Student Thinking

Elicit

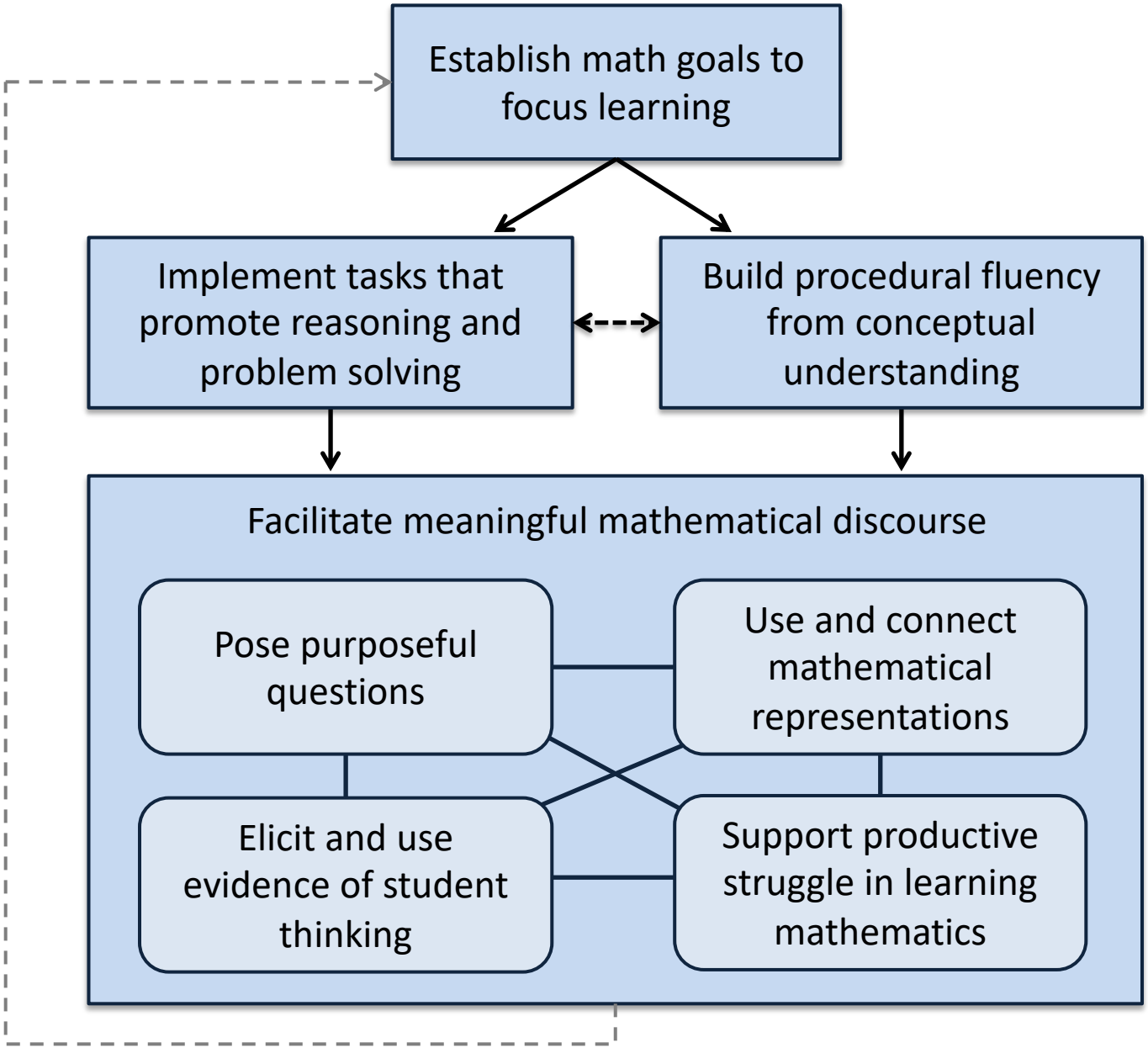
- Gave them a task that required them to think and reason and explain. So the task help elicit student thinking (10-18).
- T asked questions throughout the lesson that focused on explaining what they knew and what they thought about solutions produced by others.
- T gave an exit slip at the end of class that was intended to elicit their thinking about a graphical representation (77-80).

Use

- Ms. Peterson plans a discussion that is based on student work (lines 44-52).
- Her entire lesson appears to unfold based on what she has learned about students thinking and understanding.
- One would expect that her lesson the following day would be informed by what she learns from the exit slip.

So how do the practices fit together?

Effective Mathematics Teaching Practices
"Building a Teaching Framework"



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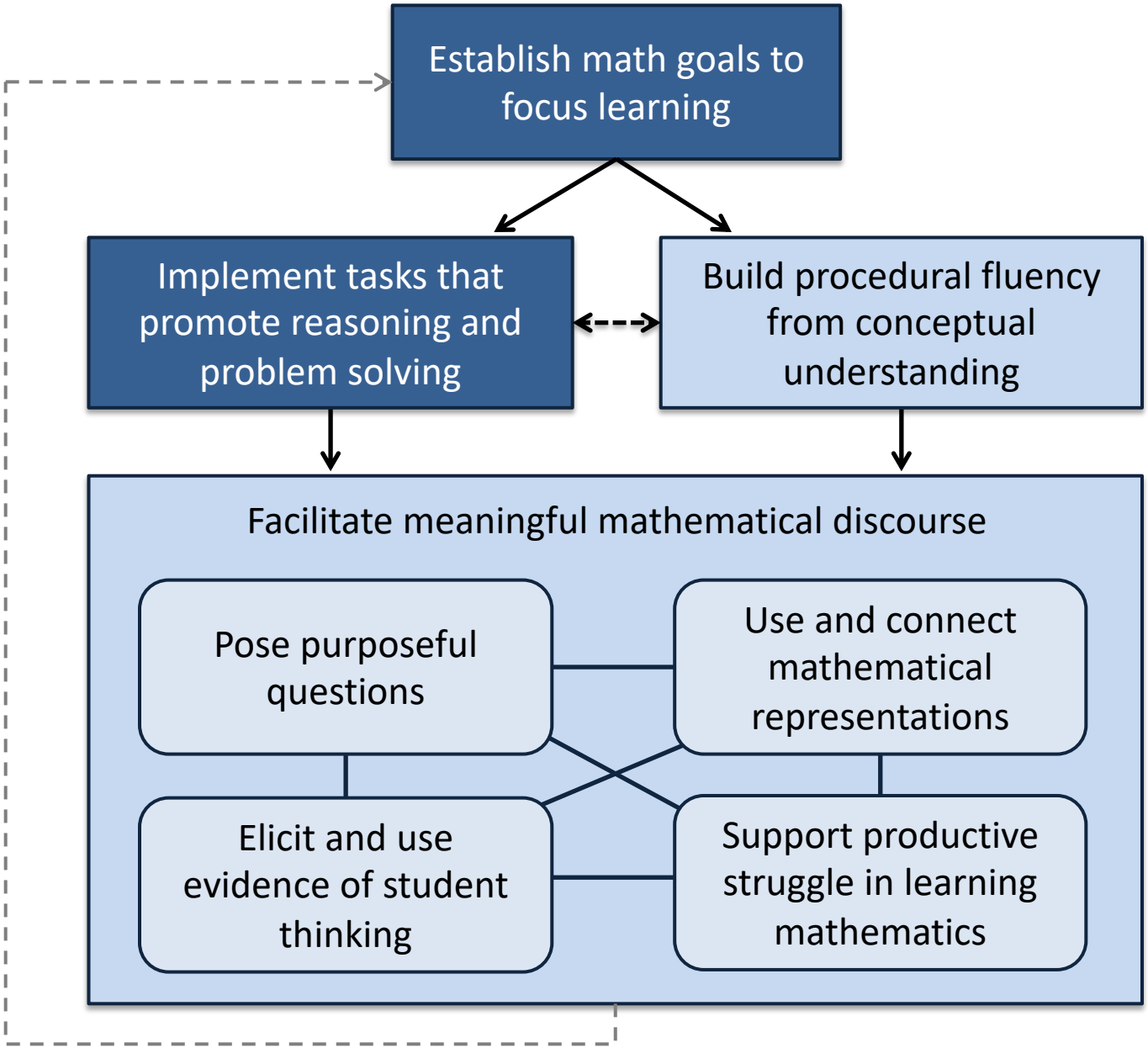


Getting Started

- Learn more about the effective teaching practices from reading PtA, exploring other resources, and talking with your colleagues and administrators.
- Engage in observations and analysis of teaching (live or in narrative or video form) and discuss the extent to which the eight practices appear to have been utilized by the teacher and what impact they had on teaching and learning.
- Co-plan lessons with colleagues using the eight effective teaching practices as a framework. Invite the math coach (if you have one) to participate.
- Observe and debrief lessons with particular attention to what practices were used in the lesson and how the practices did or did not support students' learning.

If your students are going home at the end of the day less tired than you are, the division of labor in your classroom requires some attention.

Wiliam, D. (2011)


ANY
QUESTIONS ?

Reflection

What will you do to improve the quality of teaching and learning in your classroom?

Taking Action: Bringing the Effective Teaching Practices to Life in Your Classroom Webinar Series

Opening Session

Peg Smith	November 24, 2020	7:00-8:00 pm

Breakout Sessions (Authors)

DeAnn Huinker (E)	November 30, 2020	7:00-8:00 pm
Michael Steele (M)	December 1 , 2020	7:00-8:00 pm
Melissa Boston (H)	December 3 , 2020	7:00-8:00 pm

Breakout Sessions (Teachers)

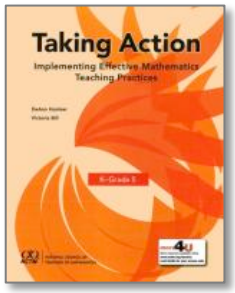
Roshanna Beard (E)	December 7 , 2020	7:00-8:00 pm
Jennifer Perego (M)	December 8 , 2020	7:00-8:00 pm
Fredrick Dillon (H)	December 10, 2020	7:00-8:00 pm
Anthony Bokar (H)		

Closing Session

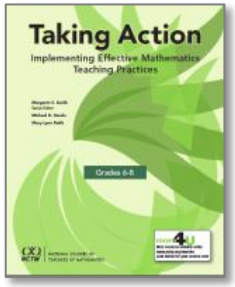
Robert Berry	December 15 , 2020	7:00-8:00 pm


danke
謝謝
ngiyabonga
teşekkür ederim
спасибо
dank je
thank you
gracias
tapadh leat
mauruuru
hvala
bedankt
mochchakkeram
dziękuję
go raibh maith agat
sagolun
sukriya
kop khun krap
obrigado
arigatō
takk
dakujem
grazie
мерси
terima kasih
감사합니다
ευχαριστώ
merci

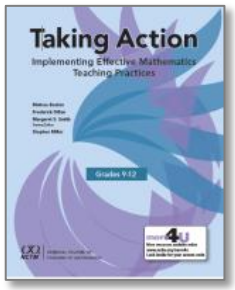
The *Taking Action* Series
Margaret (Peg) Smith, Series Editor



Huinker, DeAnn, and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017. (November 30, 2020)



Smith, Margaret, Michael Steele, and Mary Lynn Raith. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 6-8*. Reston, VA: National Council of Teachers of Mathematics, 2017. (December 1, 2020)



Boston, Melissa, Fredrick Dillon, Margaret Smith, and Stephen Miller. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades 9-12*. Reston, VA: National Council of Teachers of Mathematics, 2017. (December 3, 2020; December 10, 2020))

Levels of Demands

Lower-level demands (memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen 1996; Stein, Lane, and Silver 1996).

Pattern Task

Hexagon Task

Stein, M. K., Smith, M. S., ºHenningsen, M. A., & Silver, E. A. (2000). *Implementing Standards-based Mathematics Instruction: A Casebook for Professional Development.* New York, NY: Teachers College Press. [p.16]