In 1989 the National Council of Teachers of Mathematics (NCTM) launched the standards-based education movement with the release of *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989). Subsequent publications built on this foundation by further detailing the mathematics content of student learning (NCTM 2000, 2006; CCSSI 2010). We have learned, however, that more is needed than content standards if we are to realize the goal of high levels of mathematical understanding by each and every student. For that reason, NCTM formed a writing team in 2012 to envision and develop a document that could guide mathematics education into the future. I served as one of three lead writers on this team. After several revisions, including feedback on a public review draft, our work resulted in the publication of *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014). In this article, I provide some background on *Principles to Actions*, reflect on its initial impact, and suggest next steps for our professional work toward ambitious teaching of mathematics.

**Essential elements**

We cannot just mandate student learning. Standards documents identify rigorous student learning expectations, but it takes high-quality mathematics programs and the collaborative work of teachers, specialists, and administrators to bring about high levels of student learning. A major purpose of writing *Principles to Actions* was to fill this gap between adopting rigorous and challenging content standards and making them a reality for each and every student in our classrooms.

Toward that goal, we described six essential elements of excellent school mathematics programs: (1) Teaching and Learning, (2) Access and Equity, (3) Curriculum, (4) Tools and Technology, (5) Assessment, and (6) Professionalism. Although the first five elements reflect updates to NCTM’s (2000) principles based on more than a decade of experience and new empirical evidence, the sixth principle on professionalism was new and, from my perspective, particularly important to include. The professionalism principle states:

> In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for personal and collective professional growth toward effective teaching and learning of mathematics. (NCTM 2014, p. 99)

This principle highlights how the work before us can be accomplished only as a collective effort, addressing the isolation that many teachers still experience.

If you are just joining the conversation about *Principles to Actions*, I encourage you to start by discussing
the productive and unproductive beliefs identified for each of the six principles. For example, some individuals hold the unproductive belief that “mathematics learning should focus on practicing procedures and memorizing basic number combinations” (p. 11). The contrasting productive belief is that “mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse” (p. 11). A total of thirty-nine pairs of contrasting beliefs are included across the six principles. Note that beliefs should not be viewed as good or bad, but rather “should be understood as unproductive when they hinder the implementation of effective instructional practice or limit student access to important mathematics content and practices” (p. 11). As authors, we debated whether we should include these tables of contrasting beliefs in the book. We now realize they are necessary in prompting powerful conversations. Unless we put our beliefs on the table for examination and discussion, they can become unspoken obstacles or barriers to implementing high-quality mathematics instruction and programs.

Effective teaching as nonnegotiable

As I reflect on the initial impact of Principles to Actions as it relates to the Teaching and Learning Principle, two messages strike me as particularly important. First is our clear message that—

effective teaching engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically. (NCTM 2014, p. 7)

This is the vision that must guide our daily work with students.

To further our understanding of effective teaching, we challenged ourselves as writers to identify a core set of high-leverage practices that define “the essential teaching skills necessary to promote deep learning of mathematics” (p. 9). On the basis of a review of more than two decades of empirical evidence, we identified eight research-informed teaching practices (see fig. 1). This list starts to illustrate the complexity of teaching mathematics, and as such, has given us clearer focus and professional
language for deeper analysis of our own teaching and for engagement in collaborative work with our colleagues.

Whether you are new to the list of teaching practices or have some familiarity with it, please take a moment to read the statements in the accompanying more4u supplement (available online) and note some of the characteristics of each teaching practice. I ask you to do this because often I see just the list of bold phrases in conference sessions or in an online post, but the full sentence starts to unravel some of the details and nuances that comprise each teaching practice. I start most of my own facilitation sessions, whether working with administrators, teachers, or coaches, by reading and highlighting key phrases and then discussing and raising questions about specific characteristics associated with each teaching practice.

NCTM has developed additional resources to support your understanding and implementation of the mathematics teaching practices. A team of writers and contributors was identified, including myself, and we developed the Principles to Actions Professional Learning Toolkit. It is available on the NCTM website for members (www.nctm.org/ptatoolkit). The toolkit contains grade-band modules that include presentation materials (slides, notes, handouts, video) to support professional learning through analyzing artifacts of teaching (e.g., narrative cases, student work samples, vignettes, tasks).

Expanding on the toolkit modules, some team members went a step further and wrote the Taking Action: Implementing Effective Mathematics Teaching Practices series as a set of three grade-band books (elementary, middle, and high school). Each book presents a coherent set of professional learning experiences with a full chapter devoted to each teaching practice. Through narrative cases, vignettes, videos, task analyses, and student work (Huinker and Bill 2017), the authors of the elementary school book, myself included, tried to capture the complexity of teaching mathematics with K–grade 5 students. Whether for your own individual learning or as a book study with colleagues, the book can be a tool for delving deeper into the characteristics and nuances of each teaching practice and gaining insights toward strengthening your own skills in implementing the teaching practices in your classroom.

Access, equity, and ambitious teaching

A concern of mine in discussions of the mathematics teaching practices is the lack of attention to access and equity. These should not be separate conversations. In using the term effective, we meant that the teaching practices are effective for each and every student in the mathematics classroom. In other words, we view access and equity as threads that are interwoven within and across the eight teaching practices. As such, they promote equity-based approaches shown to strengthen mathematical learning, cultivate positive mathematical identities and agency, and position students as mathematically competent (Aguirre, Mayfield-Ingram, and Martin 2013). In the Taking Action series, we include a section in each chapter that elaborates on the connection between the teaching practices and equitable teaching (Huinker and Bill 2017).

Furthermore, the effective teaching practices support what some are calling ambitious teaching of mathematics (Lampert, Boerst, and Graziani 2011), in which teachers view each and every student as capable of engaging
in cognitively demanding work and developing deep understanding of concepts, procedures, and underlying mathematical structures. Teachers skillfully elicit, build on, and support student thinking during instruction, including the use of mistakes and partially formed ideas as important stepping stones to understanding. Ambitious teaching results in students who can use their mathematical knowledge to solve authentic problems and who come to view themselves as confident and capable mathematicians.

Framework for teaching mathematics

Since writing Principles to Actions, I have wondered how we can move from a list of eight teaching practices to viewing them as a coherent framework for teaching mathematics. Knowing the power of a graphic, the authors of the Taking Action series developed a diagram (see fig. 1) to illustrate how we envision the relationships among the teaching practices as a framework for teaching mathematics. This framework supports equitable and ambitious teaching of mathematics.

The teaching practice to establish mathematics goals to focus learning is the first level of the framework. Formulating clear goals for the mathematical ideas we want students to understand sets the stage for all teacher decisions in planning and implementing instruction. With goals as the destination for students’ learning, tasks are the vehicle to reach those goals—the second level of the framework. Note that problem-solving tasks also provide the conceptual base on which fluency is developed. Thus, the teacher might implement tasks that promote reasoning and problem solving, engage students in tasks that build procedural fluency from conceptual understanding, or use tasks that contribute to both of these teaching practices as shown by the dashed line that connects them.

The third level of the framework (the large rectangle) shows the complex interactions as teachers facilitate meaningful mathematical discourse. Discourse forms the heart of any lesson as students share ideas and clarify understanding in whole-class discussions, small-group work, or written work. Discourse is mediated by the four teaching practices highlighted within the discourse rectangle, that is, (1) pose purposeful questions, (2) use and connect mathematical representations, (3) elicit and use evidence of student thinking, and (4) support productive struggle. Discourse makes students’ thinking public and accessible and serves as formative assessment that feeds back into teachers’ instructional decisions for the next lesson. This reflective feedback loop is represented by the dashed line connecting back to goals as the teaching cycle continually repeats.

Our professional work

Since its release, Principles to Actions has become a foundational resource for professional learning. Whether it is a theme at conferences, the topic of book studies, or the focus of coaching conversations, the book has clearly given us much-needed common professional language to engage in serious discussions on strengthening mathematics teaching. Being able to name those aspects of our teaching practice that are effective for student learning allows us to focus our attention on ensuring that each teacher in our schools becomes skillful in those teaching actions needed for ambitious teaching of mathematics (Ball and Forzani 2010).

The articles published in Teaching Children Mathematics (TCM) have contributed greatly over the years in moving toward effective teaching of mathematics. The articles spark our reflections, conversations, and dialogue about effective teaching in the elementary grades. As a former chair of the editorial panel (many years ago), I still anticipate and cherish each issue of the journal and often revisit and use past articles in my professional work. This department is edited by Tonya Bartell, tbartell@msu.edu, an associate professor of mathematics at Michigan State University in East Lansing; and Anita A. Wager, Professor of the Practice, Director of Elementary and ECE Programs at Vanderbilt University, Peabody College Department of Teaching and Learning, in Nashville, Tennessee.

On the second Wednesday of each month, TCM hosts a lively discussion with authors and readers about an important topic in our field.

On December 12, 2018, at 9:00 p.m. ET, we will discuss “What Happened at Frida’s Museum?” by Jesica Peña, José Luis Cortina, and Jana Višňovská. Follow along using #TCMchat.

Unable to participate in the live chat? Follow us on Twitter @TCM_at_NCTM and watch for a link to the recap.

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Promoting productive discourse

BY ROBIN SILBEY, PD AND CAMPUS CONSULTANT

Mathematical discussions play an integral part of every math lesson. The mathematical learning of every student advances when students engage in productive talk. While planning lessons with teachers, help them prepare questions that will nurture productive math discourse. Encourage teachers to use a variety of discourse-friendly structures, including partner work, partner sharing with another pair, and small-group discussion. Discourse should be sprinkled throughout the lesson, with questions planned for each part.

During the development of a new concept, counsel the teacher to think about prior knowledge and common misconceptions.

- What questions can you ask to connect previous learning to the day’s new learning?
- What are some anticipatory questions that you can ask to help students avoid common misconceptions?
- Have teachers show an example of a problem solved incorrectly and pose questions about it: What is incorrect about this response? What error did the student make? What would you say to the student to clarify his misconception?

While students work during independent time and teachers circulate around the classroom, ask them to think about how students’ solutions can be shared and used to spark discussion.

- Which students’ responses clearly show their thinking?
- Which students’ responses show divergent thinking or a unique but reliable solution process?
- What activities or grouping structures can be used to maximize the benefit of the discussion?
- What questions help students connect the solution strategies to each other?

During the student reflection, encourage teachers to tailor questions to assess student understanding.

- What questions help students verbally articulate what they did with numbers today?
- What questions will help students anticipate the application of today’s work to future learning?

Productive discourse using authentic, real-time classwork to reinforce understanding helps students invest in their own learning. Students learn from one another and value one another as resources while developing the habit of reflecting on their thought processes and constructing understandings through rich discussions.

Questions? Comments? Contact robyn@robynsilbey.com.