Rough Drafts in Math Class: Going beyond Getting Correct Answers

Amanda (Mandy) Jansen, University of Delaware

@MandyMathEd

#NCTM100 #RoughDraftMath
You might want:

• A second device (phone or tablet) for an interactive task early in the talk
• Pencil and paper for later in the talk
Mandy Jansen

Professor
Mathematics Educator
Former junior high teacher
Researcher
Learner
Pinball player
Kayaker
Paddleboarder
Enjoys yoga
Delawarean currently (Fighting Blue Hen!)
Arizona Wildcat & Michigan State Spartan previously
Big Ideas for this Evening

Explore how to treat the process of doing mathematics as:

GENERATING a rough draft
GAINING new insights
REVISING thinking

Learning mathematics is a PROCESS of CONTINUALLY DEVELOPING new understandings.

When students' in-progress thinking is welcomed, learning math can be less threatening and more meaningful.
Brainstorming your rough draft ideas

http://tinyurl.com/NCTM100RD

Why might mathematics teachers incorporate the use of rough drafts into mathematics classrooms? How might they do this?
BE BRAVE
BE KIND
Would you rather?
If you want to cool YOUR drink, which ice cubes would you use? Explain and justify your thinking.
Teaching this task

• Would you rather? Whole group, first drafts
• Individual thinking time, then group work
• Pause group work to strategically select an early draft to share & discuss.
• Groups revise their work.
• Make a poster – group draft about which ice cubes to choose and why (justification)
  [sharing next drafts] - Gallery walk
• Individual reflection about which ice cubes to
Teaching this task **ONLINE** synchronously

• Would you rather? Whole group, first drafts [Padlet]

• Individual thinking time, then group work [Google slides – each group has a slide in the deck]

• Pause group work to strategically select an early draft to share & discuss.

• Groups revise their work. [Google slides]

• Strategically select and sequence justifications for which ice cubes to choose and why [sharing next drafts]

• Individual reflection about which ice cubes to
Teaching this task ONLINE asynchronously

- Eliciting initial ideas - Would you rather?
  [Padlet, FlipGrid]

- Sharing a draft solution or justification
  [online discussion board]

- Students submit a revised solution
  [individual work]
Rough Drafts in math class

Elicit initial ideas
Explore initial ideas by doing more mathematics together or justifying initial ideas

Revise initial ideas further
http://www.mathalicious.com/lessons/ice-cubed
If you want to get students thinking, ask a question with more than one good answer.
What is rough draft thinking?
What is rough draft thinking?

Communicating to learn.
Sharing unfinished thinking and revising emerging ideas.
Being willing to revise ideas even when "correct."
Why rough drafts in math class?

“If I don’t understand things, I’m not going to raise my hand and blurt out an answer. I think one big problem with me is that I’m afraid that the answer’s wrong. I don’t want to be embarrassed by having a wrong answer. That’s why

“Miss Cooper lets us express ourselves. And she’s a fun person to do math with because there’s always different ways to do the math problem and express ourselves with it. That’s just a great way to help with math.”
Use communication to learn rather than to perform smartness!

Final draft talk: Presentational talk to perform

Rough draft talk: Exploratory talk to learn

Rough Draft Talk

Sounds like... abrupt, bumpy, starts and stops, expressions of uncertainty.

Used for... talking to “work on understanding” (Barnes, 2008).

Any time you’re trying to understand a concept, talking it through can help.
When using rough draft thinking in the mathematics classroom, ALL LEARNERS, including the teacher, are trying to make sense of ideas over time.
## Rough Draft Talk in Math Class

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How can we foster rough drafts in math classrooms?

• Non-evaluative discussion
• Explicit revision opportunities
• Positioning students as
Non-Evaluative Discussion
Non-Evaluative Discussion

• Seek to understand how others are thinking
  • What's similar? What's different?
  • What makes sense?
• Learning to listen to one another
Non-evaluative discussion structure

A round of rough draft sharing.

Brandy Cooper, 6th grade teacher, Milford, DE

• Launch problem.
• Students engage in initial rough draft thinking about strategies together in small groups.
• Students share out rough draft thinking to the large group. Peers may ask questions about their strategies or thinking or comment on what they notice. They do not make evaluative statements about correctness.
• Revise: Second round of rough draft thinking in small groups.
• Second round of group presentations to large group.
• Repeat as needed.
Sentence Starters for non-evaluative discussions

- That makes sense to me because...
- I appreciate what you said because...
- This makes me think about...
- This is similar to [describe], because...
- I noticed that you...
- I wonder why...
Sentence Starters for non-evaluative discussions

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Other ideas?
Explicit revision opportunities
Revising in Math Class

• Not just error analysis or “fixing” your answer

Alternative goals for revising

• Explanations could become more precise
• Explanations could use more mathematical terminology
• Solutions could be more illuminating
• Generate more than one way of solving
• Generate more than one way of representing
Examples of revision structures

• Generating multiple drafts
• Stronger and Clearer Each Time
Generating multiple drafts
Leah Simon, high school teacher, Dayton, OH

My Algebra students used rough draft thinking to start off our function carnival day! #iteachmath
Generating multiple drafts
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Generating multiple drafts
Leah Simon, high school teacher, Dayton, OH

My Algebra students used rough draft thinking to start off our function carnival day!

I played a video https://youtube.be/6f2tg_O2x5ms of a human slingshot ride, and we made a list on the board of everything we noticed about the ride. Then asked the students to sketch out a height vs. time graph.
Graph of Human Slingshot Ride

Draft 1

Draft 2

Draft 3
Generating multiple drafts
Leah Simon, high school teacher, Dayton, OH

To revise the first draft, I asked the students to consider if their graph included our observations we wrote on the board. We briefly discussed those, then the worked alone or in their small groups to come up with another version.

Then I drew some of their 2nd draft graphs on the board, (one that didn't start at 0, one that was pointy instead of smooth, one that didn't...
Graph of Human Slingshot Ride

Draft 1

Draft 2

Draft 3
Generating multiple drafts

• More videos: http://www.graphingstories.com/

• Given a story problem, create a graph.

• Given a story problem, draw a picture of the situation.

• Given a story problem, write an expression, equation, number sentence to represent the situation.

• Given a graph, write a story problem.

• Given a graph, write an expression or equation.
• Give students a prompt to elicit a first draft.
• Individual think time: Record a first draft
• Pair Share
  • Trade drafts with a partner. Each partner reviews their peer’s draft
  • Each person in the pair takes turns seeking to understand the other person’s draft by asking clarifying questions
• Individual think time
  • Trade drafts back
  • Revise the draft informed by the partner’s ideas
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<td>Second draft</td>
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Inspiration – what changes did you make to your definitions and why?

Retrieved from Stanford University, UL/SCALE website: https://ell.stanford.edu/sites/default/files/u6232/ULSCALE_ToA_Principles_MLRs_Final_v2.0_030217.pdf
Positioning Students as Competent
Write a story to represent this graph — speed as a function of time.
Joey: It says you’re going, you’re riding at a constant speed up the hill. But the part that’s going down, that point is, that’s the top of the hill. How can you be above the top of the hill?
Positioning Students as Competent

Mr. Wandless: How many people also wondered about Joey’s question?

[More than half of the students in the class raised their hands.]

Mr. Wandless: See, Joey, you’re not alone.

Joey: Okay.

Mr. Wandless: Almost everybody thought about it like that. Thanks for sharing that with us.
“Once I was able to get Joey to speak to the class, I stepped back to see how this would affect the understanding for other students. Some who thought they understood realized, through Joey’s rough draft talk, that they were no longer sure if they understood. After more students did get involved in the conversation, which squared away the understanding for the class. Afterwards I explicitly pointed out how Joey’s rough draft talk allowed all of us to have a better understanding of speed vs. time.”
“When I first heard of Rough Draft Thinking, I was thinking, ‘How could it apply to my students? When I presented the idea to them and tried the concept out, they picked up on it and started trying more. It established a safe set of norms to allow the students to feel accepted and try to connect ideas. As a teacher, I feel that mathematical discussions and engagement have improved because the students know that they can try, without being wrong.”
Students’ Voices

“I feel that I can have a conversation with others and openly share my ideas. Originally, I felt that if I spoke up, I would be made fun of. Now, when we work on Rough Drafts, I feel that I am really learning.”

- Joe, 11th grade

“We have different ways to talk in class about our ideas. I had one idea and then it was shown to the whole class. I was really surprised he picked my rough draft to show everyone.
We’re all rough drafts of the people we’re still becoming.

Bob Goff
Sara Calleja @APSElemLiteracy · 3m
Love this phrase. All my thinking is rough draft thinking these days!

Amanda Jansen @MandyMathEd · 5h
Tonight! Oct. 5, 7 p.m. Eastern. I'll give a webinar about #RoughDraftMath -- also can be viewed on NCTM's Facebook live. I've shared one of tonight's slides in this Tweet. Thanks to @stenhousepub for sponsoring. #NCTM100 Please come!

nctm.zoom.us/webinar/regist...

What is rough draft thinking?
- Communicating to learn.
- Sharing unfinished thinking and revising emerging ideas.
- Being willing to revise ideas even when “correct.”
Something ‘Shooty shooty shooty shooty shooty’

‘Shot!’

Haven’t figured out how it works yet

Lin-Manuel Miranda

"Shooty shooty shooty shot."-rough draft, 2014
Rough-Draft Talk in Mathematics Classrooms

Amanda Jansen, Brandy Cooper, Stefanie Vascellaro, and Philip Wandless

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ROUGH DRAFT MATH

REVISING TO LEARN

Amanda Jansen

Foreword by Robert Q. Berry, III
I used to think...
And now I think...

(regarding rough draft thinking in mathematics classrooms)
Thank you for attending!

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@MandyMathEd
#RoughDraftMath