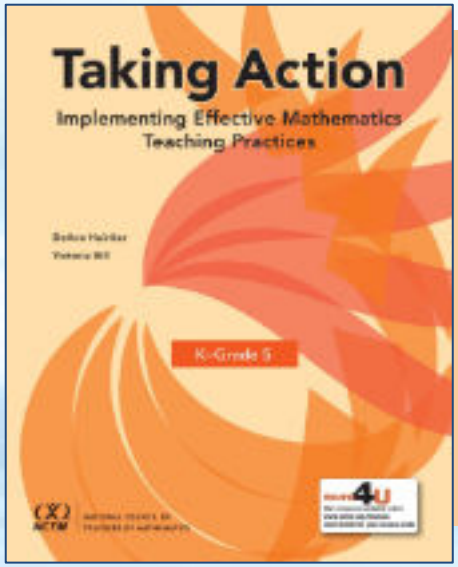
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

# *Taking Action: Bringing the Effective Teaching Practices to Life in your Classroom*




## Focus on Elementary School: Connecting Representations and Posing Purposeful Questions

DeAnn Huinker, University of Wisconsin–Milwaukee
Paige Richards, Mathematics Institute of Wisconsin

NCTM Webinar Series, November 30, 2020

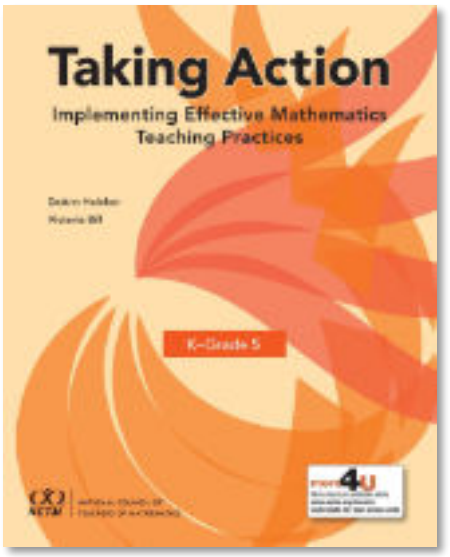
DeAnn Huinker
University of Wisconsin–Milwaukee
huinker@uwm.edu
@dh11235

Paige Richards
Mathematics Institute of Wisconsin
paige.richards@mathinstitutewi.org
@mathinst_Paige

The work we will do together draws from NCTM Resources:

- *Principles to Actions*
- *Taking Action* series

(NCTM, 2014)




(Huinker & Bill, 2017)

Yes! Slides and other resources will be shared through a link to a Google Folder at the end of the session.

**Framework for teaching mathematics**

- Eight effective math teaching practices

**Teaching with a focus on representations**

- Classroom artifact: Student work samples

**Becoming more purposeful with our questions**

- Classroom artifact: Excerpts from a class discussion

**Taking Action**

# *Principles to Actions*

# Background

In *Principles to Actions*, NCTM identified eight effective mathematics teaching practices that "represent a core set of **high-leverage** practices and **essential teaching skills** necessary to **promote deep learning** of mathematics" (p. 9).

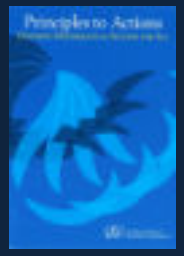
National Council of Teachers of Mathematics. (2014). *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: NCTM.

# *Principles to Actions: Ensuring Mathematical Success for All*

A research-informed consensus on what constitutes effective teaching accumulated over the prior 25 years.

# Eight Effective Mathematics Teaching Practices

- Establish mathematics **goals** to focus learning.
- Implement **tasks** that promote reasoning & problem solving.
- Use and connect mathematical **representations.**
- Facilitate meaningful mathematical **discourse.**
- Pose purposeful **questions**.
- Build procedural **fluency** from conceptual **understanding.**
- Support productive **struggle** in learning mathematics.
- Elicit and use **evidence** of student thinking.

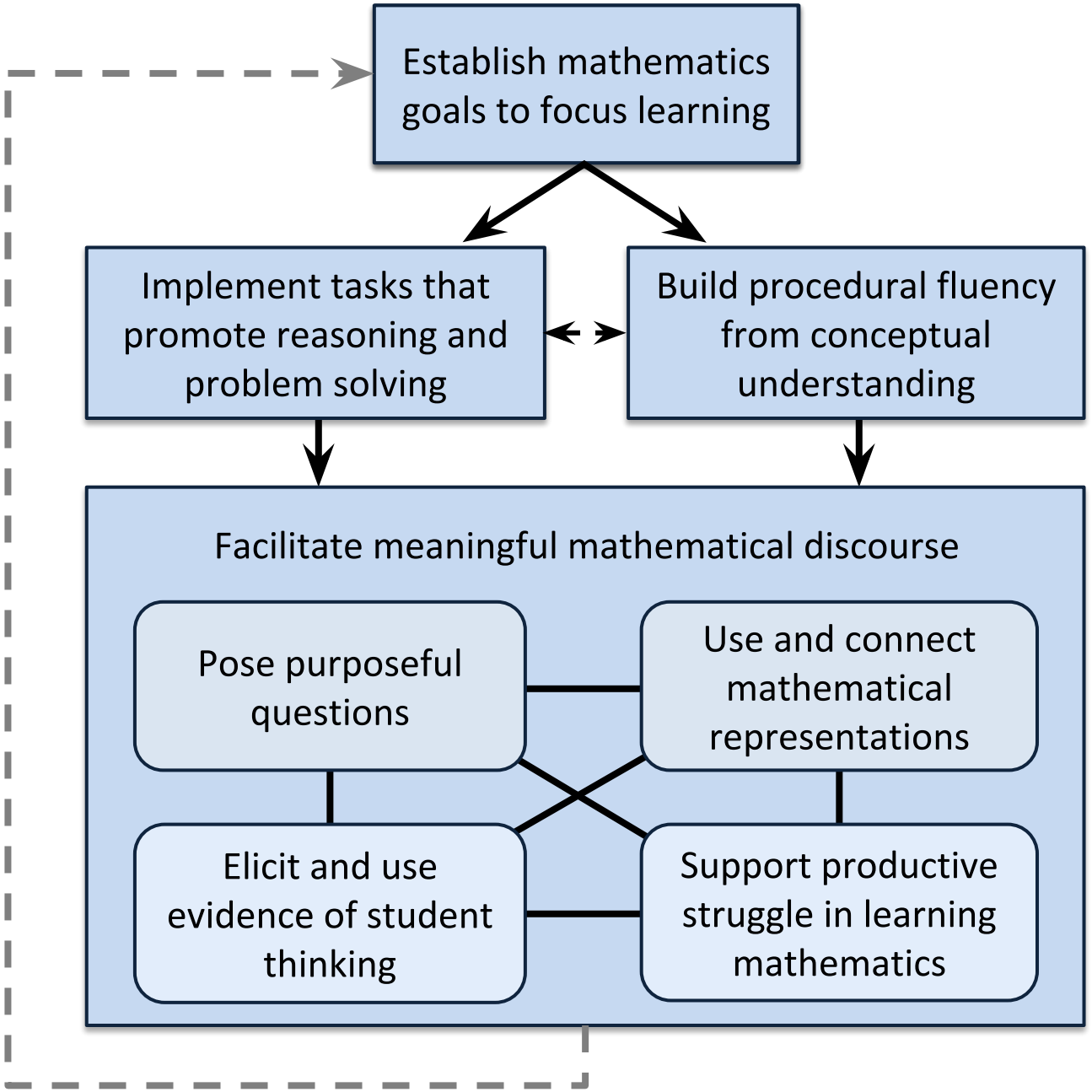


Move from a list of eight teaching practices for mathematics

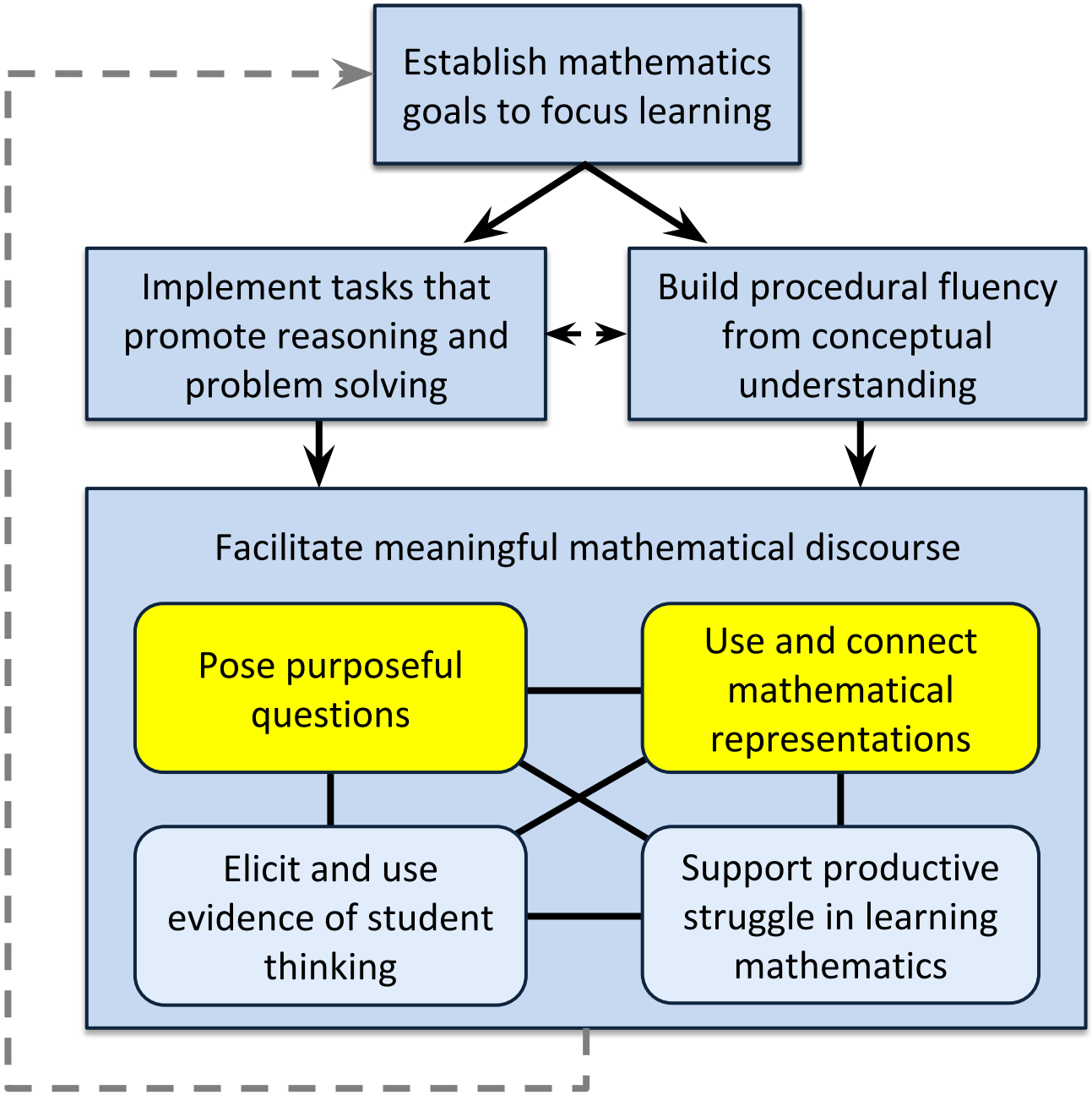
Coherent and connected framework for the teaching of mathematics

# Mathematics Teaching Framework

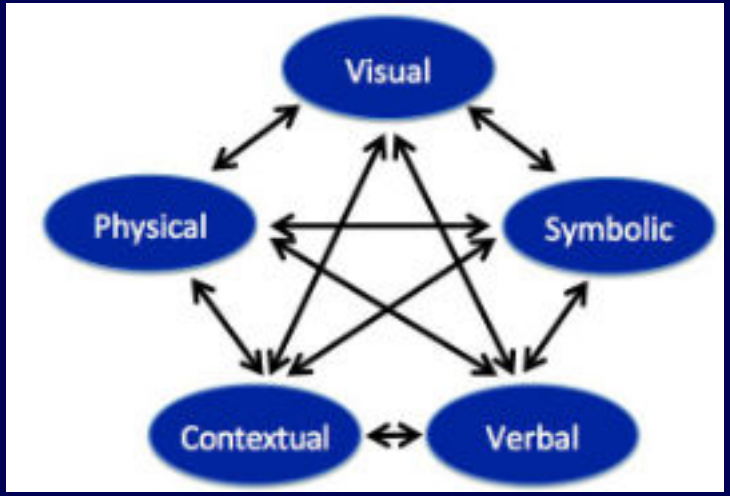
(p. 245)



# Mathematics Teaching Framework



# Teaching with a Focus on Representations

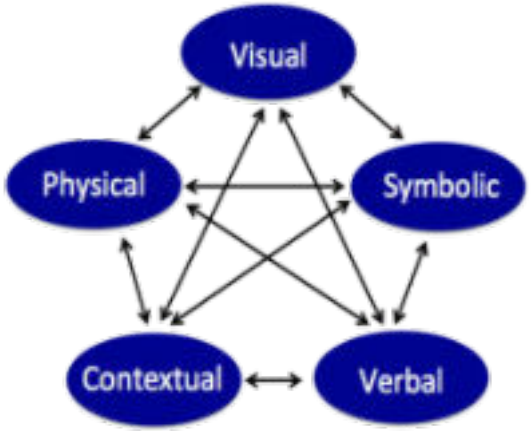


Math Teaching Practice

# Use and connect mathematical representations

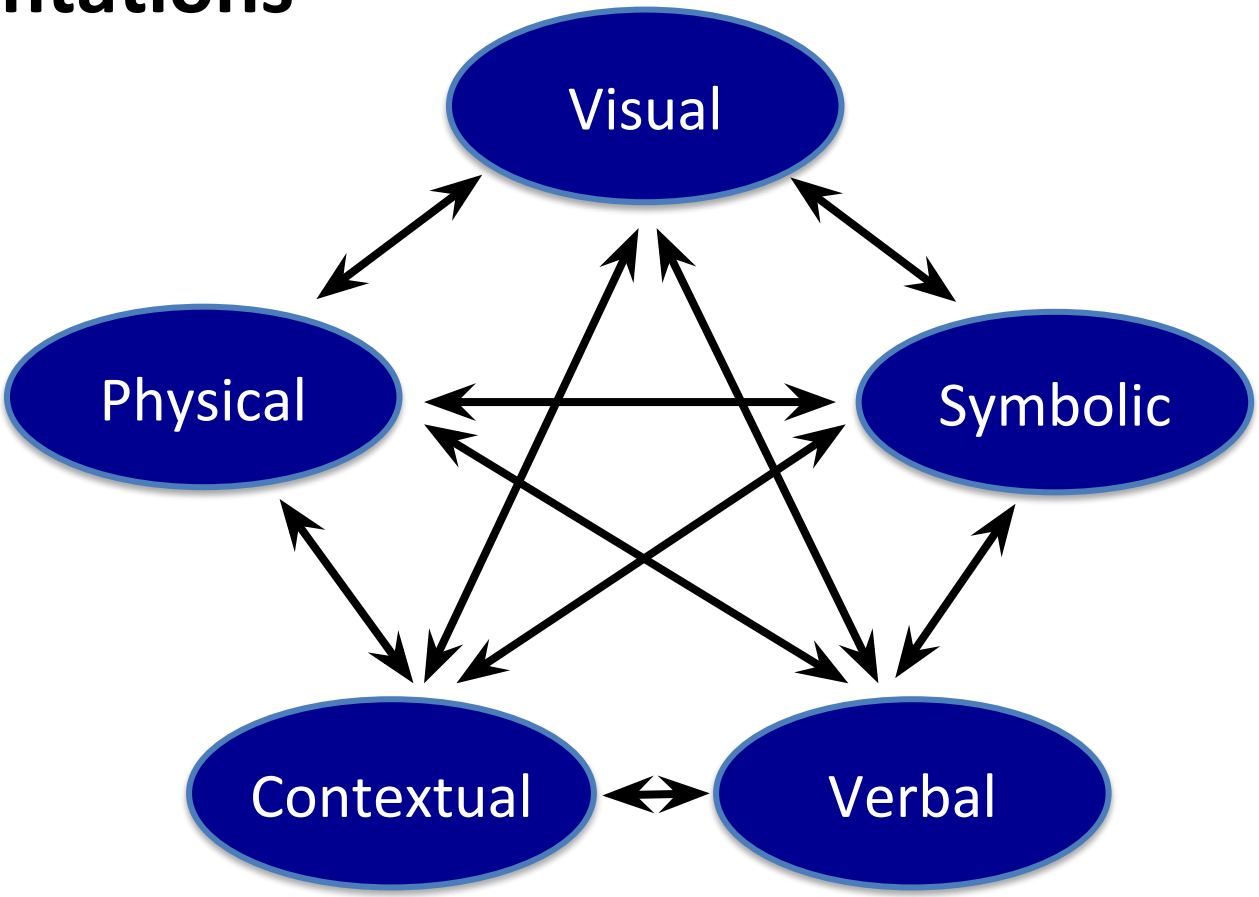
Effective teaching of mathematics engages students in making connections among mathematical representations:

- to **deepen understanding** of mathematical concepts and procedures.
- as **tools** for problem solving.



National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

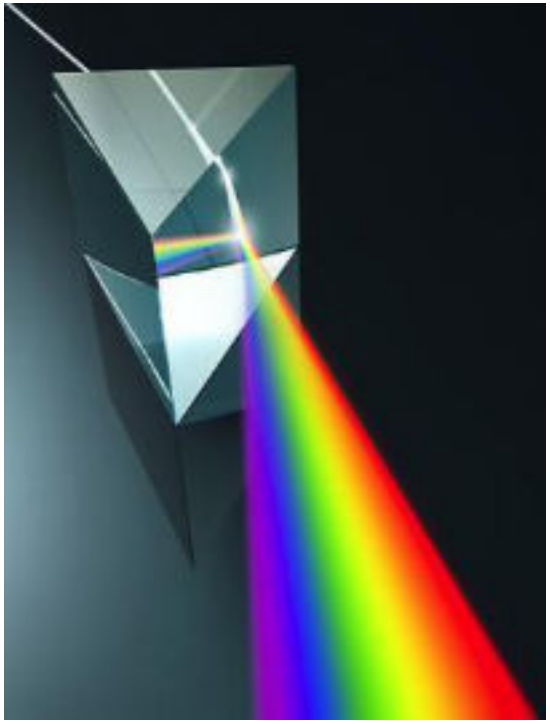
# Five types of representations




Adapted from: Lesh, R., Post, T., & Behr, M. (1987). Representations and Translations among Representations in Mathematics Learning and Problem Solving. In C. Janvier, (Ed.), *Problems of Representations in the Teaching and Learning of Mathematics* (pp. 33-40). Hillsdale, NJ: Erlbaum.

# Why focus on multiple images to represent a mathematical idea?

"Using different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture (math idea) richer and deeper."

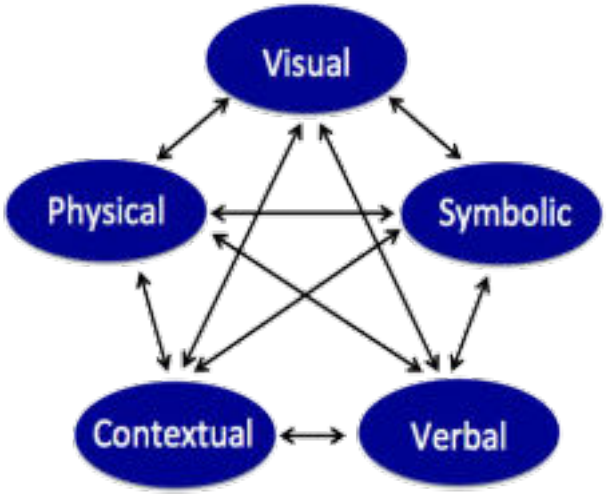


(Tripathi 2008, p. 439)

# Teaching with Representations

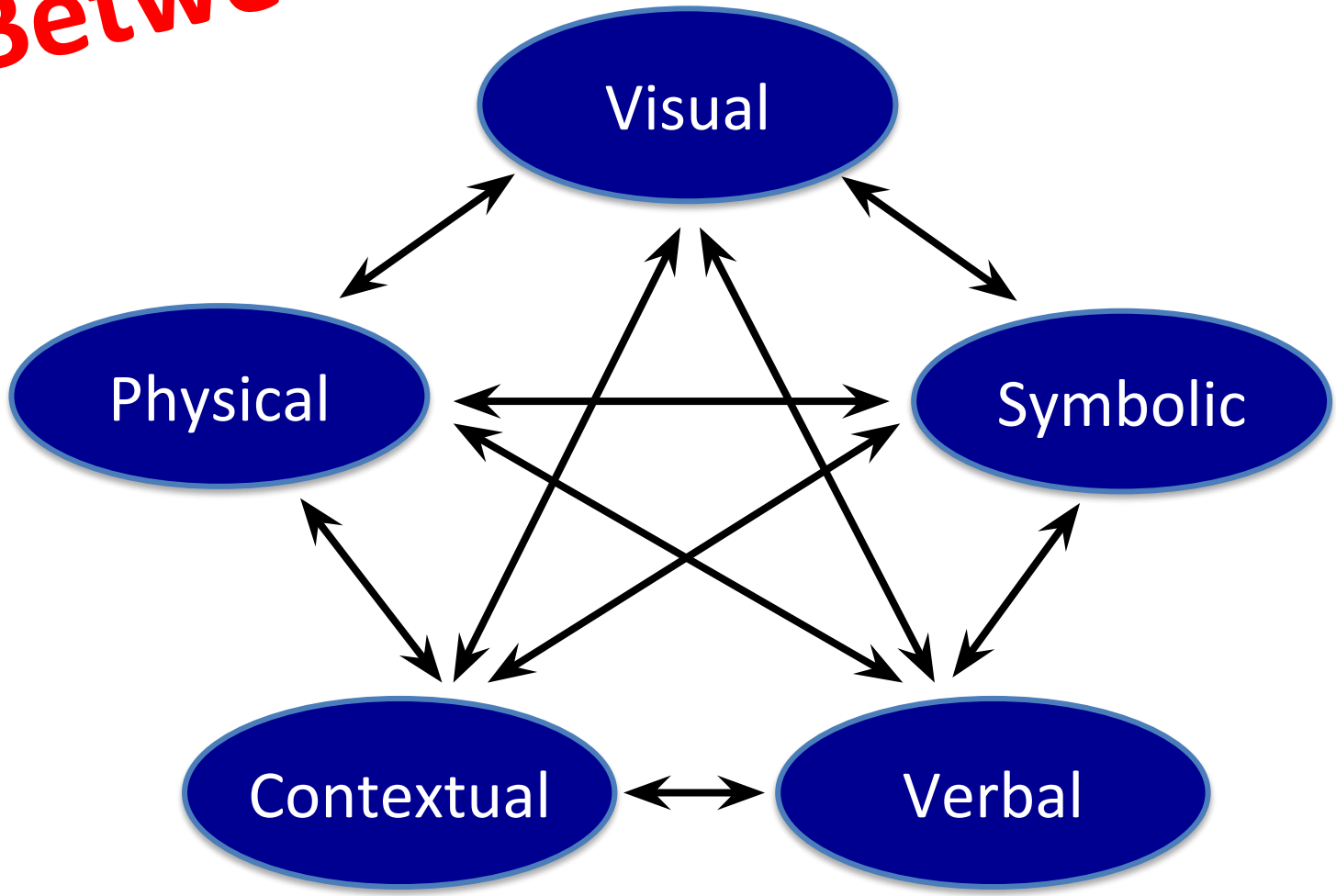
"Not only are the distinct types of representations important, but translations between them and transformations within them, are also important."

(Lesh, Post, & Behr, 1997, p. 34)

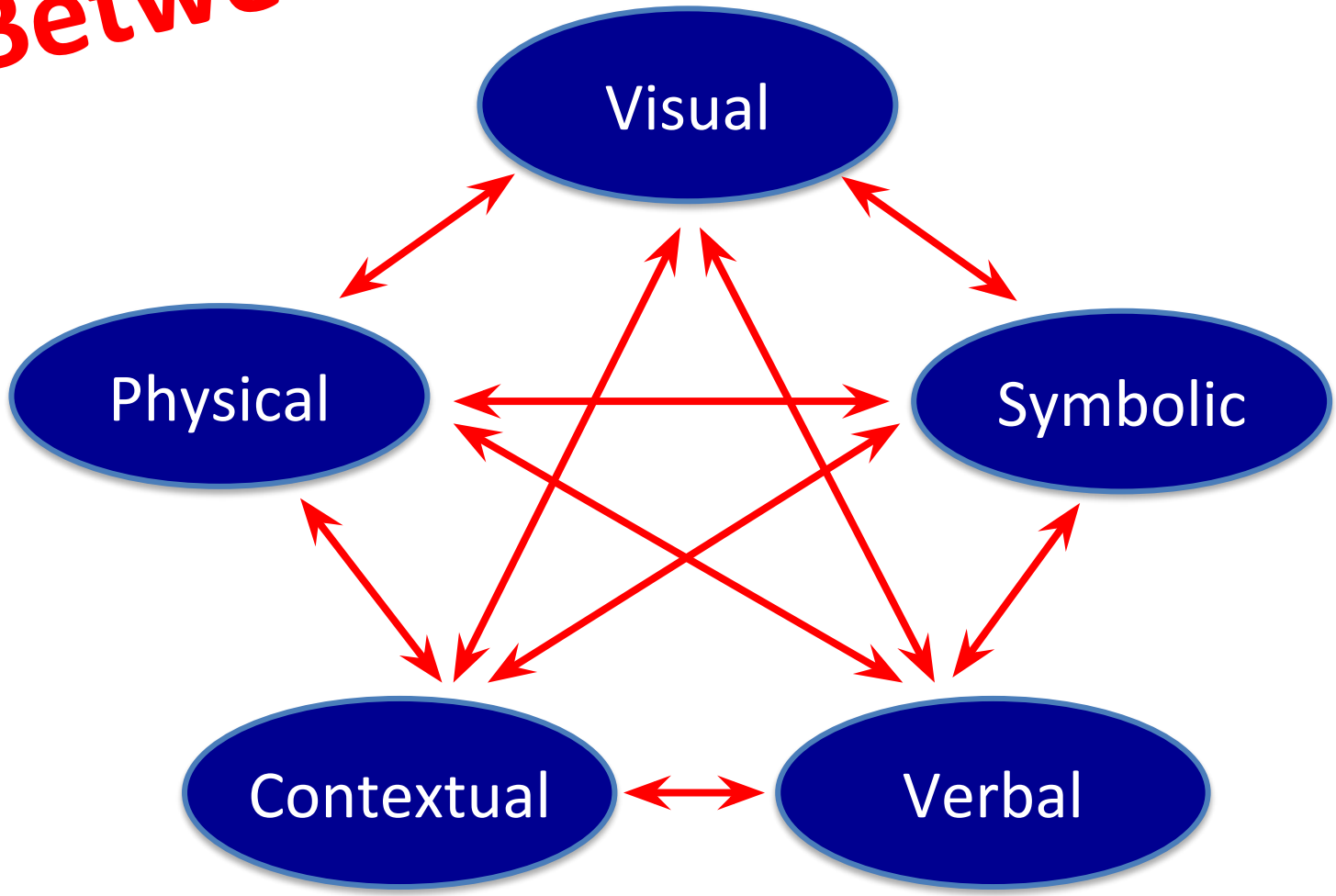



Lesh, R., Post, T., & Behr, M. (1987). Representations and Translations among Representations in Mathematics Learning and Problem Solving. In C. Janvier, (Ed.), *Problems of Representations in the Teaching and Learning of Mathematics* (pp. 33-40). Hillsdale, NJ: Erlbaum.

# Between

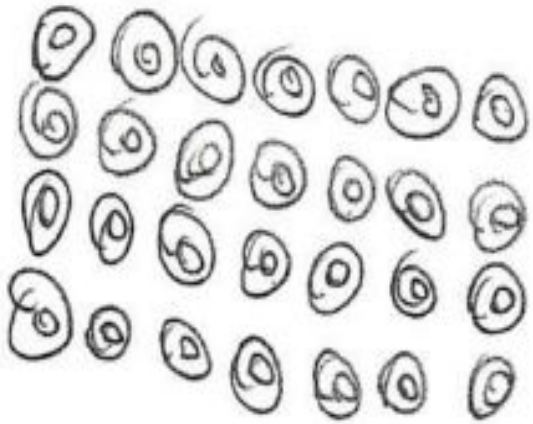


# Between



# Connections between Representations

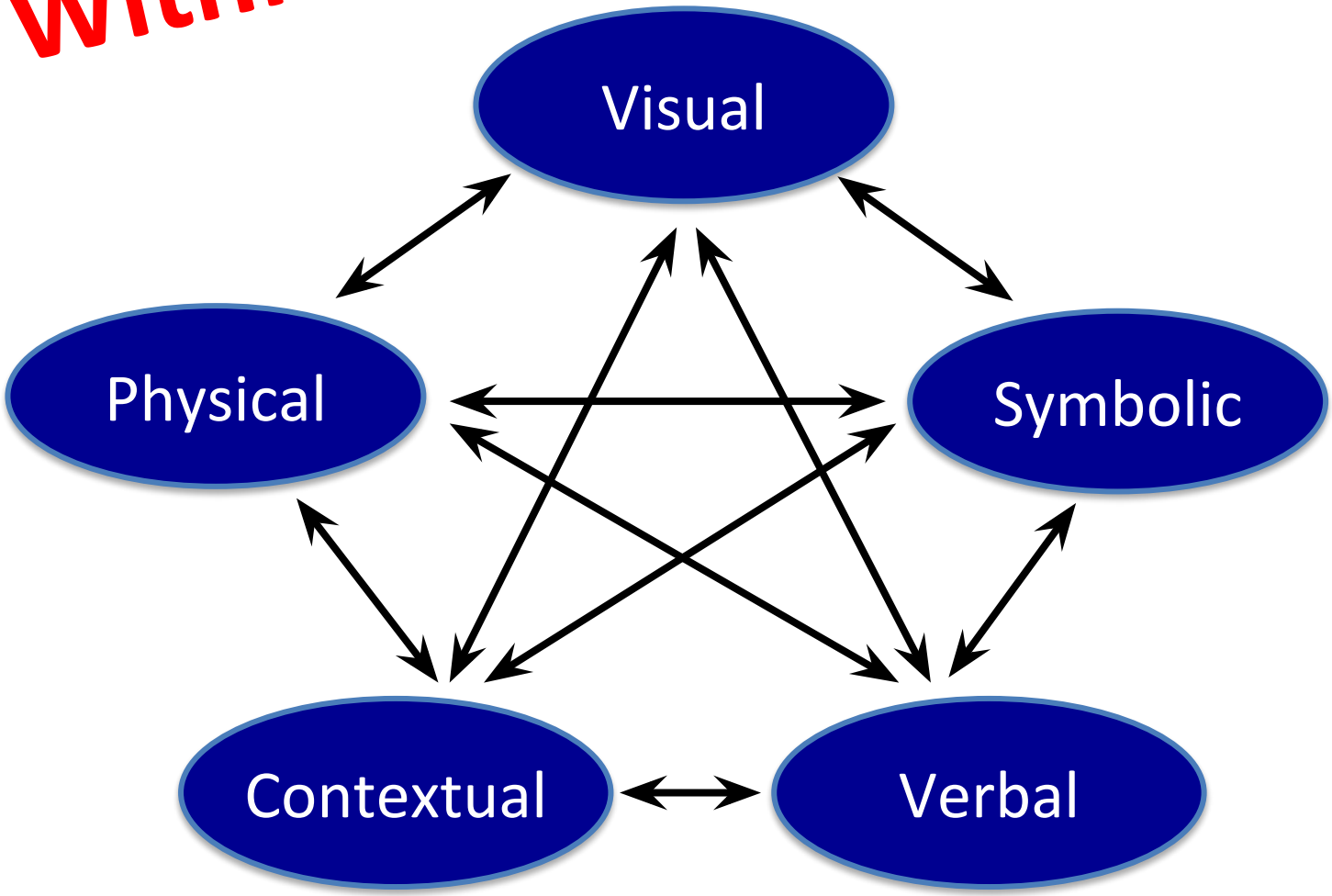
4 x 7

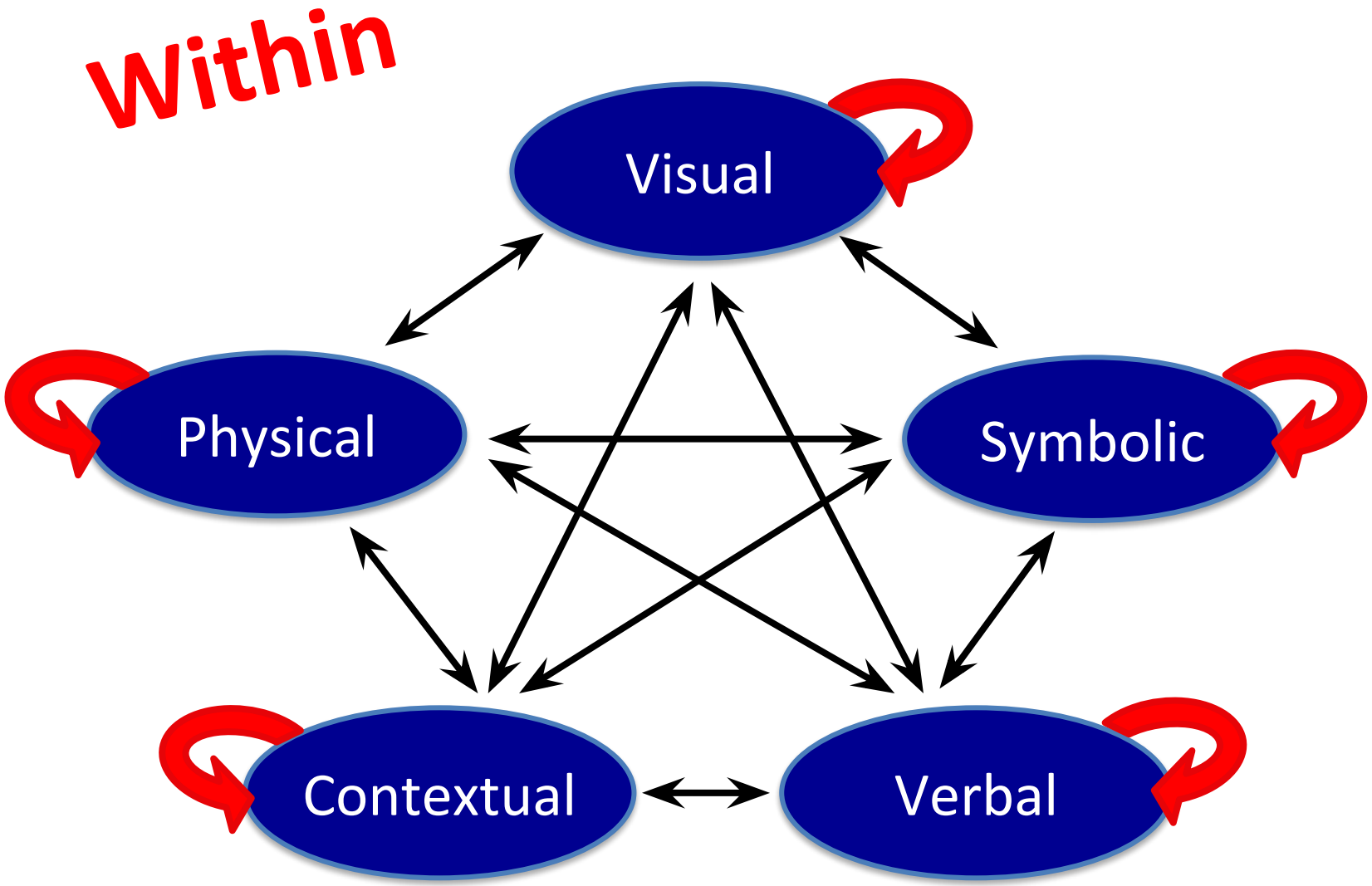


Symbolic ⟷ Visual

- What does the 4 tell us about the visual representation?
- Where is 7 in the picture?
- What does “x” have to do with the picture?

# Within

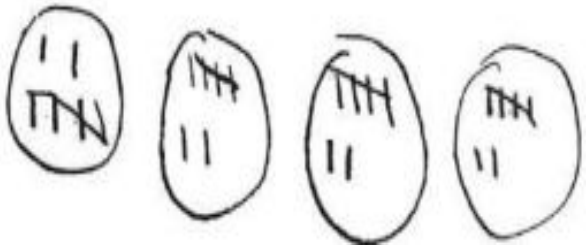


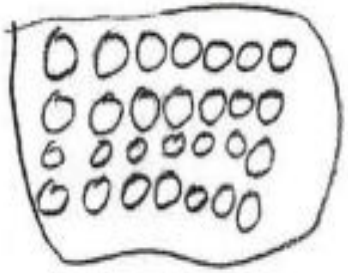

Within
Visual
Physical
Symbolic
Contextual
Verbal

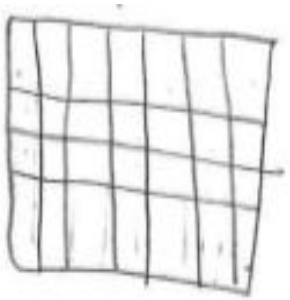
# Connections within Representations.

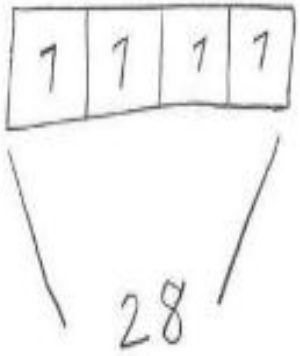
Visual

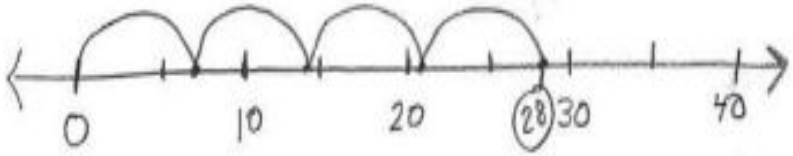
## 4 x 7

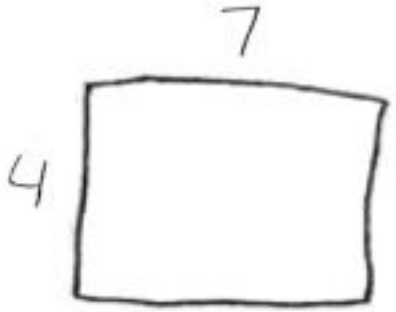









1. What is the same about each visual representation?
2. What are some of the differences?
3. Where is the math idea easier to see? harder to see?

# Assessing Connections among Representations

# Analyzing Teaching and Learning (ATL) 6.1

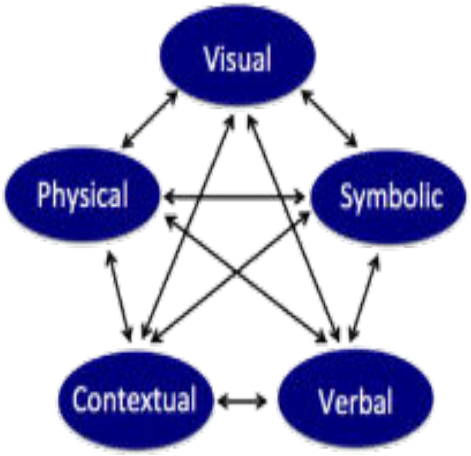
## Multiplication Check-In

1. Find the answer: 4 x 7 = ?
2. Explain how you figured out the answer.
3. Draw a picture that shows what 4 x 7 means.
4. Write a story problem for 4 x 7.

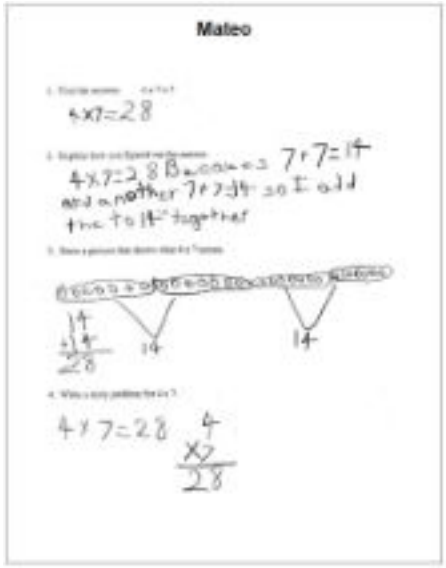
Huinker, DeAnn, and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017. (pp. 121-125)

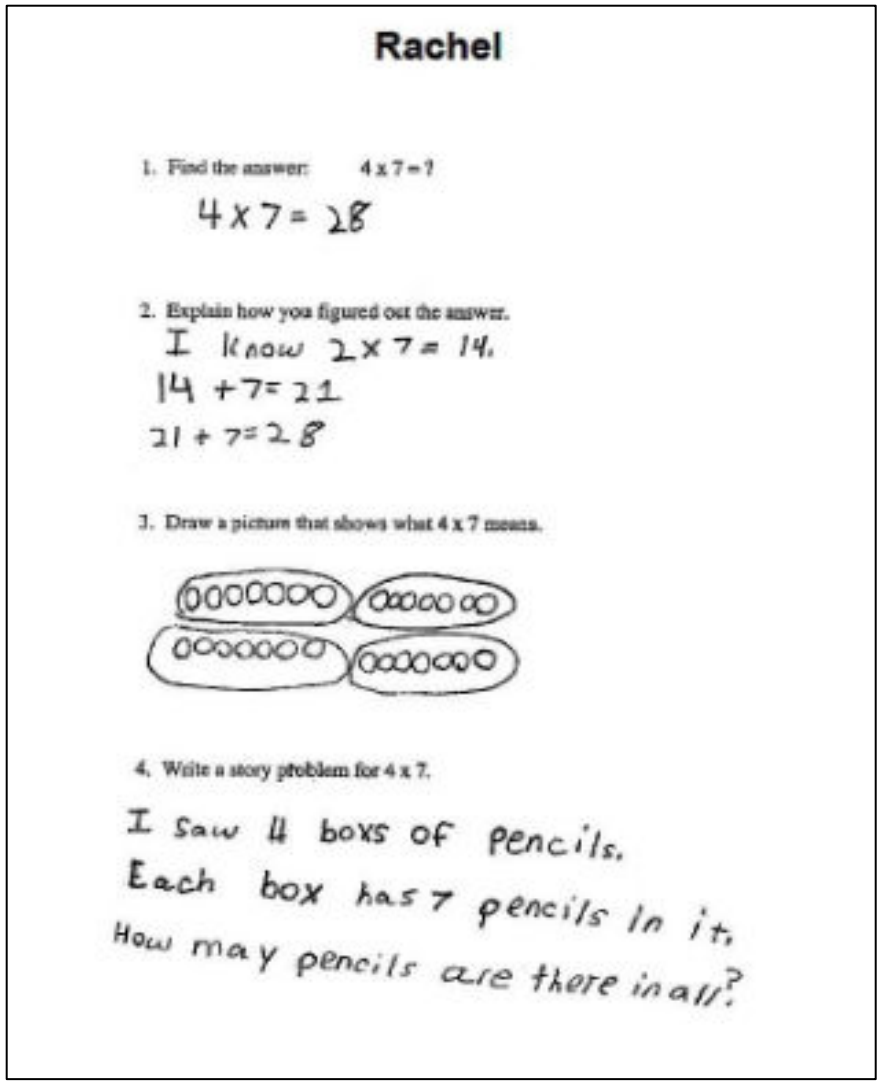
Breakout Rooms

# Grade 3 Multiplication Check-In




- Introduce yourselves to each other.
- Study all 4 student work samples.
- What does each reveal about the child's ability to connect representations for multiplication?
- What might be some implications for instruction?





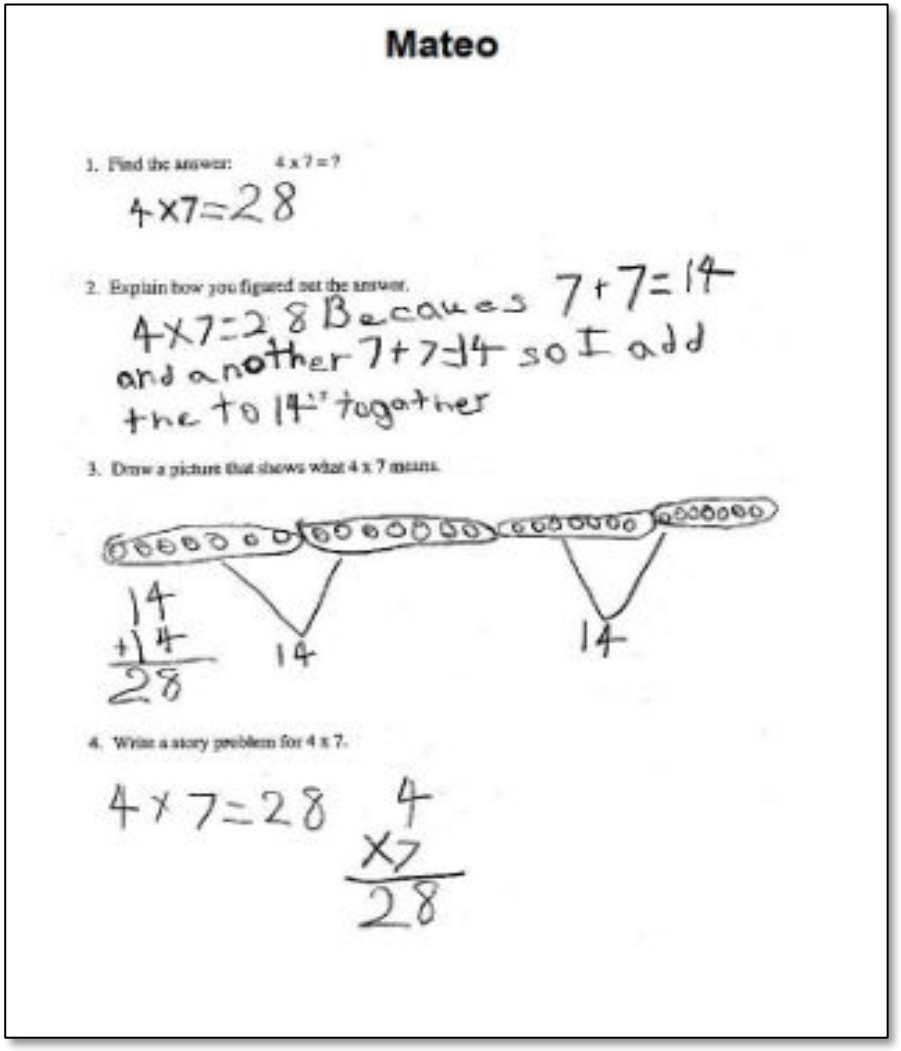

# Rachel

Correct visual & correct context.

Understands equal group situations and can connect an equation (symbolic) to both visual and contextual representations.

**Next Steps**

Focus on arrays, tape diagrams, number lines, and stories with measurement contexts.



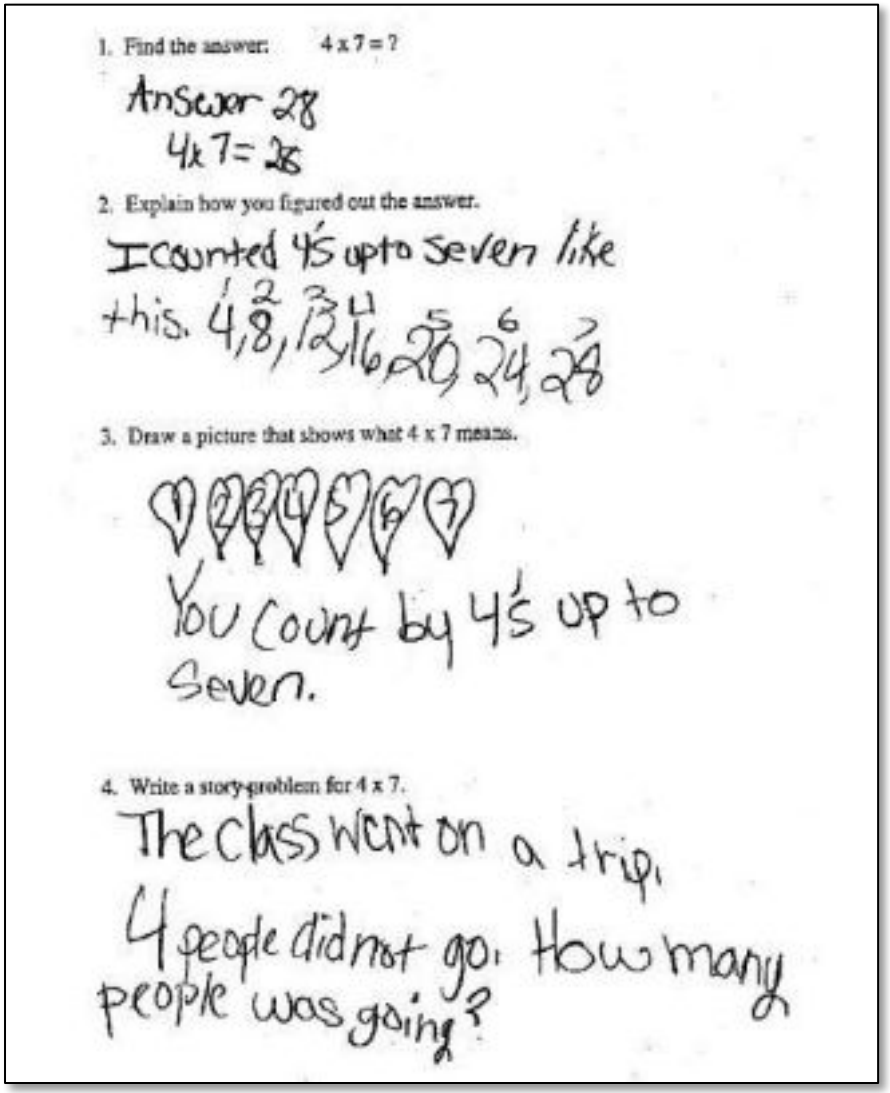

# Mateo

Correct visual & no context.

He seems to know multiplication involves combining equal groups, but likely has limited experiences with contexts for multiplication.

## Next Steps

Connecting to real-world contexts and to other visual representations, such as arrays and tape diagrams.



1. Find the answer: $4 \times 7 = ?$

Answer 28

$4 \times 7 = 28$

2. Explain how you figured out the answer.

I counted 4's upto seven like this. 4, 8, 12, 16, 20, 24, 28

3. Draw a picture that shows what 4 x 7 means.



You count by 4's up to seven.

4. Write a story problem for 4 x 7.

The class went on a trip. 4 people did not go. How many people was going?

# Olivia

Incorrect visual & incorrect context.

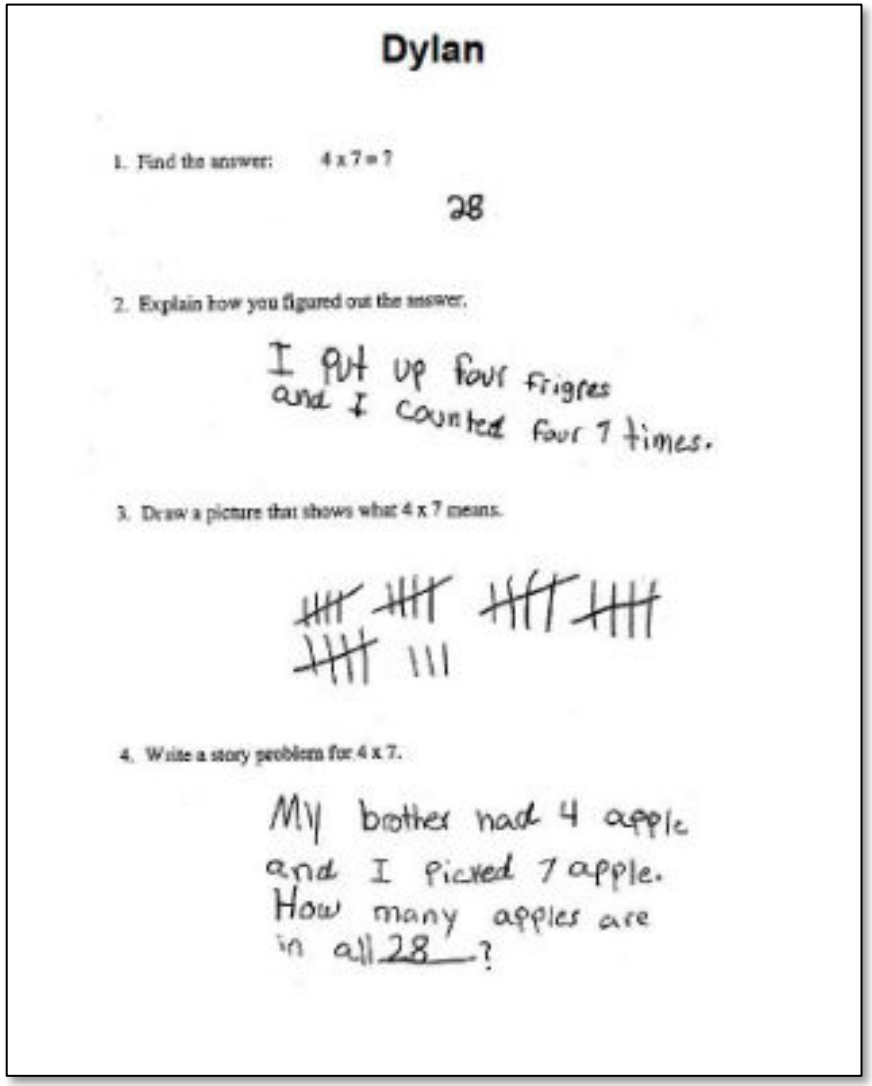
She can skip count but may not understand multiplication as combining equal groups.

**Caution**

Skip counting can present a false sense of understanding.

**Next Steps**

More work with visual representations and story contexts.



Dylan

1. Find the answer: 4 x 7 = ?

28

2. Explain how you figured out the answer.

I put up four frigres and I counted four 7 times.

3. Draw a picture that shows what 4 x 7 means.



4. Write a story problem for 4 x 7.

My brother had 4 apple and I picked 7 apple. How many apples are in all 28 ?

# Dylan

Incorrect visual & incorrect context.

Dylan likely counted by ones, and he lacks connections to visual diagrams showing equal groups and to real-world contexts.

**Next Steps**

More experiences with physical and visual representations for story contexts about equal groups.

**Grade 3**

6 classrooms

6 different schools

Total of 133 students

November Check-in

# Multiplication Connections Check-in: 4 x 7 = ?
# Grade 3 Results: Percent Correct

| Class | Number of Students | Answer | Visual (Picture) | Context (Story) |
|---|---|---|---|---|
| Class A | 20 | | | |
| Class B | 26 | | | |
| Class C | 24 | | | |
| Class D | 15 | | | |
| Class E | 24 | | | |
| Class F | 24 | | | |
| **Totals** | **133** | | | |

# Multiplication Connections Assessment: 4 x 7 = ?
# Grade 3 Results: Percent Correct

| Class | Number of Students | Answer | Picture (Visual) | Story (Context) |
|---|---|---|---|---|
| Class A | 20 | 80% | 85% | 50% |
| Class B | 26 | 88% | 54% | 4% |
| Class C | 24 | 88% | 75% | 67% |
| Class D | 15 | 80% | 67% | 27% |
| Class E | 24 | 79% | 33% | 17% |
| Class F | 24 | 92% | 75% | 25% |
| **Totals** | **133** | **85%** | **64%** | **31%** |

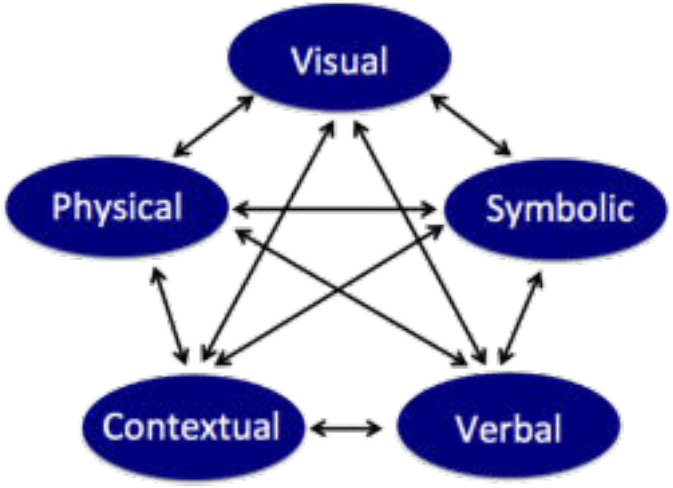
**Some observations . . . 133 students . . .**

- 85% of students had the right answer.
- 64% drew an appropriate picture.

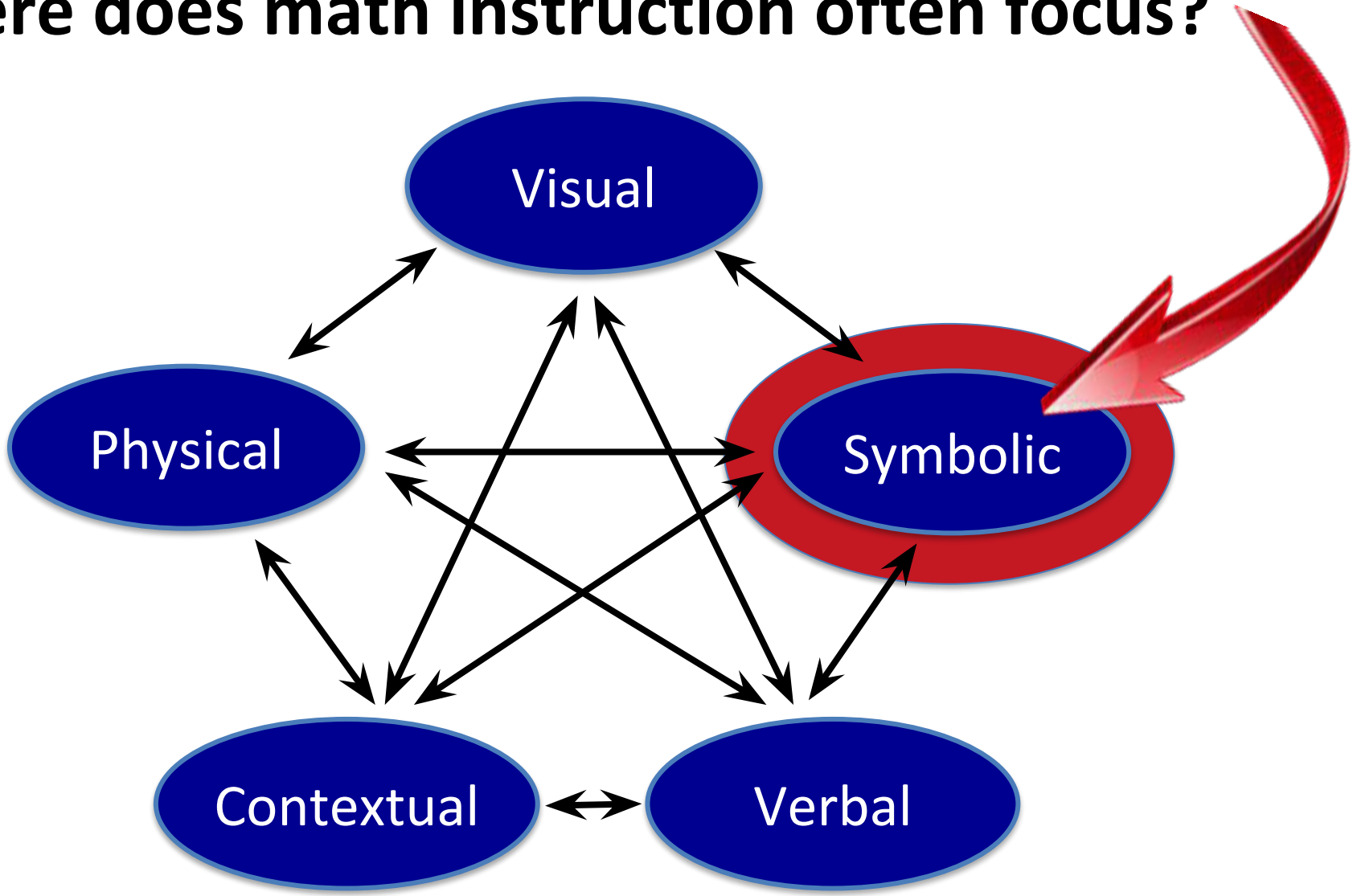
  ***But 48 students did not!***

- 31% had an appropriate context.

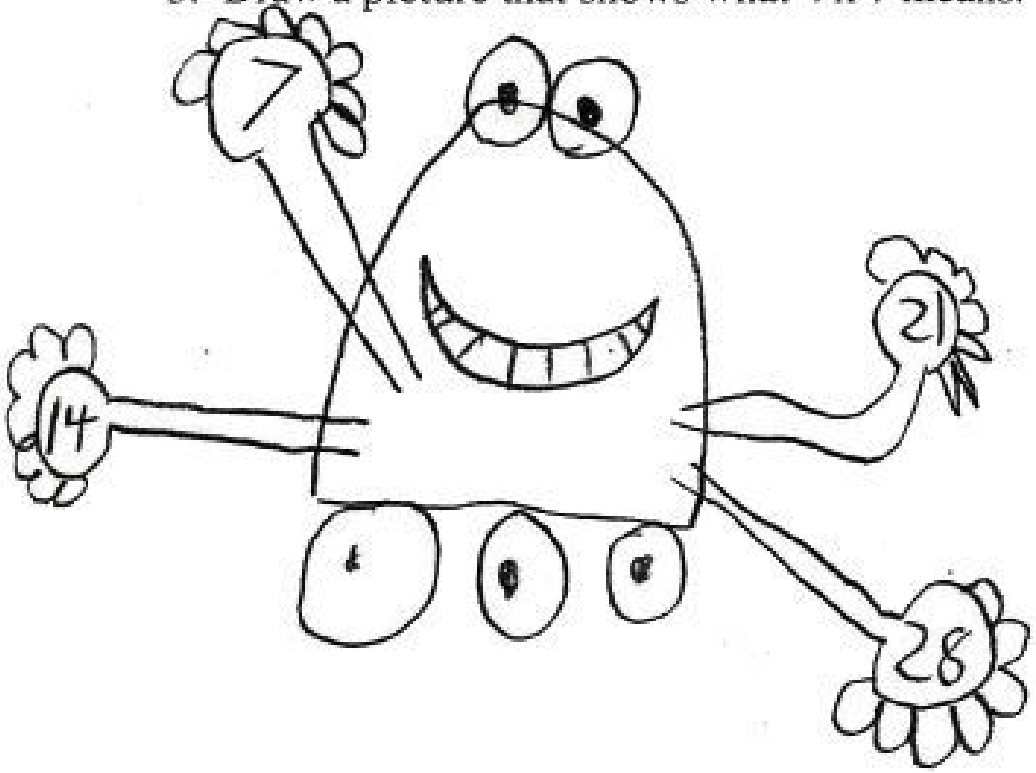
  ***But 92 students did not!***



# Where does math instruction often focus?

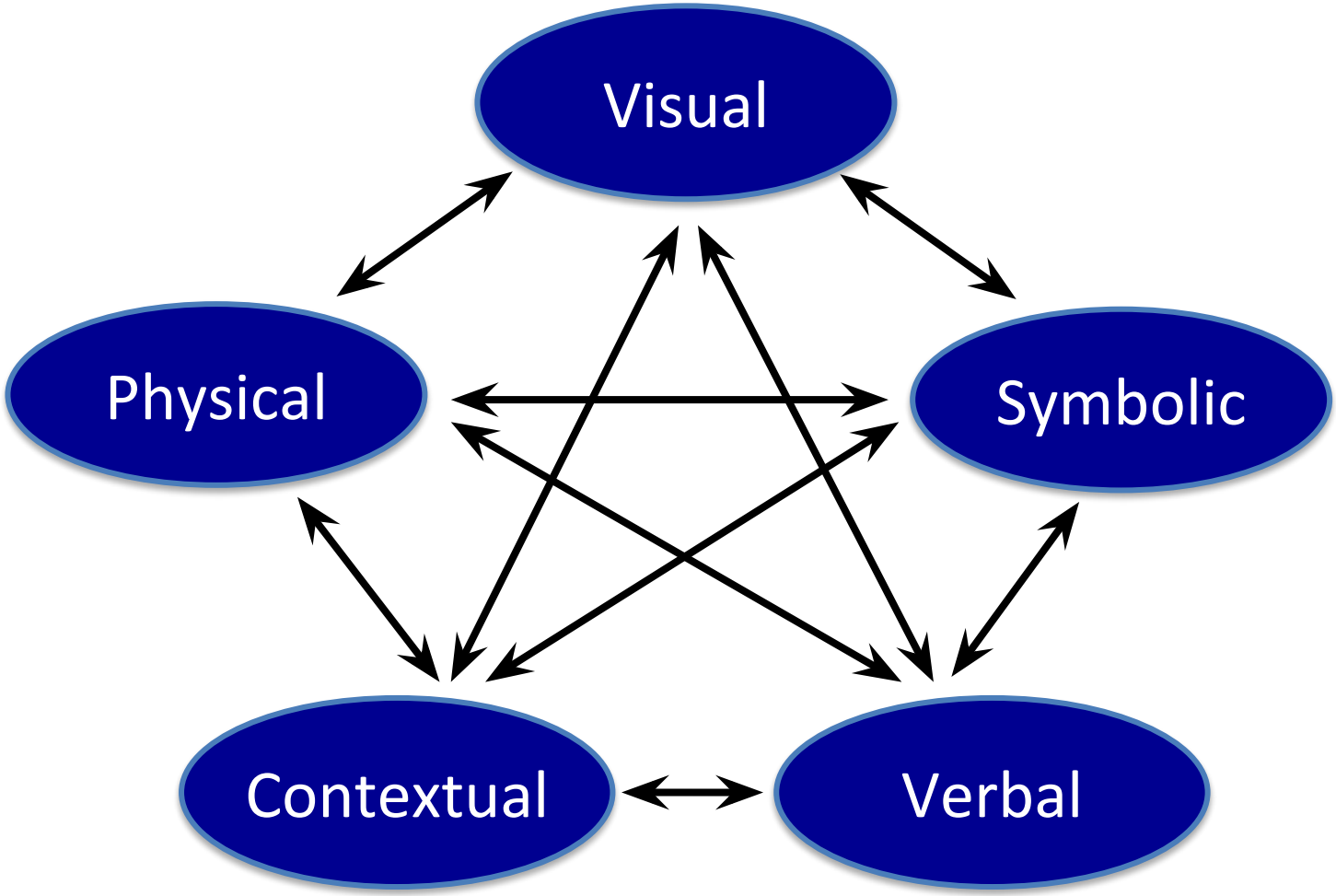


3. Draw a picture that shows what 4 x 7 means.



4. Write a story problem for 4 x 7.

I have 4 hands. Each hand has 7 figers. I have 28 figers in all.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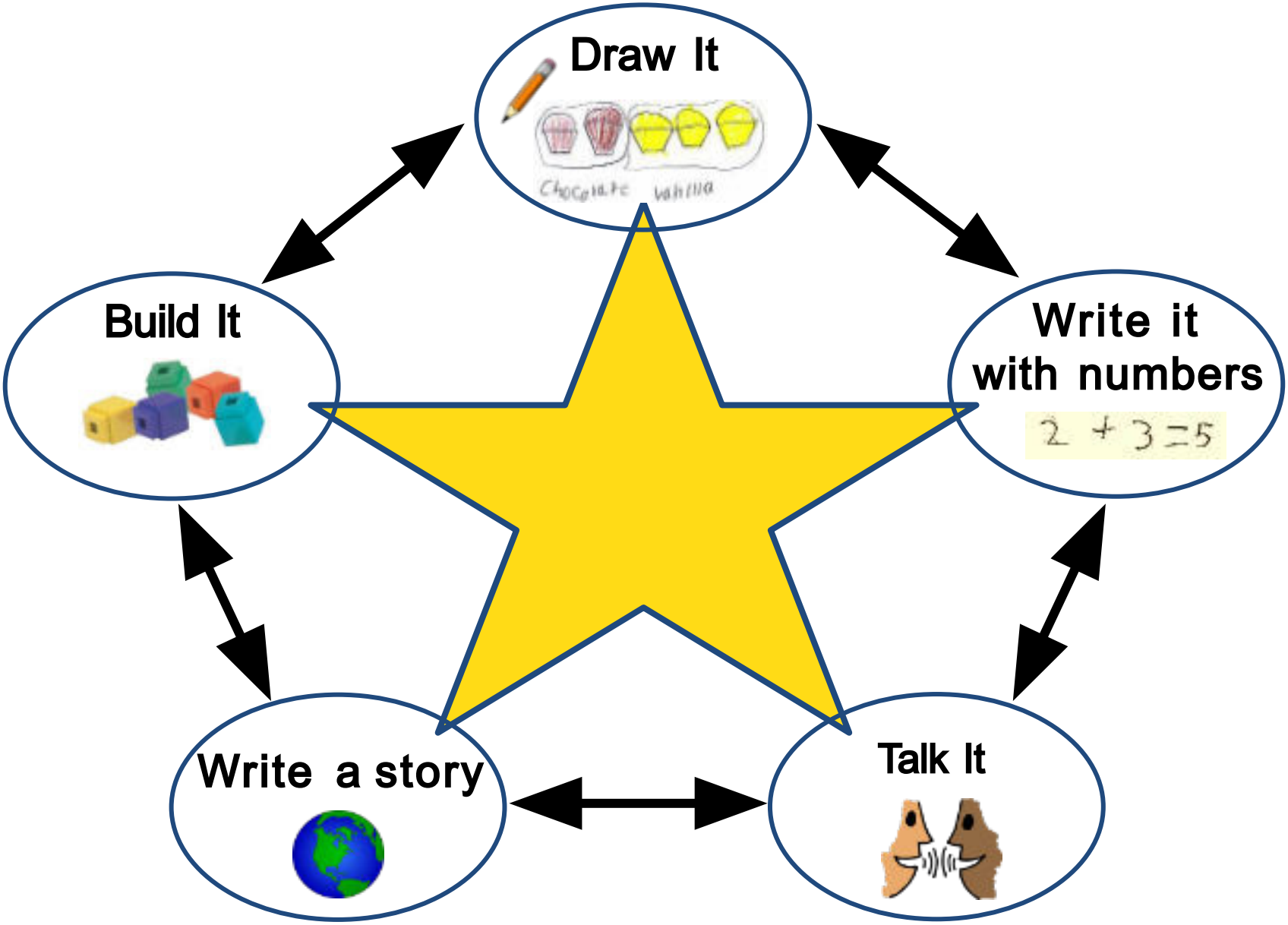
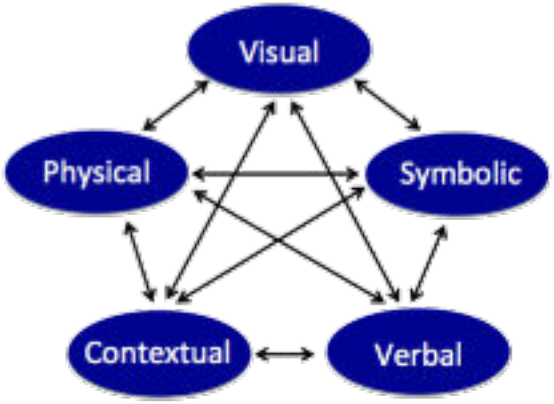



Developed by Beth Schefelker (South Milwaukee School District) and DeAnn Huinker (University of Wisconsin-Milwaukee), November 2015.

# Key Messages




- Teaching with representations is a high-leverage, effective teaching practice.
- Our goal for students is to develop the ability to move with ease and flexibility between and within different types of representations.
- Teaching and assessment must attend to the “arrows” by asking students to translate in both directions (alternate directionality) among representations.

# What does the research say?

**Representational . . .**

**Flexibility** (Greer, 2009)

**Fluency** (Nathan et al., 2010)

**Thinking** (Pape & Tschoshanov, 2001)

**Versatility**(Graham & Thomas, 2008)

**GOAL for students**:

*An ease in moving back and forth among representations.*

**Role of the teacher**:
Intentional and purposeful questioning.


Dreyfus, T. & Eisenberg, T. (1996). On different facets of mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), *Nature of Mathematical Thinking* (pp. 253-284). Erlbaum.

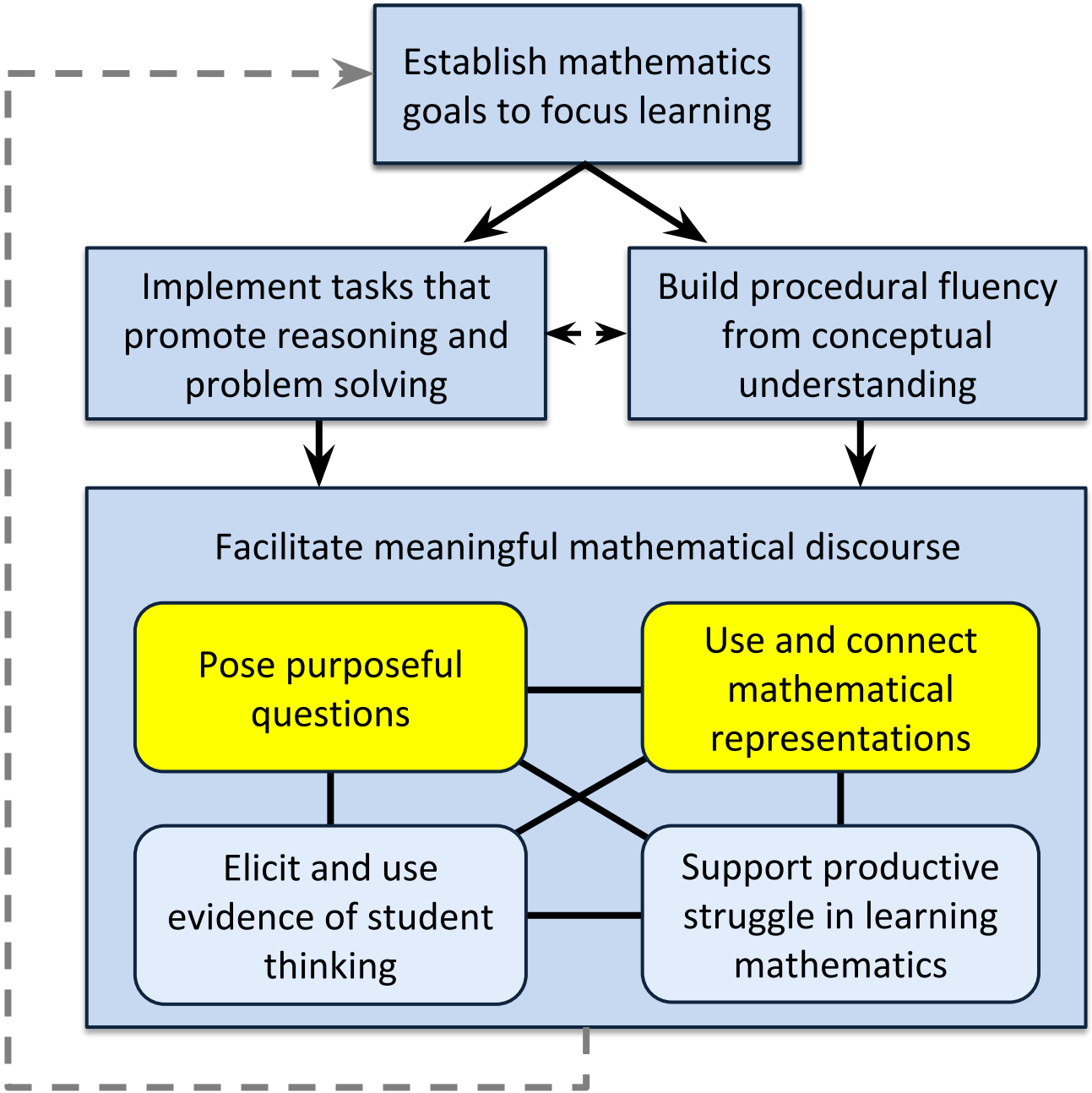
Graham, A. T., & Thomas, M. O. (2005). Representational Versatility in Learning Statistics. *International Journal for Technology in Mathematics Education*, *12*(1), 3-14.

Greer, B. (2009). Representational flexibility and mathematical expertise. *ZDM*, *41*(5), 697-702.

Nathan, M. J., Alibali, M. W., Masarik, K., Stephens, A. C., & Koedinger, K. R. (2010). *Enhancing middle school students' representational fluency: A classroom-based study*.

# Becoming more Purposeful with our Questions

# Mathematics Teaching Framework



Math Teaching Practice

# Pose Purposeful Questions

Effective teaching of mathematics uses purposeful questions to ***assess and advance*** students' reasoning and sense making about important mathematical ideas and relationships.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: NCTM.

# Hungry Caterpillar Task

On Monday, the hungry caterpillar ate through one apple, but he was still hungry. On Tuesday he ate through two pears, but he was still hungry. On Wednesday he ate through three plums. On Thursday he ate through four strawberries. On Friday he ate through five oranges.

How many pieces of fruit did the hungry caterpillar eat during the week?

Huinker, DeAnn, and Victoria Bill. *Taking Action: Implementing Effective Mathematics Teaching Practices in Grades K-5*. Reston, VA: National Council of Teachers of Mathematics, 2017. (p. 98)

# Ms. Bouchard and The Hungry Caterpillar Task Grade 1

# Analyzing Teaching and Learning 5.1

## *Investigating the Effect of Teacher Questions*

Read Ms. Bouchard's class discussion excerpt:

- What purpose did each question appear to serve?
- Which questions reveal insights into students' understanding and strategies for addition?

Note: *A question can have more than one purpose!*

# Types of Questions in Mathematics Teaching

1. Gathering information.
2. Probing thinking.
3. Making the mathematics visible.
4. Encouraging reflection and justification.

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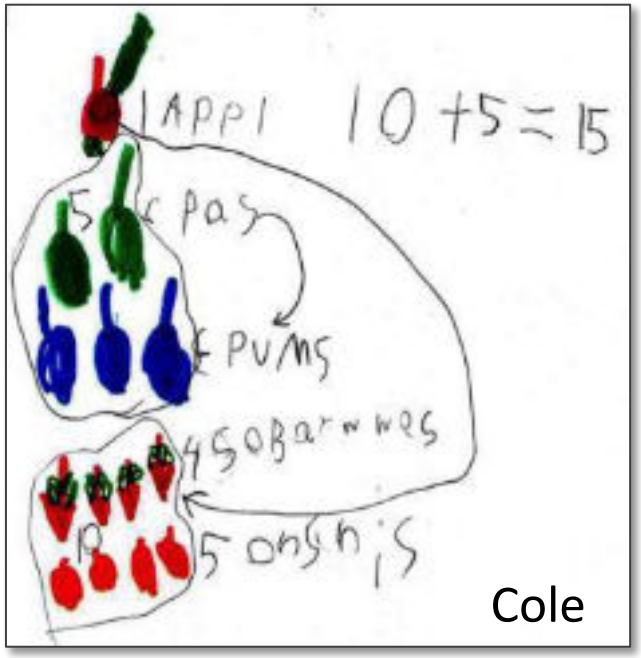
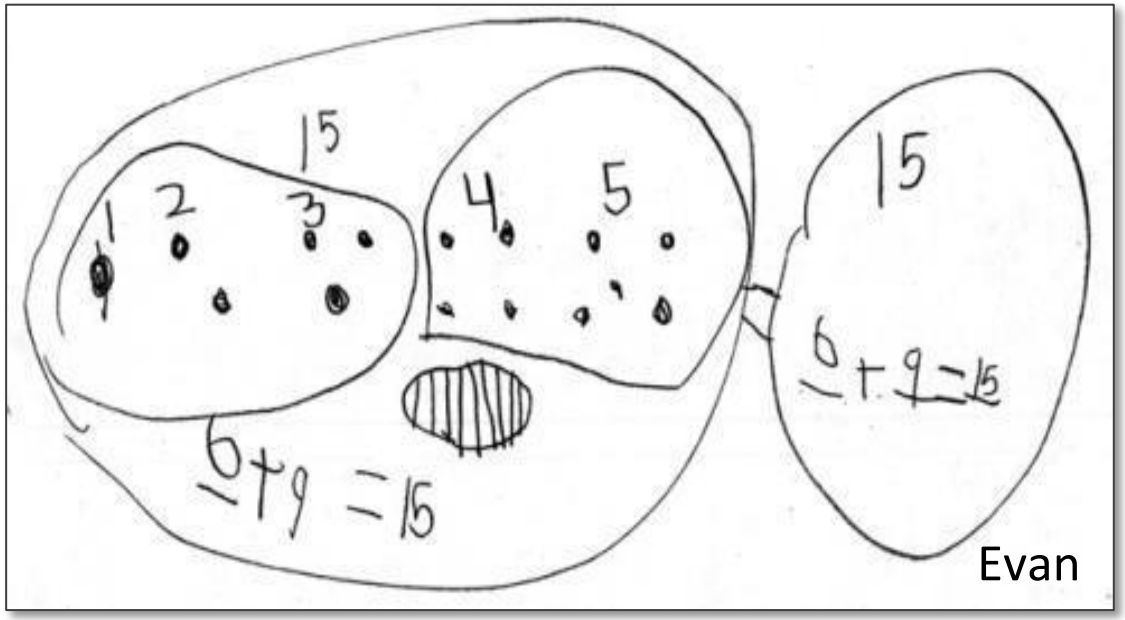
5. Engaging with the reasoning of others.




**Probe**

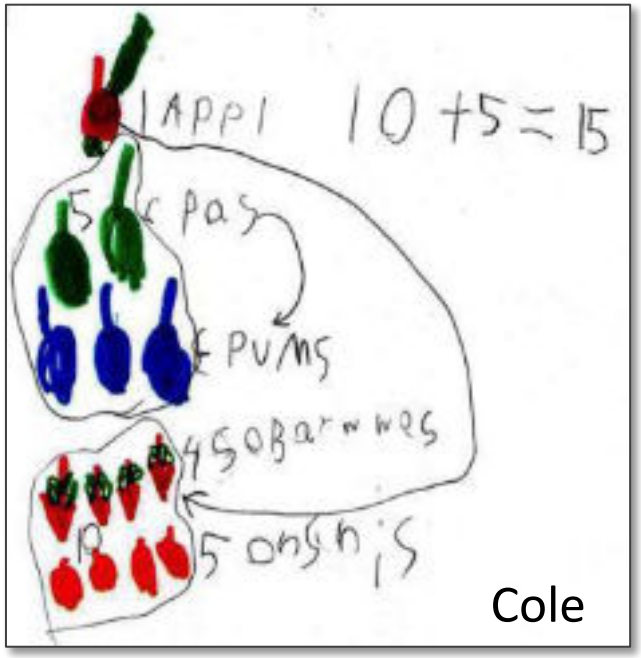
Line 1: How did you solve?

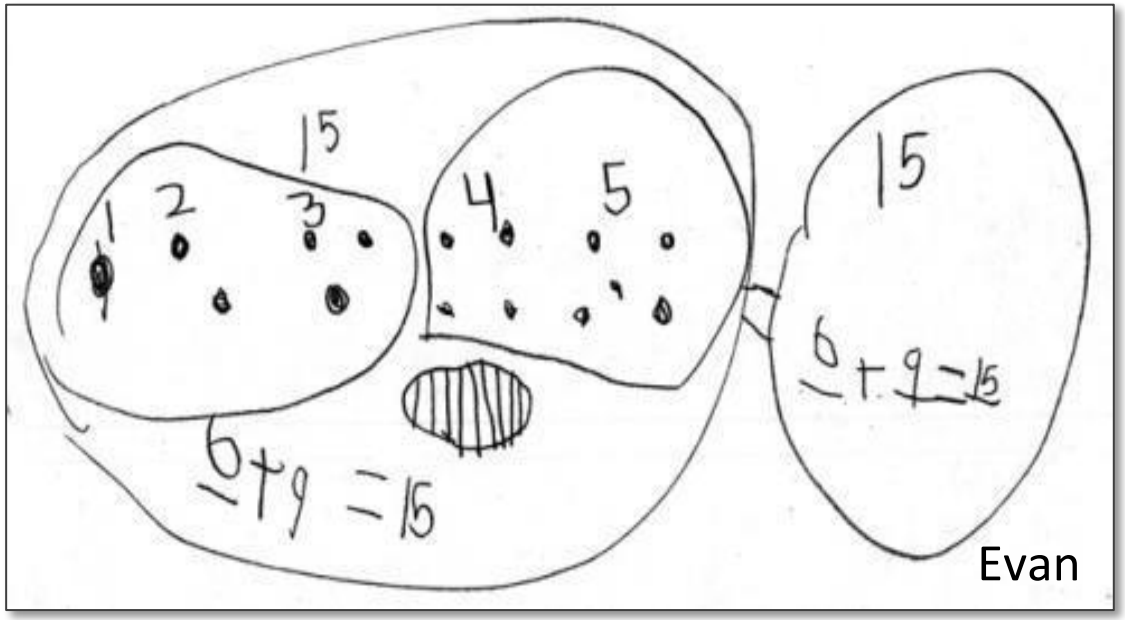
**Probe & Engage**

Line 4: Rosa, can you say back what Cole did?

**Engage**

Line 7: Can anyone else say back what you heard?




**Make the Math Visible & Reflect and Justify**

Lines 20-21: How can both Cole and Evan each get 15 pieces of fruit when they each wrote and solved a different equation?

Lines 27-28: Who can use what you know about 10 + 5 to think about the sum of 6 + 9?

| Assessing Questions "Uncover" | Advancing Questions "Push & Press" |
| --- | --- |
| • Based closely on student's current work and approach.<br>• Clarify aspects of student work and what the student understands.<br>• Provide information to the teacher about student math understanding. | • Use student work as basis for moving toward the lesson goal or beyond.<br>• Press students to extend what they know to a new situation.<br>• Prompt students to think further about a mathematical idea they may not be currently considering. |
| Teacher **STAYS** to hear the answer to the question. | Teacher **WALKS AWAY** (PAUSES), allowing students to figure out how to proceed. |

(p. 109-110)

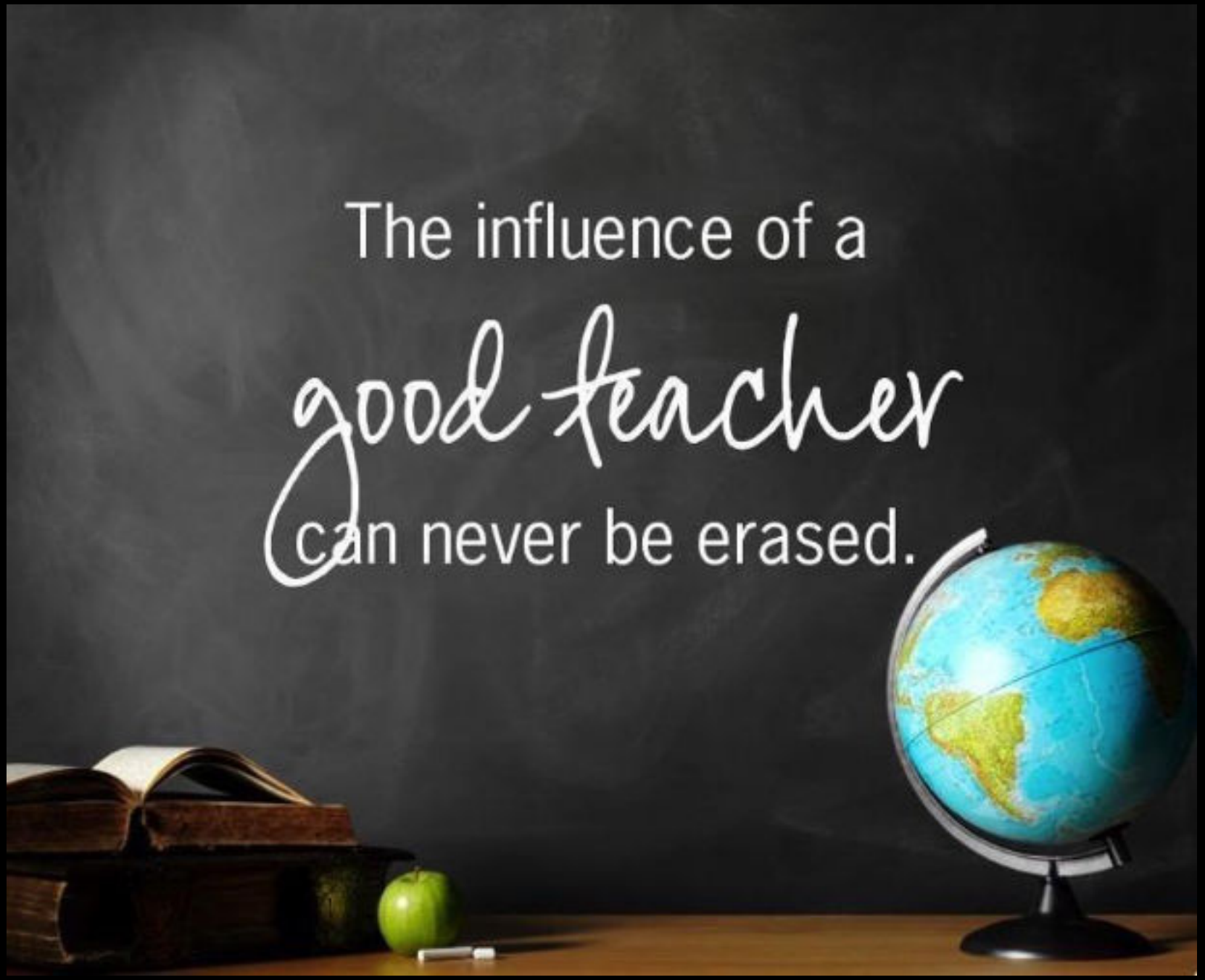
# Key Messages

- Use a **balance of question types** to surface different aspects of students' math reasoning.

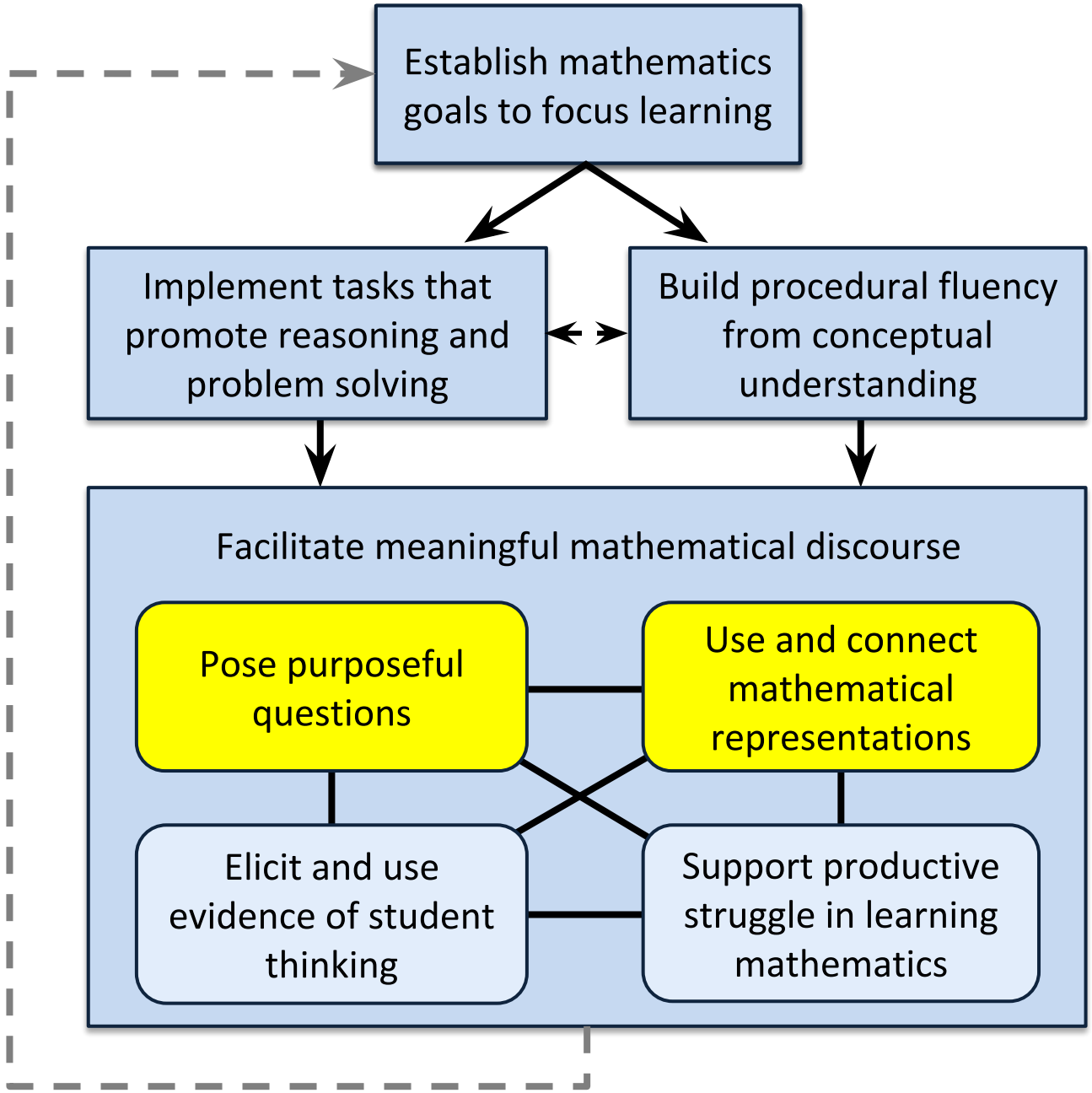
- **Questions are the most important tool** that we as teachers have to further student learning; they should be pre-planned and not left to chance.

- Be intentional and purposeful in **assessing** (uncovering) and **advancing** (pushing/pressing) toward important math ideas.

# Taking Action


The influence of a
good teacher
can never be erased.

# Mathematics Teaching Framework



# The *Taking Action* Series

www.nctm.org/store/takingaction/

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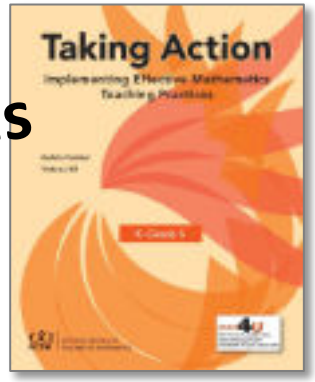
# Taking Action Series

**Grades K-5**




Authors:

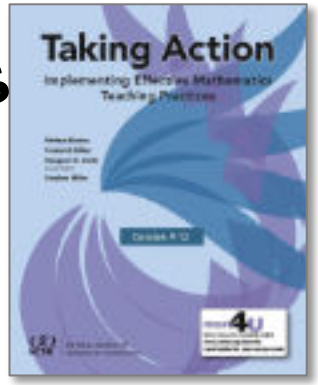
DeAnn Huinker and Victoria Bill

**Grades 6-8**

Authors:

Margaret (Peg) Smith, Michael Steele, and Mary Lynn Raith

**Grades 9-12**




Authors:

Melissa Boston, Fredrick Dillon, Margaret (Peg) Smith, and Stephen Miller

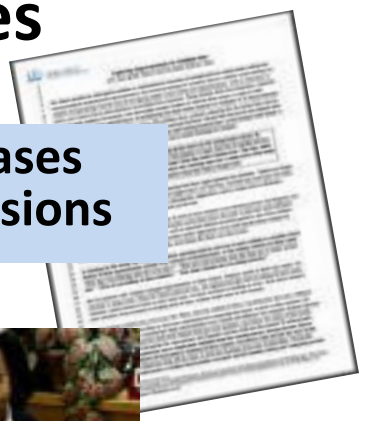
Published by NCTM in 2017.

# The *Taking Action* Series

1. Setting the Stage
2. Establish Mathematics Goals to Focus Learning
3. Implement Tasks that Promote Reasoning and Problem Solving
4. Build Procedural Fluency from Conceptual Understanding
5. Pose Purposeful Questions
6. Use and Connect Mathematical Representations
7. Facilitate Meaningful Mathematical Discourse
8. Elicit and Use Evidence of Student Thinking
9. Support Productive Struggle in Learning Mathematics
10. Pulling it Altogether

# Contents of Chapters 2-9

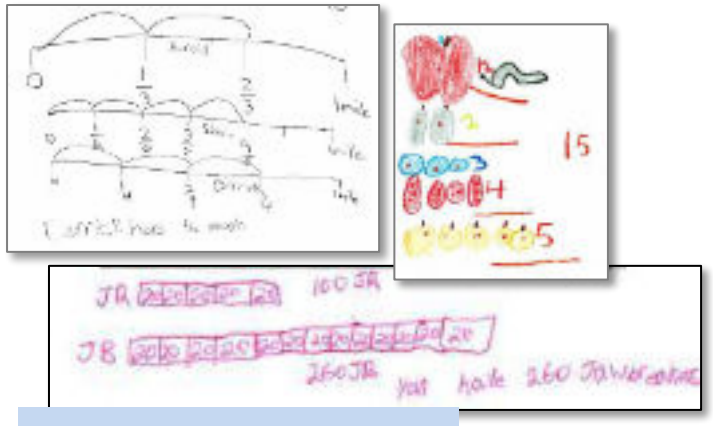
- **Analyzing Teaching and Learning (ATL) Activities**
  - Use classroom artifacts to study each teaching practice.
  - A detailed analysis of the artifact as it relates to effective teaching.



**Narrative Cases**
**Class Discussions**

**Video Clips**

- **What the Research Says** about the specific teaching practice
- **Promoting Equity** with the specific teaching practice
- **Taking Action in the Classroom**



**Student Work**

# *Taking Action Webinar* Series

https://www.nctm.org/online-learning

Nov 24: Opening Session - Peg Smith

Nov 30: Focus on Elementary - DeAnn Huinker & Paige Richards

Dec 01: Focus on Middle School - Mike Steele

Dec 03: Focus on High School - Melissa Boston

Dec 07: Focus on Elementary - Roshanna Beard

Dec 08: Focus on Middle School - Jennifer Perego

Dec 10: Focus on High School - Fred Dillon & Anthony Bokar

Dec 15: Closing Session - Robert Q. Berry, III

# Thank You!

**DeAnn Huinker**
University of Wisconsin–Milwaukee
huinker@uwm.edu
@dh11235

Paige Richards
Mathematics Institute of Wisconsin
paige.richards@mathinstitutewi.org
@mathinst_Paige

https://tinyurl.com/TA-Webinar-Elementary

**Shared Google Folder: Focus on Elementary Webinar**

https://tinyurl.com/TA-Webinar-Elementary

- Slides
- Resources
- Articles and Readings

# NCTM Resources

***Principles to Actions**: Ensuring Mathematical Success for All* (NCTM, 2014)
www.nctm.org/PtA/

## NCTM Taking Action series

3 Grade Band Books: ES, MS, & HS
www.nctm.org/store/takingaction