RESOURCES AND TOOLS FOR ENGAGING FAMILIES IN FACT FLUENCY

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Share something you are feeling good about related to teaching basic fact fluency.
Our Plan

1. Families and Facts
2. Basic Fact Fluency Big Ideas
3. Addition & Subtraction Games
4. Multiplication & Division Games
5. Educating and Supporting Families
6. Summing Up: Resources & Tools
FAMILIES & FACTS
ANXIETY AWARENESS

A parent’s emotions are connected to the student’s emotions, and positive emotions are connected to better performance.

Else-Quest et al., 2008

When parents are more math anxious and they help their child with homework, their child learns significantly less math over the school year and have more math anxiety by the school year’s end.

Meta-analysis by Maloney, et al., 2015
a) What do parents most value about their child’s learning of mathematics?
b) What expectations do families have about how math is taught?
c) What were parents’ experiences in learning the basic facts?
It is not sufficient nor effective to justify a change in math because of...

- Standards
- Assessments
- District policies
- “Research says”

It must be because the change will better meet the needs of their child.
STRATEGIZING

• Why strategies?
• What strategies?
• How will my child learn the strategies and basic facts?
1. Mastery must focus on Fluency

**Figure 1.1** What Procedural Fluency Is and What It Looks Like

<table>
<thead>
<tr>
<th>Computing with...</th>
<th>What It Looks Like</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>Correct answer</td>
</tr>
<tr>
<td>Efficiency</td>
<td>Time needed to solve is reasonable.</td>
</tr>
<tr>
<td>Flexibility</td>
<td>Selected strategy ‘fits’ the numbers in the problem (i.e., appropriate strategy selection).</td>
</tr>
<tr>
<td></td>
<td>Strategy is applied to new problems.</td>
</tr>
</tbody>
</table>

The four components (bolded) are interrelated. Appropriate strategy selection is required for efficiency and flexibility.

Figure from *Math Fact Fluency*, p. 2
1. Strategy Groups Outperform Non-strategy Groups on Post-Assessments

Studies include:
• Baroody, Purpura, Eiland, Read, & Paliwal, 2016
• Locuniak & Jordan, 2008
• Purpura, Baroody, Eiland, & Reid, 2016
• Tournaki, 2003
RESEARCH SAYS...

2. Strategy Groups RETAIN facts better than non-strategy groups

Studies include:
• Baroody, Bajwa, & Eiland, 2009
• Henry & Brown, 2008
• Hiebert & Carpenter, 1992
• Hiebert & Lefevre, 1986
• Jordan, Kaplan, Olah, & Locuniak, 2006

Studies include:
- Geary, 2011
- Jordan, Kaplan, Ramineni, & Locuniak, 2009
- Jordan, Kaplan, Locuniak, & Ramineni, 2007
- Vasilyeva, Laski, & Shen, 2015
THESE FINDINGS MAKE SENSE...

**FIGURE 8.3 Parent Quizzes**

<table>
<thead>
<tr>
<th>Basic Facts</th>
<th>Beyond the Basics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 5 + 7 =  ____</td>
<td>1. 95 + 7 =  ____</td>
</tr>
<tr>
<td>2. 5 × 9 =  ____</td>
<td>2. 5 × 49 =  ____</td>
</tr>
<tr>
<td>3. 9 + 6 =  ____</td>
<td>3. 90 + 60 =  ____</td>
</tr>
<tr>
<td>4. 6 × 8 =  ____</td>
<td>4. 8 × 15 =  ____</td>
</tr>
<tr>
<td>5. 7 × 3 =  ____</td>
<td>5. 35 × 3 =  ____</td>
</tr>
</tbody>
</table>
Fundamental #2. Fluency Develops in 3 Phases

Phase 1: Counting
(counts or skip counts)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

Phase 3: Mastery (Automaticity)
(efficient production of answers)

Based on Baroody, 2006
2. Fluency Develops in 3 Phases

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(counts or skip counts)

Phase 2: Deriving
(uses reasoning strategies based on known facts)

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(efficient production of answers)

Based on Baroody, 2006

$6 + 7$
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Based on Baroody, 2006
Fundamental #3. Knowing foundational facts must precede derived facts.


- Doubles
  - Use a double to find the sum.
  - $6 + 8 = 6 + 6 + 2$

- Combos of 10
  - Move some from one addend to the other to make a 10.
  - $6 + 8 = 10 + 4 = 14$

- Near Doubles
  - Think of an 8 or 9 as a 10, and adjust answer.
  - $6 + 8 \rightarrow 6 + 10 = 16$
  - $16 - 2 = 14$

*Also called Compensation and Use 10; we have found that young learners remember the strategy and distinguish it from Making 10 when we use this name. Research indicates that this strategy is more accessible than Making 10, and therefore should be explicitly taught (Baroody, Eiland, Reid, & Paliwal, 2016).
Fundamental #3. Knowing foundational facts must precede derived facts.

*We acknowledge that all the derived fact strategies are break apart (distributive property) strategies. We focus on specific ways to break apart (e.g., adding a group) and move towards generalizing the Break Apart strategy.
# 5 Fundamentals: Family Messaging

<table>
<thead>
<tr>
<th>#1: Mastery must focus on fluency.</th>
<th>Families must understand that while we do want every child to master their facts for life, the way to reach this goal is through a focus on fluency, not rote memorization.</th>
</tr>
</thead>
<tbody>
<tr>
<td>#2: Fluency develops in three phases.</td>
<td>Parents need to learn strategies (Phase 2), why the strategies matter, and questions to ask to help their children progress through the phases.</td>
</tr>
<tr>
<td>#3: Knowing foundational facts must precede derived facts.</td>
<td>Because traditionally facts are taught in order from smallest number to largest, families need to have, understand, and use these progressions.</td>
</tr>
<tr>
<td>#4: Timed tests do not assess fluency.</td>
<td>Parents may themselves not like timed tests, but think they have some value. We need to help them see that timed tests and quick flashing of cards are not as effective as working on strategies.</td>
</tr>
<tr>
<td>#5: Students need substantial and enjoyable practice</td>
<td>Practice at home is critical, but it also needs to be meaningful and stress-free! Games, strategy talk, and self-assessing progress are important “home work.”</td>
</tr>
</tbody>
</table>
Fundamental #5: Students need substantial and enjoyable practice

1. Stories
2. Quick Looks
3. Games!
ADDITION & SUBTRACTION
Addition Facts
Flexible Learning Progression

- \( +/- 0,1,2 \)
  - Doubles
    - Near Doubles
  - Combos of 10
    - Making 10
  - 10+
    - Pretend-a-10

Foundational Fact Sets
Derived Fact Strategies
Fundamental #5: Students need substantial and enjoyable practice

1. Stories
2. Quick Looks
3. Games!

Round 1: Foundational Facts
Game: One More Cover It

Players: Partners

How to Play:
1. Each partner has a different colored chip to cover board.
2. Partners take turns. Each player rolls the number cube and covers a square that is one more than what was rolled on the number cube.
3. Players try to cover a complete column, row or diagonal.
4. Optional: The player who first gets a column, row or diagonal wins.

You need:
- Game Board
- Die
- Chips to cover board.
## One More Cover It

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<td>4</td>
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<td>7</td>
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<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>
REFLECTING

① How might the game be adapted?

② How might you involve families in the strategy or the game?
Subtract 1, numerals
____ minus 1 is ____.

Materials: One ten-sided die, and counters in two colors.

How to play: On your turn, roll the die. If you roll a zero, it means ten. Say the number you rolled minus one, and the answer. Cover a circle that shows the answer. Hint: Remember that subtracting one is just like saying the next number when you are counting backward. You count “10, 9, 8, 7, 6, 5, 4, 3, 2, 1.” Right after 5, it is 4. That means 5 minus 1 is 4. So if you roll a 5, say “5 minus 1 is 4” and put a counter on a 4. The first player to get four in a row wins.
Addition Facts
Flexible Learning Progression

Doubles

Combos of 10

Near Doubles
Making 10
Pretend-a-10

 +/- 0, 1, 2

Foundational Fact Sets

Derived Fact Strategies
Fundamental #5: Students need substantial and enjoyable practice

Round 2: Derived Fact Strategies

1. Stories
2. Quick Looks
3. Games!
Add 5 and 6

___ + ___ is the same as ___ + ___ + ___

Materials: The 5, 6 cards from a deck of ten-frame cards, and two counters. How to play: Both players put a counter on START. On your turn, draw two cards and place them face up so both players can see them. Move to the next circle with an amount that is the same as the sum of the two cards. Hint: With 5+6, notice how the six dots are the same as five dots plus one more. Imagine that the five is going together with the other five. You already know 5+5 is 10. That leaves just one dot left over, and you know that 10+1 is 11. The first player to land on END wins.
**Game: Fixed Addend War**

**Players:** Partners

**How to Play:**
1. Identify an addend (e.g., 9) and place a card with that number face up in the middle.
2. Partners split the rest of the cards equally, shuffled and face down.
3. At the same time, partners turn up their top card and say their sum.
4. Each partner takes turns saying their full addition sentence, and both decide if sums are correct.
5. The player with the larger (correct) sum gets the cards.
6. If there is a tie (War), repeat steps 3 – 5.

Player with the most cards wins.
Game: Fixed Addend War

My sum is _____ . I added ___ to 10 and got ____, then subtracted 1 and got ________.

My sum is _____. I __________________________.  
Answer How you thought about it.
Game: Fixed Addend War
Game in Action:

The 8 is 2 away from 10. I moved 2 over to make a 10. 10 + 3 equals 13!

I am going to pretend the 9 is a 10. I know 10 + 4 equals 14. I have to take one away from my answer. One less than 14 is 13!
LUCKY 13

Lucky 13

2-4 players

How to Play:

1. Each player is dealt 4 cards face up.
2. Each player picks two cards which, when added together, produces a sum as close to 13 as possible.
3. Players record the sums on their score card.
4. Players determine how far their sum is away from “Lucky 13” and record that difference on the score card. The player’s score for each round is the difference between the sum and 13.
5. Players discard all their cards, and the dealer gives each player 4 new cards.
6. Play repeats for five rounds.
7. Players add their scores from each round.
8. The player with the lowest score is the winner.
LUCKY 13 [4 CARDS]

Game in Action:

I am going to choose the 7 and 5 because 7 plus 5 equals 12. My score is 1 because 12 is only 1 away from 13!

I am going to choose the 9 and 7. 9 plus 7 equals 16. My score for this round is 3 because 16 is 3 away from 13!
LUCKY 13

Game in Action:

I am going to choose the 7 and 5 because 7 plus 5 equals 12. My score is 1 because 12 is only 1 away from 13!

I am going to choose the 9 and 7. 9 plus 7 equals 16. My score for this round is 3 because 16 is 3 away from 13!

Score Card

<table>
<thead>
<tr>
<th></th>
<th>Sum</th>
<th>Difference</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>6 + 5 = 11</td>
<td>13 – 11 = 2</td>
<td>2</td>
</tr>
<tr>
<td>Draw 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draw 2</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Draw 3</td>
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<tr>
<td>Draw 4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draw 5</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# First to 20

## Player 1

<table>
<thead>
<tr>
<th>Roll</th>
<th>6</th>
<th>5</th>
<th>3</th>
<th>5</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>6</td>
<td>11</td>
<td>14</td>
<td>19</td>
<td>15</td>
</tr>
</tbody>
</table>

(must subtract 4 because 19 + 4 is greater than 20)

## Player 2

<table>
<thead>
<tr>
<th>Roll</th>
<th>5</th>
<th>3</th>
<th>7</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>5</td>
<td>8</td>
<td>15</td>
<td>9</td>
</tr>
</tbody>
</table>

(must subtract 6 because 15 + 6 is greater than 20)
<table>
<thead>
<tr>
<th>Player 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Player 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roll</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

Dice
FAMILY CONNECTIONS
DEBRIEF

What questions or statements might you ask after playing the game?
DEBRIEF

• What did you notice about this game?
• How many facts might your child solve in playing 5 rounds of Lucky 13?
• What are the benefits of this type of practice over a worksheet?
• What downsides does this type of practice have?
Dear Parent,

In math class your child is exploring a thinking strategy known as Pretend-a-10. We have worked hard to discover that when we imagine a ten, we can make addition problems easier to think about.

When exploring the 10+___ facts (e.g., 10 + 2, 10 + 6, 10 + 9, etc.), our class began to realize that adding any single-digit number to 10 is quite simple. Therefore, if we are adding 9 or 8 to a single-digit number, we can pretend that the 9 or 8 is a 10, find the total, and then adjust the actual sum.

The stories and ten-frames here show how this thinking strategy works.

Nine red jelly beans and six yellow jelly beans are in the jar. How many beans in the jar?

\[
9 + 6 \quad \Rightarrow \quad 10 + 6 = 16 \quad \Rightarrow \quad 16 - 1 = 15
\]

A pod of whales has 8 adults and 7 calves. How many whales in the pod?

\[
8 + 7 \quad \Rightarrow \quad 10 + 7 = 17 \quad \Rightarrow \quad 17 - 2 = 15
\]

If you would like to review this thinking strategy with your child, please encourage your child to use the attached ten-frame to model similar sums with everyday objects from around your home.

Please also challenge your child to play Fixed Addend War and Sum War with you, a family member, or a friend.

Thank you for your consideration and energetic support.

With appreciation,
Dear Parent,

In math class your child is exploring a thinking strategy known as **Pretend-a-10.** We have worked hard to discover that when we imagine a ten, we can make addition problems easier to think about.

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\[
\begin{align*}
9 + 6 & \quad 10 + 6 = 16 \\
16 - 1 & = 15 \\
8 + 7 & \\
10 + 7 & = 17 \\
17 - 2 & = 15
\end{align*}
\]
MULTIPLICATION & DIVISION
Multiplication Facts
Flexible Learning Progression

2s

- Doubling (4s, 6s, 8s)
- Break Apart (3s, 4s, 6s, 7s, 8s, 9s)

10s

- Adding a Group (3s and 6s)

5s

- Subtracting a Group (9s, 4s)

0s

- Squares

Foundational Fact Sets

Derived Fact Strategies

Near Squares
Fundamental #5: Students need substantial and enjoyable practice

Round 1: Foundational Facts

1. Stories
2. Quick Looks
3. Games!
On the Double

Players: Partners

How to play:
1. Place all 15 counters on the game board on whichever numbers you want. For example, if you think an 8 will occur most often, place more counters on the 8. Once counters have been placed, they cannot be moved to a new location.
2. Take turns rolling the die, doubling the number, saying the corresponding multiplication fact aloud (e.g., “I double 4 and it equals 8” “2 times 4 equals 8”), and removing a counter from the space on the board for the doubled fact. If you do not have a counter on that number, you do not get to remove any counters.
3. The first player to remove all their counters wins!

You need:
✓ 10-sided Die or Deck of Cards, (No Js or Ks)
✓ Game Board
✓ 15 counters
Possible Game Boards

For Cards:

0  2  4  6  8  10  12  14  16  18  20

For 10-sided dice

0  2  4  6  8  10  12  14  16  18

For 10-sided dice

0  1  2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17  18
Let’s Play: ‘Place’ 15 Chips

<table>
<thead>
<tr>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
</tr>
</thead>
</table>

![Image of 15 chips placed on the board]
<table>
<thead>
<tr>
<th></th>
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<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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### FIGURE 4.8  Multiplication Table with Foundational Facts Highlighted

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Fundamental #5: Students need substantial and enjoyable practice

1. Stories
2. Quick Looks
3. Games!

Round 2: Derived Fact Strategies
For Halloween, Jay got two packs of gum, each pack had 6 pieces of gum. How many pieces of gum does Jay have?
Story #1

Jay’s sister got four packs of the same gum. How many pieces of gum does she have?

Use your work from the first problem to help you solve this problem.
Fundamental #5: Students need substantial and enjoyable practice

Round 2: Derived Fact Strategies

1. Stories
2. Quick Looks
3. Games!
Fundamental #5: Students need substantial and enjoyable practice

1. Stories
2. Quick Looks
3. Games!

Round 2: Derived Fact Strategies
**Fixed Factor War**

- **Fixed Factor Card** →
  
  (Does not change.)

- **Player 1**
  
  "3 doubled is 6 and 6 doubled is 12, so four 3s is 12”

- **Player 2**
  
  "Two times 8 is 16. I double 16 and its 32. Four times eight is 32.”
TRIOS

**Players:** Partners

**How to Play:**
1. Give each player a set of counters or different color of marker.
2. Take turns. On your turn pick a card (or roll the die).
3. Select a place on the board after multiplying your card (roll) times 5.

**Goal:** Get as many 3-in-a-row (Trios) as you can (horizontally, vertically, or diagonally) while blocking your partner from getting.

**Scoring:** 5 points for every ‘Trio’ 3-in-a-row. Person with highest score wins. Trios can overlap.

---

You need:
- 10-sided Die or Deck of Cards, (No Js or Ks)
- Game Board
TRIOS

TRIOS Game Board: Multiples of Four

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<tr>
<td>16</td>
<td>40</td>
<td>36</td>
<td>28</td>
<td>12</td>
</tr>
</tbody>
</table>

My card is _____.

I double it and get ______ .

I double again and it equals _____.

Four times ______ is __________.
# TRIOS

TRIOS Game Board: Multiples of Six

<table>
<thead>
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</tbody>
</table>
FAMILY CONNECTIONS
**Extending Doubling: Beyond Basic Facts**

Doubling and halving turns problems into ones that can be solved mentally – very useful!

+ Examples:

- **12 x 15**
  - **Half:** 6 x 30
  - **Double:** 180

- **5 x 380**
  - **Double:** 1000
  - **Half:** 500 + 80

- **4.5 x 8**
  - **Double:** 9 x 4
  - **Half:** 9 x 4

---

**Multiplication Strategy Brief: Doubling**

Research-based learning facts:

1. Students start learning multiplication facts by skip counting. That is natural, but they must progress to more efficient reasoning strategies.

2. Implementing reasoning strategies may initially be slower than counting, but eventually it is faster and will lead to quick recall (automaticity), with the added (critical) benefit of long-term retention (rather than forgetting a fact and having to drop back to skip counting).

3. Visuals and stories help students to understand the reasoning strategy.

4. Mathematical reasoning emerges as children notice patterns and relationships through repeated opportunities. Playing purposeful math games is a great way to do this.

5. Reasoning strategies themselves are important to learn because they generalize to larger numbers. Learning the strategies builds stronger math skills!

---

*Thank you for your support in developing fact fluency with your child!*
EDUCATING & SUPPORTING FAMILIES
## Figure 8.3 Parent Quizzes

<table>
<thead>
<tr>
<th>Basic Facts</th>
<th>Beyond the Basics</th>
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<tbody>
<tr>
<td>1. $5 + 7 =$</td>
<td>1. $95 + 7 =$</td>
</tr>
<tr>
<td>2. $5 \times 9 =$</td>
<td>2. $5 \times 49 =$</td>
</tr>
<tr>
<td>3. $9 + 6 =$</td>
<td>3. $90 + 60 =$</td>
</tr>
<tr>
<td>4. $6 \times 8 =$</td>
<td>4. $8 \times 15 =$</td>
</tr>
<tr>
<td>5. $7 \times 3 =$</td>
<td>5. $35 \times 3 =$</td>
</tr>
</tbody>
</table>
Automaticity

Apply a strategy with ease

Just know it
Automaticity ≠ Memorization

*PISA study of 250,000 15-year-olds: Students’ use of memorization/rehearsal strategies are almost universally negatively associated with learning (OECD, 2015, 2010).
Fundamental #3. Knowing foundational facts must precede derived facts.

Addition Fact Fluency Flexible Learning Progression

- **Doubles**
  - Use a double to find the sum.
  - $6 + 8 = 6 + 6 + 2$

- **Combos of 10**
  - Move some from one addend to the other to make a 10.
  - $6 + 8 = 10 + 4 = 14$

- **10+**
  - Think of an 8 or 9 as a 10, and adjust answer.
  - $6 + 8 \rightarrow 6 + 10 = 16$
  - $16 - 2 = 14$

**Near Doubles**

**Making 10**

**Pretend-a-10**

*Also called Compensation and Use 10; we have found that young learners remember the strategy and distinguish it from Making 10 when we use this name. Research indicates that this strategy is more accessible than Making 10, and therefore should be explicitly taught (Baroody, Eiland, Reid, & Paliwal, 2016).*
When is doubling useful? What does doubling look like?

**Doubling: x 4**

It works great for the 4s facts, and is sometimes called Double and Double Again. Have a look!

Marvin arranged his new set of Legos into four rows. Each row has seven pieces.

1. I know...
2. groups of 7 is
   14:
   \[2 \times 7 = 14\]

**Doubling** 14 is 28:

\[14 + 14 = 28\]

Thinking about 14 + 14 is more efficient than skip counting, 7 + 7 + 7 + 7.

**Extending Doubling: x 6 and x 8**

Doubling works for other even numbers, like 6 facts and 8 facts. Once your child knows their 3s facts, they double to solve for 6s; once they know their 4s facts, they double to solve 8s. Have a look at a fact that is commonly difficult for children: 7 x 6 or 6 x 7.

6 x 7 means 6 groups of 7.
I know 3 x 7 = 21. I double 21 to get 42.

**Extending Doubling: Beyond Basic Facts**

Doubling and halving turns problems into ones that can be solved mentally – very useful!

**Examples:**

- 12 x 15
  - 6 x 30
  - 180

- 5 x 380
  - 10 x 190
  - 1,900
  - Double
  - 380 + 80
  - 460
  - Half
  - 90 + 40
  - 130

- 4.5 x 8
  - 9 x 4
  - 36

**Multiplication Strategy Brief: Doubling**

Research-based learning facts:

1. Students start learning multiplication facts by skip counting. That is natural, but they must progress to more efficient reasoning strategies.

2. Implementing reasoning strategies may initially be slower than counting, but eventually it is faster and will lead to quick recall (automaticity), with the added (critical) benefit of long-term retention (rather than forgetting a fact and having to drop back to skip counting).

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5. Reasoning strategies themselves are important to learn because they generalize to larger numbers. Learning the strategies builds stronger math skills!

Thank you for your support in developing fact fluency with your child!
Provide Bookmarks

Supporting Basic Fact Fluency

Fluency Goals:
• Efficiency (strategy choice & time)
• Flexibility (uses different strategies for different problems)
• Accuracy

Questions to Ask:
• How did you solve it?
• How do you know it is correct?
• Is there another way you could solve it?
• If a friend didn’t know the answer to _____, what strategy would you suggest they use?
• How is ________ like ________? (e.g., How is $2 \times 7$ like $4 \times 7$?) How are they different?

Adapted from: Math Fact Fluency (2019) by Jennifer Bay-Williams & Gina Kling
Math Fluency Card

I demonstrate fluency when I…
• am flexible in how I solve problems
• Choose an efficient strategy for the problem
• Use that strategy accurately and get an accurate answer.

I will:
• Estimate
• Look at the numbers in the problem and choose a strategy. I will ask,
  o Can I do it in my head?
  o Do I know an efficient strategy?
• Try a different strategy if my first choice isn’t going well
• Check to see if my answer is close to my estimate
Share
Games

☑️ One More Cover It
☑️ Fixed Addend War
☑️ Sum War
☑️ Lucky 13
☑️ First to 20
☑️ Fixed Factor War
☑️ Product War
☑️ On the Double
☑️ Trios
Softball Hits

How to Play:
The goal of the game is to “hit” to every position, beginning with pitcher (position 1) and going around the field in order.

1. Player 1 takes turns rolling three dice.
2. Use any two or all three of the numbers and any combinations of operations to create an equation equal to 1.
3. If Player 1 can successfully create an equation equal to 1, they can try to use the same numbers to create an equation equal to 2, and so on.
4. When Player 1 cannot create an equation for the next desired number, they pass the dice to the next player.
5. On the next turn, Player 1 picks up where they left off.
6. First player to make equations for 1, 2, 3, 4, 5, 6, 7, 8, and 9 in order wins.
SOFTBALL HITS

Softball Hits Score Card

<table>
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WOULD YOU RATHER?

A. [Image of playing cards]

B. [Table with addition problems]

Addition

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@JBayWilliams
SUMMING UP: RESOURCES & TOOLS
On this site, you will find about 40% of the basic fact games and assessment tools found in the *Math Fact Fluency* book in easy-to-use, printable formats. Just click on a chapter at the left to reveal its associated games or assessment tools or use the Games Alphabetical Index or Assessment Tools Alphabetical Index to find what you need.
What ideas might you use with your students and/or with families?
Thank You