Cultivating Reasoning

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Imagine this task

- Instead of:

  18 + [] + [] = 30, I ask:

Imagine this task

- The sum of 3 whole numbers is 30.
- Is it true that at least one number is even?

Imagine this task

- Or I ask :
- The sum of 3 whole numbers is 30. Which are true?
- A. At least one number is greater than 10.
- B. All numbers must be even.
- C. At least one number is a multiple of 3.
- D. The difference between the greatest and least of the numbers is a multiple of 3.

Responses

- A. True
- B. False (25, 2, 3)
- C. False (14, 14, 2)
- D. False (25, 2, 3)

Reasoning is encouraged when analyzing patterns

- We really don’t just want students to observe patterns.
- The math is the explanation that exposes the underlying structure.

For example

- Is 100 in this pattern or not?
- 4, 8, 12, 16,….
- How do you know?

For example

- Is 480 in this pattern?
- 40, 51, 62, 73,...
- How do you know?

One struggle students face

- is that they (and their teachers) struggle with what constitutes evidence of how we know something is true. There is often "sloppiness" about evidence.

For example…

- A teacher asks why you get an even number when you multiply an odd by an even.
- A student says : It works for 3 x 6 and 5 x 8 and 9 x 10.

For example…

- A response should be "good start, but how do you know it works for other odds and evens?"

Eventually we need

- Suppose there are an even number of copies of an odd number.

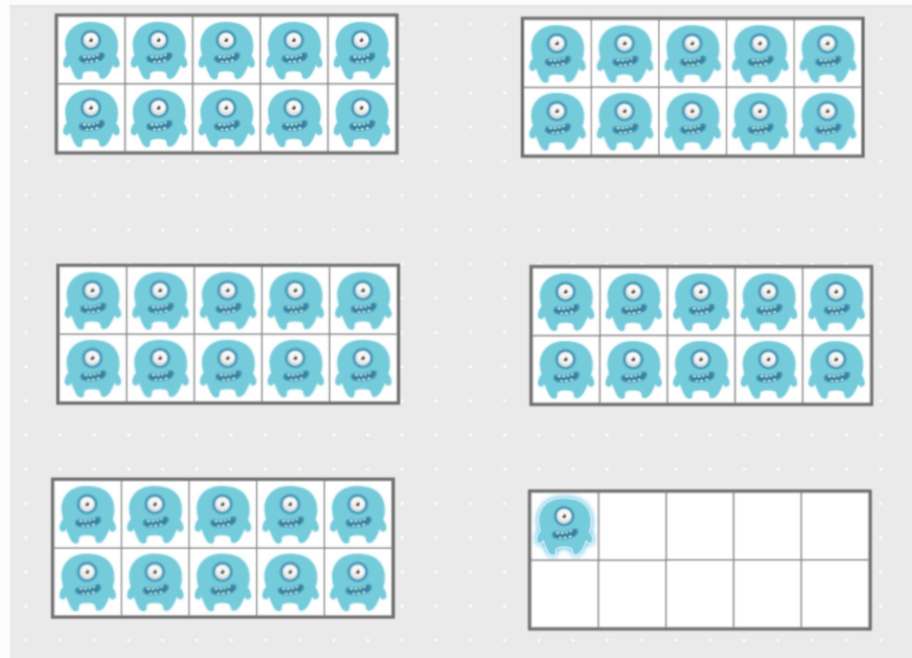
- Even + 1
- Even + 1
- Even + 1
- Even + 1

Let’s try with this

- You show a number with 5 full ten-frames and less than half of another ten-frame.

- What could it be?

5 full ten-frames and less than half of another



Or

- The sum of the digits of a 3-digit number is 15. What is the greatest possible number? How do you know?

Maybe

- It has to start with 9 to be as great as possible.
- The other two digits must add to 6.
- The greatest must be 960.

Or

- A ratio is equivalent to 9:50.
- Could it be equivalent to another ratio with whole number terms where the second term is 120?

The reasoning might be…

- If 9:50 = ? :120,then you’d have to have multiplied 9 by 2.4 and that’s not a whole number.

Here’s another idea to propose.

- There is no prism with 25 edges.
- True or false?

Students might think….

- Notice that a rectangular prism has 12 edges.
- Notice that a pentagonal prism has 15 edges.

Students might think….

- So the number of edges is 12, 15, …. and I don't think there could be 25.

Or it might be this proposal

- There is only one pair of whole numbers with a sum of 100 and a difference of 8.

- How would you convince someone that is true, or is it?

Maybe I'll think of numbers that add to 100

- 100 + 0 are 100 apart apart.
- 50 + 50 are 0 apart.
- 60 + 40 are 20 apart.
- 55 + 45 are 10 apart.
- 54 + 46 are 8 apart.

Maybe

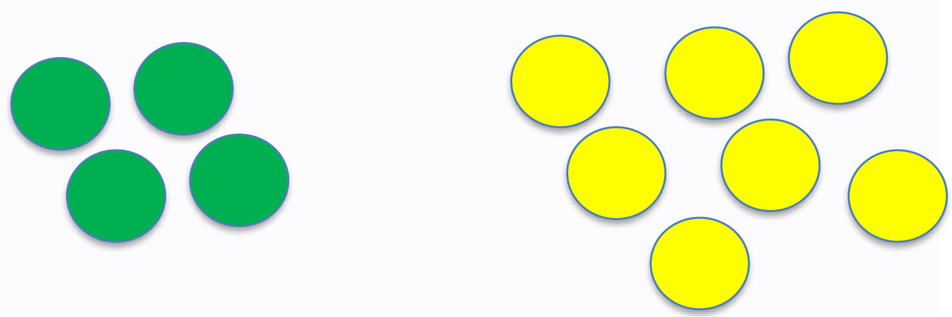
- $a + b = 100$
- $a - b = 8$

- $a = 100 - b$
- $a = 8 + b$

- $100 - b = 8 + b$
- $2b = 92$

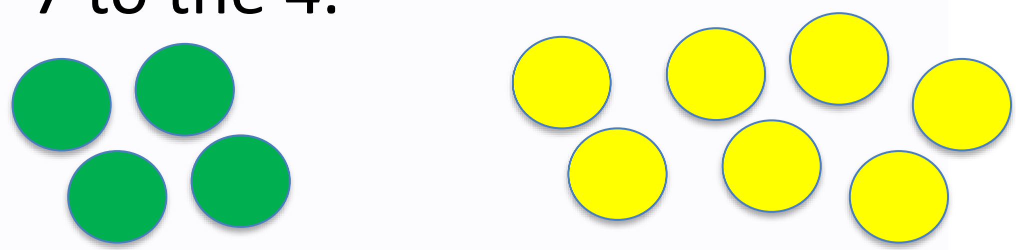
It might be something like

- **Without getting the answers for these questions**, how do you know that 4 + 7 will have to be less than 6 + 6?

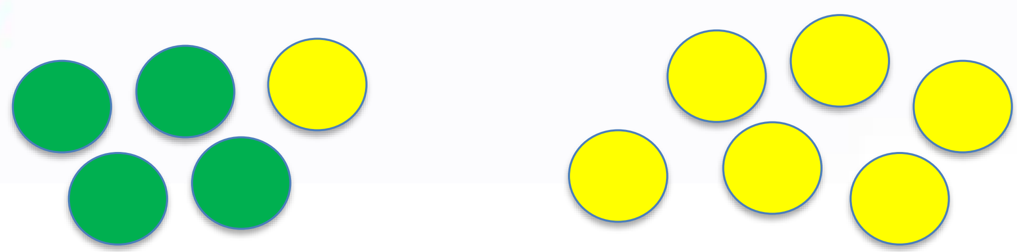


I hope students think

- 4 + 7 is the same as 5 + 6 since you just move one from the 7 to the 4.



- And 5 + 6 has to be less than 6 + 6.



Or you could propose:

- Any whole number starting with 3 appears in one, but only one, of these patterns.
- 3, 6, 9, 12,…
- 4, 7, 10, 13,…
- 5, 8, 11, 14,….
- Do you agree or not? Why?

You might think…

A 3, 6, 9, 12,…

B 4, 7, 10, 13,…

C 5, 8, 11, 14,….

Where are 30, 31, 32,.. 39?

Or

A 3, 6, 9, 12,…

B 4, 7, 10, 13,…

C 5, 8, 11, 14,….

Every number has a remainder of 0, 1, or 2 when you divide by 3.

You might propose:

- A square's area is always one unit greater than the area of a rectangle that is one unit longer and one unit less wide than the square.

One might draw a bunch of squares and check

- Use a 3 x 3 square.

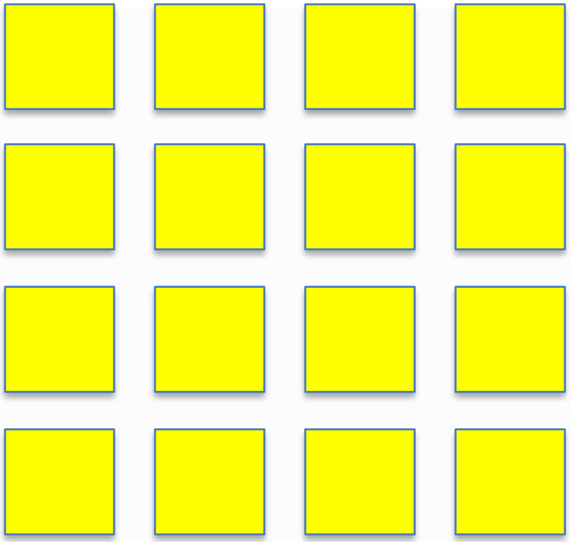
3 x 3 is 1 more than 2 x 4.

- Use a 5 x 5 square.

5 x 5 is 1 more than 4 x 6.

- Etc.

Maybe

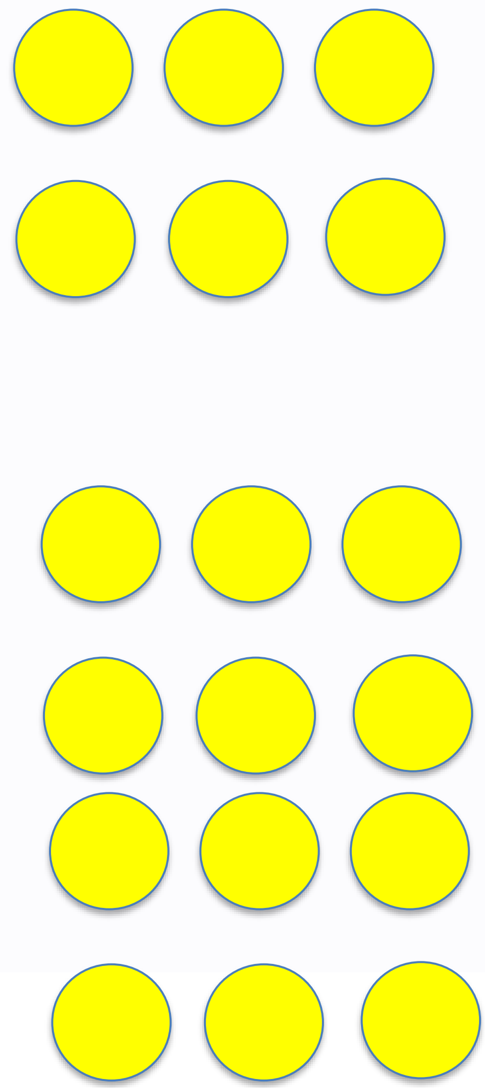


Maybe

You could propose

- The sum of two multiples of 3 has to be a multiple of 3.
- Convince us!

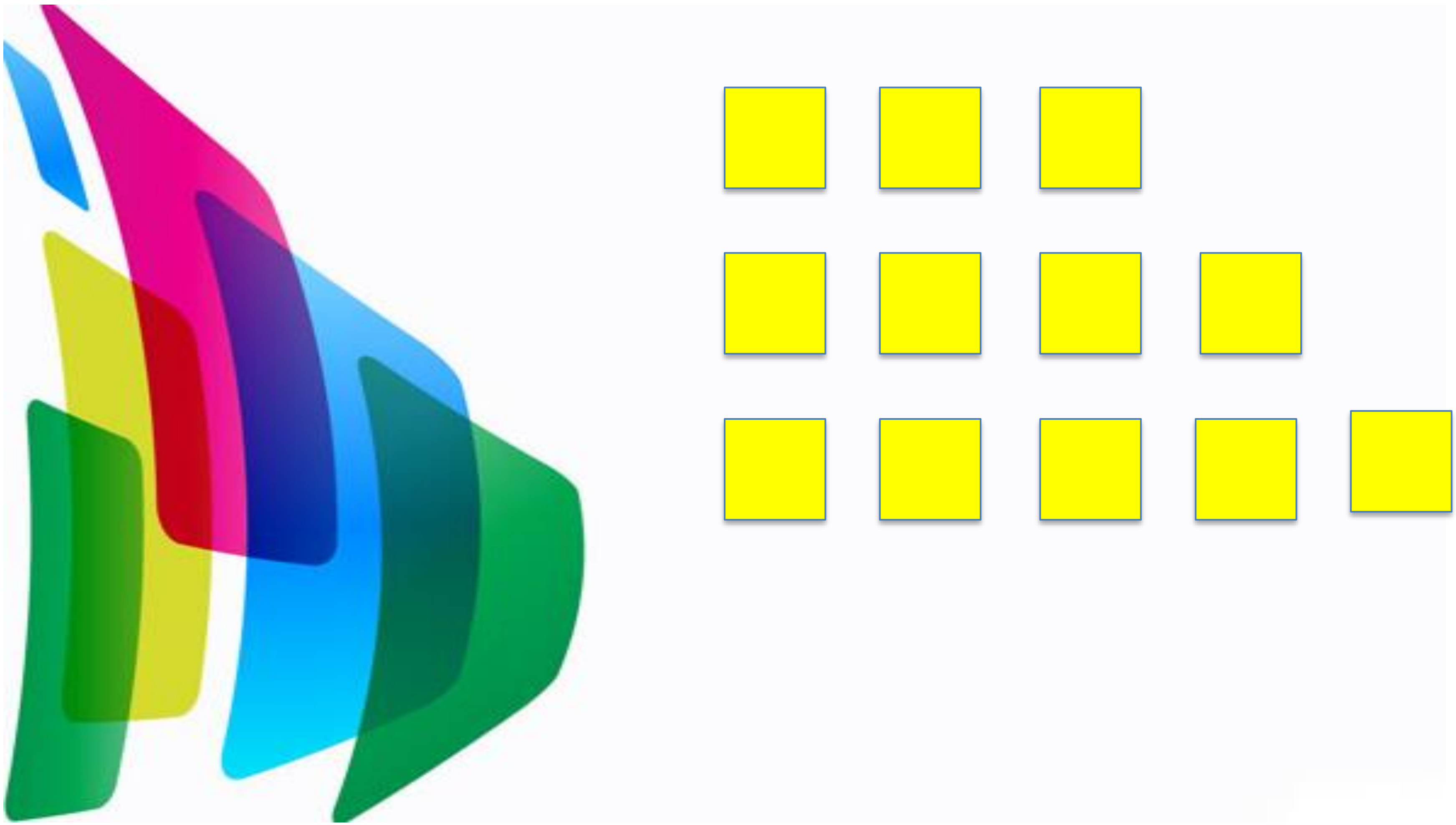
I might...



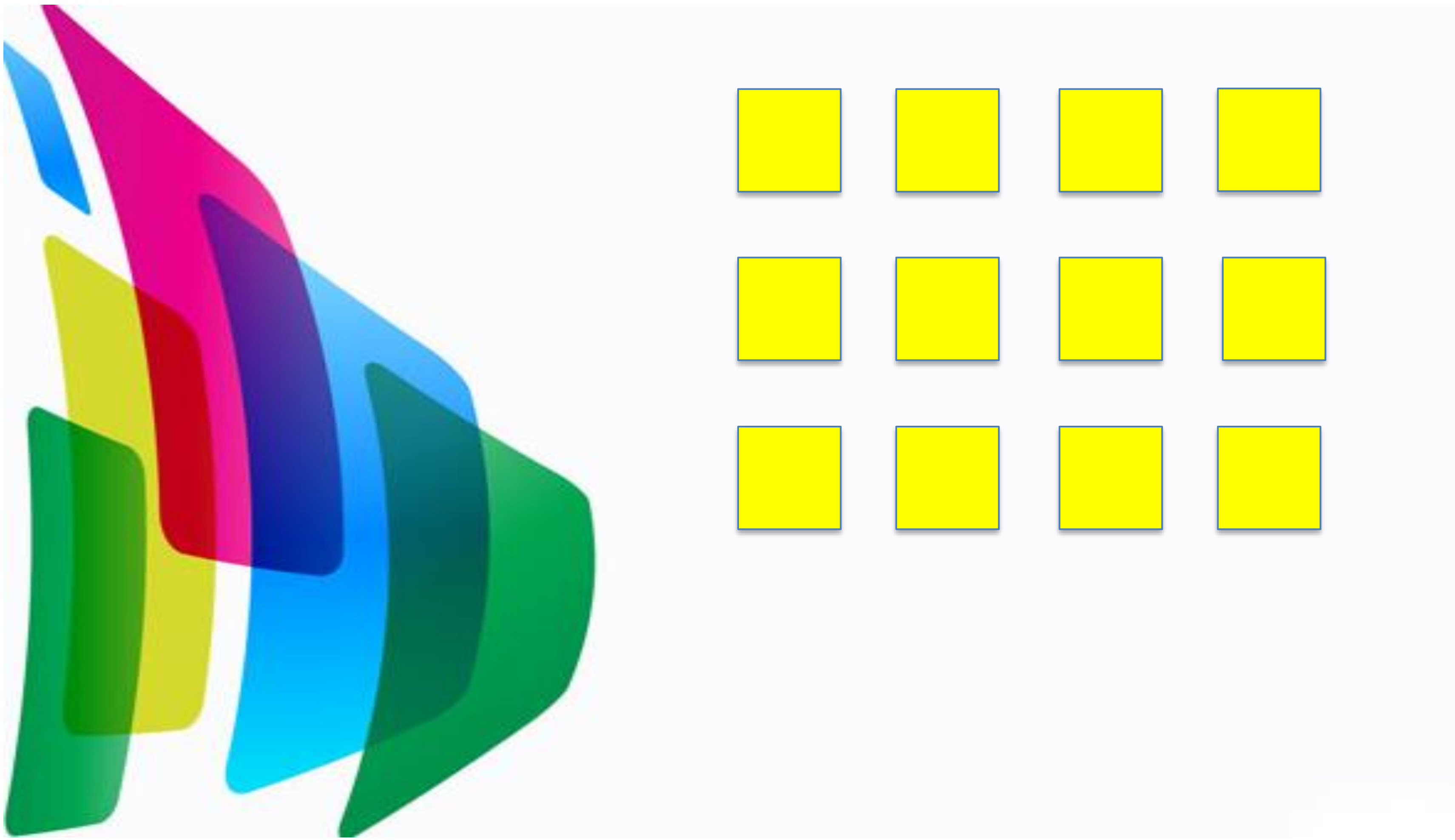
Or you could propose:

- The sum of 3 consecutive numbers is a multiple of 3.

I might



I might



2 truths and a lie

- 68 can be represented with 32 base ten blocks.
- 148 with 43 blocks
- 502 with 142 blocks.

- Which do you think is the lie?

It turns out that

- 68 can be 4 rods and 28 ones, or 32 base ten blocks.
- 148 with 43 blocks [It could be 40 blocks- 12 rods and 28 ones.]
- 502 can be 40 tens and 102 ones, or 142 blocks.

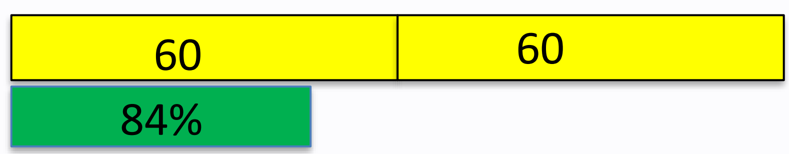
2 truths and a lie

- 48% of 50 is the same as 50% of 48.
- 120% of 80 is 60% of 40.
- 84% of 60 is 42% of 120.

- Which do you think is the lie?

It turns out that

- 0.48 x 50 = 0.50 x 48
- Lie since 120% of 80 is more than 80 but 60% of 40 is less than 40.



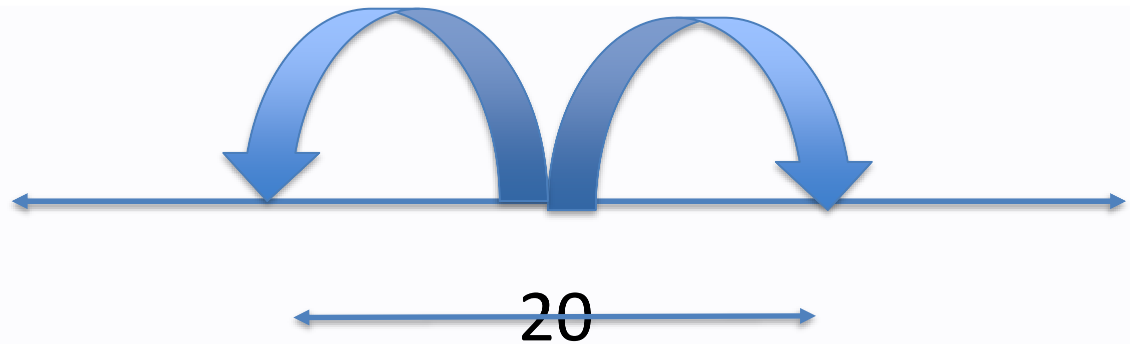
What do you think?

- You add two numbers and also subtract them.
- The sum is 20 more than the difference.
- What could the numbers be?

What do you think?

- Just try, but then

What do you think?



What do you think?

- You add two numbers and also subtract them.
- The sum is double the difference.
- What could the numbers be?

You can just try

- You learn that some possibilities are:

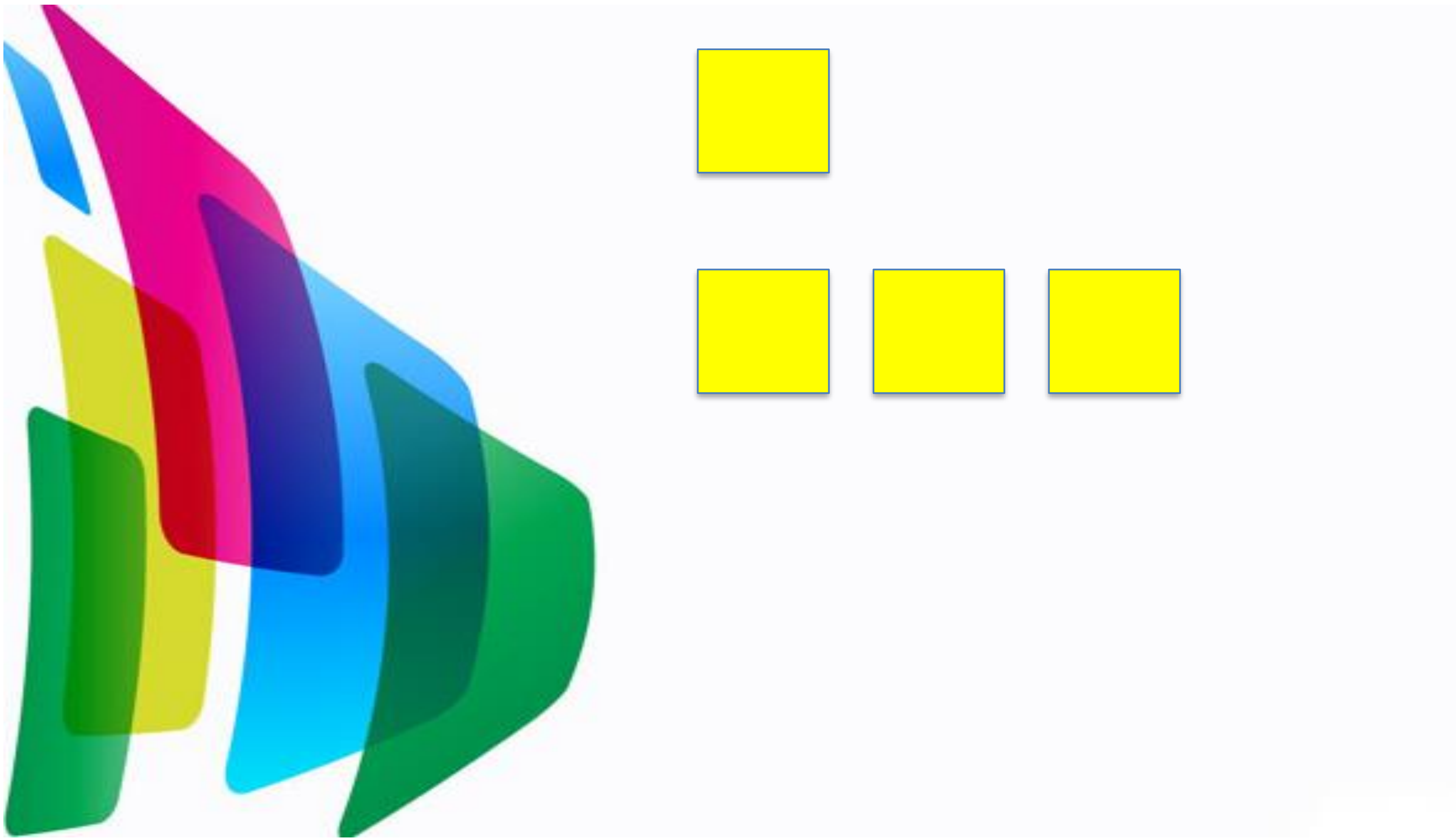
  9 and 3

  15 and 5

  12 and 4.

You start to guess what's going on.

But then you think about why



As you can see

- A lot of this is about getting past examples and looking at the structure, often visually.

You could

- Encourage students, when they notice things, to try to prove why.

Your questions

- Are there still issues you wish to raise?