Taking Action: Bringing the Effective Teaching Practices to Life in your Classroom

Focus on Elementary School: Connecting Representations and Posing Purposeful Questions

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NCTM Webinar Series, November 30, 2020
The work we will do together draws from NCTM Resources:

- *Principles to Actions*
- *Taking Action* series

Yes! Slides and other resources will be shared through a link to a Google Folder at the end of the session.
Framework for teaching mathematics
   – Eight effective math teaching practices

Teaching with a focus on representations
   – Classroom artifact: Student work samples

Becoming more purposeful with our questions
   – Classroom artifact: Excerpts from a class discussion

Taking Action
Principles to Actions

Background
In *Principles to Actions*, NCTM identified eight effective mathematics teaching practices that “represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics” (p. 9).

Principles to Actions: Ensuring Mathematical Success for All

A research-informed consensus on what constitutes effective teaching accumulated over the prior 25 years.
Eight Effective Mathematics Teaching Practices

- Establish mathematics **goals** to focus learning.
- Implement **tasks** that promote reasoning & problem solving.
- Use and connect mathematical **representations**.
- Facilitate meaningful mathematical **discourse**.
- Pose purposeful **questions**.
- Build procedural **fluency** from conceptual **understanding**.
- Support productive **struggle** in learning mathematics.
- Elicit and use **evidence** of student thinking.
Move from a list of eight teaching practices for mathematics

Coherent and connected framework for the teaching of mathematics
Mathematics Teaching Framework

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding

Facilitate meaningful mathematical discourse

- Pose purposeful questions
- Elicit and use evidence of student thinking
- Use and connect mathematical representations
- Support productive struggle in learning mathematics
Mathematics Teaching Framework

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Build procedural fluency from conceptual understanding
- Facilitate meaningful mathematical discourse
  - Pose purposeful questions
  - Use and connect mathematical representations
  - Elicit and use evidence of student thinking
  - Support productive struggle in learning mathematics
Teaching with a Focus on Representations
Use and connect mathematical representations

Effective teaching of mathematics engages students in making connections among mathematical representations:

- to **deepen understanding** of mathematical concepts and procedures.
- as **tools** for problem solving.

Five types of representations

Why focus on multiple images to represent a mathematical idea?

“Using different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective that makes the picture (math idea) richer and deeper.”

(Tripathi 2008, p. 439)
Teaching with Representations

“Not only are the distinct types of representations important, but translations between them and transformations within them, are also important.”

(Lesh, Post, & Behr, 1997, p. 34)

Between

- Visual
- Physical
- Symbolic
- Contextual
- Verbal
Connections **between** Representations

4 x 7

- What does the 4 tell us about the visual representation?
- Where is 7 in the picture?
- What does “x” have to do with the picture?
Connections **within** Representations.

4 x 7

1. What is the same about each visual representation?
2. What are some of the differences?
3. Where is the math idea easier to see? harder to see?
Assessing Connections among Representations
Multiplication Check-In

1. Find the answer: $4 \times 7 = ?$
2. Explain how you figured out the answer.
3. Draw a picture that shows what $4 \times 7$ means.
4. Write a story problem for $4 \times 7$. 
Grade 3 Multiplication Check-In

- Introduce yourselves to each other.
- Study all 4 student work samples.
- What does each reveal about the child’s ability to connect representations for multiplication?
- What might be some implications for instruction?
Rachel

Correct visual & correct context.
Understands equal group situations and can connect an equation (symbolic) to both visual and contextual representations.

Next Steps
Focus on arrays, tape diagrams, number lines, and stories with measurement contexts.
Mateo

Correct visual & no context.
He seems to know multiplication involves combining equal groups, but likely has limited experiences with contexts for multiplication.

Next Steps
Connecting to real-world contexts and to other visual representations, such as arrays and tape diagrams.
1. Find the answer: $4 \times 7 = ?$
   Answer: 28
   $4 \times 7 = 28$

2. Explain how you figured out the answer.
   I counted 4's up to seven like this: 4, 8, 12, 16, 20, 24, 28

3. Draw a picture that shows what $4 \times 7$ means.
   ![Picture](image)
   You count by 4's up to seven.

4. Write a story problem for $4 \times 7$.
   The class went on a trip.
   4 people did not go. How many people was going?

Olivia

Incorrect visual & incorrect context.

She can skip count but may not understand multiplication as combining equal groups.

Caution
Skip counting can present a false sense of understanding.

Next Steps
More work with visual representations and story contexts.
Dylan

Incorrect visual & incorrect context.
Dylan likely counted by ones, and he lacks connections to visual diagrams showing equal groups and to real-world contexts.

Next Steps
More experiences with physical and visual representations for story contexts about equal groups.
Grade 3
6 classrooms
6 different schools
Total of 133 students
November Check-in
### Multiplication Connections Check-in: 4 x 7 = ?

**Grade 3 Results: Percent Correct**

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Students</th>
<th>Answer</th>
<th>Visual (Picture)</th>
<th>Context (Story)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class A</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class B</td>
<td>26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class C</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class D</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class E</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Class F</td>
<td>24</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>133</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Multiplication Connections Assessment: $4 \times 7 = ?$

**Grade 3 Results: Percent Correct**

<table>
<thead>
<tr>
<th>Class</th>
<th>Number of Students</th>
<th>Answer</th>
<th>Picture (Visual)</th>
<th>Story (Context)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class A</td>
<td>20</td>
<td>80%</td>
<td>85%</td>
<td>50%</td>
</tr>
<tr>
<td>Class B</td>
<td>26</td>
<td>88%</td>
<td>54%</td>
<td>4%</td>
</tr>
<tr>
<td>Class C</td>
<td>24</td>
<td>88%</td>
<td>75%</td>
<td>67%</td>
</tr>
<tr>
<td>Class D</td>
<td>15</td>
<td>80%</td>
<td>67%</td>
<td>27%</td>
</tr>
<tr>
<td>Class E</td>
<td>24</td>
<td>79%</td>
<td>33%</td>
<td>17%</td>
</tr>
<tr>
<td>Class F</td>
<td>24</td>
<td>92%</td>
<td>75%</td>
<td>25%</td>
</tr>
<tr>
<td><strong>Totals</strong></td>
<td><strong>133</strong></td>
<td><strong>85%</strong></td>
<td><strong>64%</strong></td>
<td><strong>31%</strong></td>
</tr>
</tbody>
</table>
Some observations . . . 133 students . . .

• 85% of students had the right answer.

• 64% drew an appropriate picture.

  **But 48 students did not!**

• 31% had an appropriate context.

  **But 92 students did not!**
Where does math instruction often focus?
3. Draw a picture that shows what $4 \times 7$ means.

4. Write a story problem for $4 \times 7$.

I have 4 hands. Each hand has 7 fingers. I have 28 fingers in all.
Draw It

Write it with numbers

Build It

Write a story

Talk It

Developed by Beth Schefelker (South Milwaukee School District) and DeAnn Huinker (University of Wisconsin-Milwaukee), November 2015.
Key Messages

- Teaching with representations is a high-leverage, effective teaching practice.

- Our goal for students is to develop the ability to move with ease and flexibility between and within different types of representations.

- Teaching and assessment must attend to the “arrows” by asking students to translate in both directions (alternate directionality) among representations.
What does the research say?

Representational . . .

**Flexibility** (Greer, 2009)

**Fluency** (Nathan et al., 2010)

**Thinking** (Pape & Tschoshanov, 2001)

**Versatility** (Graham & Thomas, 2008)

GOAL for students:

An *ease in moving back and forth among representations*.

Role of the teacher:

Intentional and *purposeful* questioning.

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Erlbaum.


Becoming more Purposeful with our Questions
Establish mathematics goals to focus learning

Implement tasks that promote reasoning and problem solving

Build procedural fluency from conceptual understanding

Facilitate meaningful mathematical discourse

Pose purposeful questions

Use and connect mathematical representations

Elicit and use evidence of student thinking

Support productive struggle in learning mathematics
Effective teaching of mathematics uses purposeful questions to *assess and advance* students’ reasoning and sense making about important mathematical ideas and relationships.
Hungry Caterpillar Task

On Monday, the hungry caterpillar ate through one apple, but he was still hungry. On Tuesday he ate through two pears, but he was still hungry. On Wednesday he ate through three plums. On Thursday he ate through four strawberries. On Friday he ate through five oranges.

How many pieces of fruit did the hungry caterpillar eat during the week?
Ms. Bouchard and
The Hungry Caterpillar Task
Grade 1
Investigating the Effect of Teacher Questions

Read Ms. Bouchard’s class discussion excerpt:

- What purpose did each question appear to serve?
- Which questions reveal insights into students’ understanding and strategies for addition?

Note: A question can have more than one purpose!
Types of Questions in Mathematics Teaching

1. Gathering information.
2. Probing thinking.
3. Making the mathematics visible.
4. Encouraging reflection and justification.
5. Engaging with the reasoning of others.
**Probe**

Line 1: How did you solve?

**Probe & Engage**

Line 4: Rosa, can you say back what Cole did?

**Engage**

Line 7: Can anyone else say back what you heard?
Make the Math Visible & Reflect and Justify

Lines 20-21:  How can both Cole and Evan each get 15 pieces of fruit when they each wrote and solved a different equation?

Lines 27-28:  Who can use what you know about 10 + 5 to think about the sum of 6 + 9?
<table>
<thead>
<tr>
<th>Assessing Questions “Uncover”</th>
<th>Advancing Questions “Push &amp; Press”</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Based closely on student’s current work and approach.</td>
<td>• Use student work as basis for moving toward the lesson goal or beyond.</td>
</tr>
<tr>
<td>• Clarify aspects of student work and what the student understands.</td>
<td>• Press students to extend what they know to a new situation.</td>
</tr>
<tr>
<td>• <strong>Provide information</strong> to the teacher about student math understanding.</td>
<td>• <strong>Prompt students</strong> to think further about a mathematical idea they may not be currently considering.</td>
</tr>
</tbody>
</table>

**Teacher **STAYS** to hear the answer to the question.**

**Teacher **WALKS AWAY (PAUSES), allowing students to figure out how to proceed.**

*(p. 109-110)*
Key Messages

• Use a **balance of question types** to surface different aspects of students’ math reasoning.

• **Questions are the most important tool** that we as teachers have to further student learning; they should be pre-planned and not left to chance.

• Be intentional and purposeful in **assessing** (uncovering) and **advancing** (pushing/pressing) toward important math ideas.
Taking Action
The influence of a good teacher can never be erased.
Mathematics Teaching Framework

Establish mathematics goals to focus learning

Implement tasks that promote reasoning and problem solving

Build procedural fluency from conceptual understanding

Facilitate meaningful mathematical discourse

Pose purposeful questions

Use and connect mathematical representations

Elicit and use evidence of student thinking

Support productive struggle in learning mathematics
The *Taking Action* Series

[Links]
- [Visit the website](http://www.nctm.org/store/takingaction/)
- [PTA](http://www.nctm.org/pta)
Taking Action Series

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1. Setting the Stage

2. Establish Mathematics Goals to Focus Learning

3. Implement Tasks that Promote Reasoning and Problem Solving

4. Build Procedural Fluency from Conceptual Understanding

5. Pose Purposeful Questions

6. Use and Connect Mathematical Representations

7. Facilitate Meaningful Mathematical Discourse

8. Elicit and Use Evidence of Student Thinking


10. Pulling it Altogether
Contents of Chapters 2-9

• Analyzing Teaching and Learning (ATL) Activities
  – Use classroom artifacts to study each teaching practice.
  – A detailed analysis of the artifact as it relates to effective teaching.

• What the Research Says about the specific teaching practice

• Promoting Equity with the specific teaching practice

• Taking Action in the Classroom
Taking Action Webinar Series

https://www.nctm.org/online-learning

Nov 24: Opening Session - Peg Smith

Nov 30: Focus on Elementary - DeAnn Huinker & Paige Richards

Dec 01: Focus on Middle School - Mike Steele

Dec 03: Focus on High School - Melissa Boston

Dec 07: Focus on Elementary - Roshanna Beard

Dec 08: Focus on Middle School - Jennifer Perego

Dec 10: Focus on High School - Fred Dillon & Anthony Bokar

Dec 15: Closing Session - Robert Q. Berry, III
Thank You!

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https://tinyurl.com/TA-Webinar-Elementary
Shared Google Folder: Focus on Elementary Webinar

https://tinyurl.com/TA-Webinar-Elementary

- Slides
- Resources
- Articles and Readings
NCTM Resources

*Principles to Actions*: Ensuring Mathematical Success for All (NCTM, 2014)
www.nctm.org/PtA/

NCTM Taking Action series
3 Grade Band Books: ES, MS, & HS
www.nctm.org/store/takingaction