Cultivating Reasoning

Marian Small
May 2023
Imagine this task

• Instead of:
  18 + [] + [] = 30, I ask:
Imagine this task

• The sum of 3 whole numbers is 30.
• Is it true that at least one number is even?
Imagine this task

- Or I ask:
- The sum of 3 whole numbers is 30. Which are true?
- A. At least one number is greater than 10.
- B. All numbers must be even.
- C. At least one number is a multiple of 3.
- D. The difference between the greatest and least of the numbers is a multiple of 3.
Responses

• A. True
• B. False (25, 2, 3)
• C. False (14, 14, 2)
• D. False (25, 2, 3)
Reasoning is encouraged when analyzing patterns

• We really don’t just want students to observe patterns.

• The math is the explanation that exposes the underlying structure.
For example

- Is 100 in this pattern or not?

- 4, 8, 12, 16,....

- How do you know?
For example

• Is 480 in this pattern?

• 40, 51, 62, 73,…

• How do you know?
One struggle students face

- is that they (and their teachers) struggle with what constitutes evidence of how we know something is true. There is often “sloppiness” about evidence.
For example...

- A teacher asks why you get an even number when you multiply an odd by an even.
- A student says: It works for 3 x 6 and 5 x 8 and 9 x 10.
For example...

- A response should be “good start, but how do you know it works for other odds and evens?”
Eventually we need

- Suppose there are an even number of copies of an odd number.

- Even + 1
- Even + 1
- Even + 1
- Even + 1
Let’s try with this

• You show a number with 5 full ten-frames and less than half of another ten-frame.

• What could it be?
5 full ten-frames and less than half of another
Or

• The sum of the digits of a 3-digit number is 15. What is the greatest possible number? How do you know?
Maybe

• It has to start with 9 to be as great as possible.
• The other two digits must add to 6.
• The greatest must be 960.
Or

• A ratio is equivalent to 9:50.
• Could it be equivalent to another ratio with whole number terms where the second term is 120?
The reasoning might be...

• If 9:50 = ?, then you’d have to have multiplied 9 by 2.4 and that’s not a whole number.
Here’s another idea to propose.

- There is no prism with 25 edges.

- True or false?
Students might think....

- Notice that a rectangular prism has 12 edges.
- Notice that a pentagonal prism has 15 edges.
Students might think....

• So the number of edges is 12, 15, .... and I don’t think there could be 25.
Or it might be this proposal

• There is only one pair of whole numbers with a sum of 100 and a difference of 8.

• How would you convince someone that is true, or is it?
Maybe I’ll think of numbers that add to 100

- 100 + 0 are 100 apart.
- 50 + 50 are 0 apart.
- 60 + 40 are 20 apart.
- 55 + 45 are 10 apart.
- 54 + 46 are 8 apart.
Maybe

- $a + b = 100$
- $a - b = 8$
- $a = 100 - b$
- $a = 8 + b$
- $100 - b = 8 + b$
- $2b = 92$
It might be something like

- Without getting the answers for these questions, how do you know that $4 + 7$ will have to be less than $6 + 6$?
I hope students think

• 4 + 7 is the same as 5 + 6 since you just move one from the 7 to the 4.

• And 5 + 6 has to be less than 6 + 6.
Or you could propose:

• Any whole number starting with 3 appears in one, but only one, of these patterns.
  • 3, 6, 9, 12,…
  • 4, 7, 10, 13,…
  • 5, 8, 11, 14,…

• Do you agree or not? Why?
You might think...

A 3, 6, 9, 12, ...
B 4, 7, 10, 13, ...
C 5, 8, 11, 14, ...

Where are 30, 31, 32, .. 39?
Or

A 3, 6, 9, 12,…
B 4, 7, 10, 13,…
C 5, 8, 11, 14,…

Every number has a remainder of 0, 1, or 2 when you divide by 3.
You might propose:

- A square’s area is always one unit greater than the area of a rectangle that is one unit longer and one unit less wide than the square.
One might draw a bunch of squares and check

- Use a 3 x 3 square.  
  3 x 3 is 1 more than 2 x 4.
- Use a 5 x 5 square.  
  5 x 5 is 1 more than 4 x 6.
- Etc.
Maybe
Maybe
You could propose

- The sum of two multiples of 3 has to be a multiple of 3.
- Convince us!
I might...
Or you could propose:

• The sum of 3 consecutive numbers is a multiple of 3.
I might
I might

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2 truths and a lie

• 68 can be represented with 32 base ten blocks.
• 148 with 43 blocks
• 502 with 142 blocks.

• Which do you think is the lie?
It turns out that

- 68 can be 4 rods and 28 ones, or 32 base ten blocks.
- 148 with 43 blocks [It could be 40 blocks - 12 rods and 28 ones.]
- 502 can be 40 tens and 102 ones, or 142 blocks.
2 truths and a lie

• 48% of 50 is the same as 50% of 48.
• 120% of 80 is 60% of 40.
• 84% of 60 is 42% of 120.

• Which do you think is the lie?
It turns out that

- $0.48 \times 50 = 0.50 \times 48$
- Lie since 120% of 80 is more than 80 but 60% of 40 is less than 40.
What do you think?

- You add two numbers and also subtract them.
- The sum is 20 more than the difference.
- What could the numbers be?
What do you think?

• Just try, but then
What do you think?
What do you think?

- You add two numbers and also subtract them.
- The sum is double the difference.
- What could the numbers be?
You can just try

• You learn that some possibilities are:
  9 and 3
  15 and 5
  12 and 4.

You start to guess what’s going on.
But then you think about why
As you can see

• A lot of this is about getting past examples and looking at the structure, often visually.
You could

• Encourage students, when they notice things, to try to prove why.
Your questions

• Are there still issues you wish to raise?