Building Students' Mathematical Competence

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Good Mathematics Teaching

Conceptualizing “good” teaching in mathematics: teacher knowledge and instructional practices are the two most commonly cited components (Wilson, Cooney, & Stinson, 2005)

Mathematics teachers of students of color are more likely to:

• Disconnect taught procedures from students’ thinking
• Teach fragmented or unexplained procedures
• Teach mathematics vocabulary out of context
• Assess students based on following
• Use less resources such as manipulatives, even when available

## Defining Our Terms

<table>
<thead>
<tr>
<th>Antiracist Ideas/Practices</th>
<th>Racist Ideas/Practices</th>
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<tbody>
<tr>
<td>An idea that suggests racial groups are equals in all their apparent differences—that there is nothing right or wrong with any racial group. Any practice that produces or sustains racial equity between racial groups.</td>
<td>An idea that suggests one racial group is inferior or superior to another racial group in any way. Any practice that produces or sustains racial inequity between racial groups.</td>
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<td><strong>ASSIMILATIONIST</strong></td>
<td><strong>SEGREGATIONIST</strong></td>
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<td>An idea that a racial group is culturally or behaviorally inferior and is supporting cultural or behavioral enrichment programs to develop that racial group.</td>
<td>An idea that a permanently inferior racial group can never be developed and is supporting policy that segregates that racial group.</td>
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(Kendi, 2019)
Racist ideas about Black and Brown Students

- Aggressive or Violent Behavior
- Lack of Intelligence or Mathematics Ability
- “Limited” Language Proficiency Aligns with Intelligence
- Lack of Interest or Motivation in Education
- Cultural or Family Deficiencies
Racist Practices in Education

How might you see racist (assimilationist & segregationist) ideas play out in mathematics classrooms?

– What types of interactions might you expect?
– How might you expect instruction to be modified?
– How might you expect the content to change?
– How could you expect students be organized?
– In what ways might you expect teachers to talk to or about students?

How might you see antiracist ideas play out in mathematics classrooms?
Racist Ideas in Teaching Math

• Teaching to the basics, only providing opportunities to engage in disconnected procedures, vocabulary out of context, and constraining access to challenging mathematics
• Hyper-focusing on misbehavior
• Punishing behavior harshly or removing students from instruction
• Missing the mathematical contributions of students
• Dismissing incorrect answers as having no value
• Discussing intellectual limitations of students
• Barring students’ home language/disparaging student experiences
Relational Interactions that Reveal (Anti)Racist Ideas

Classroom interactions can be seen as embodying these ideas.

“A communicative action or episode of moment-to-moment interaction between teachers and students, occurring through verbal and nonverbal behavior that conveys meaning” (Battey, 2013).

5 Types of Relational Interactions (RIs):

- Addressing Behavior
- Framing Student Ability
- Acknowledging Student Contributions
- Attending to Culture or Language
- Setting the Emotional Tone
Relational Interactions

Addressing Behavior -
- Positive (Low Intensity): “Some people are doing mathematics very quietly in front.” Mr. L
- Negative (High): “Don’t touch, don’t turn it over or you will be disqualified. Don’t touch.” Mr. T*

Framing Student Ability -
- Positive (Low): “Please think about that, I know you can understand it, I know you can get it.” Ms. S
- Negative (Low): “The original problem said 20 but I cut it down, um I didn’t want to make it too difficult.” Mr. D

*Italics indicates speaker’s emphasis
Relational Interactions (cont.)

Acknowledging Student Contributions -

- Positive (Low): “Ok, I see a couple of people have the answer... pretty good. A couple of people have the answers for both parts.” Mr. Jones
- Negative (High): “No, wrong answer... Thomas, that is incorrect.” (italics – speaker’s emphasis) M. B

Attending to Culture or Language -

- Positive (High): Teacher introduces the problem in the context of the world cup, something many students were discussing as the class began. Mr. L
- Negative (High): Teacher corrects African American Vernacular English, never addressing the mathematical contribution of the student. Ms. S
Relational Interactions (cont.)

Setting the Emotional Tone -

• Positive (High): "Is there anybody that doesn't understand this problem still? Don't be ashamed. You know you can't be ashamed in this class because I tell you all the time, there's a lot of stuff I don't understand in math and math, I'm going to make it one of my... It's one of my goals. If you're working on that too, it's ok." Mr. G

• Negative (Low): “If I see you are arguing and this and that, I may have to step in and help you.” Ms. S
Why can’t we just focus on content instruction?

• No correlation between quality of content instruction and quality of relational instruction

• However, the quality of content instruction does predict the frequency of relational interactions
Which has better content instruction?

Teacher 1

Teacher 2

Frequency (interactions per lesson)

Addressing Behavior
Framing Math Ability
Acknowledging Student Contributions
Setting the Emotional Tone
(Anti)racist ideas around Behavior

Addressing Behavior

Frequency (interactions per 4 lesson)

- White Suburban Teachers
- White Urban Teachers
- Black Urban Teachers
(Anti)racist ideas around Ability

Frequency (interactions per 4 lesson)

Framing Math Ability

- White Suburban Teachers
- White Urban Teachers
- Black Urban Teachers

High Negative | Low Negative | Low Positive | High Positive
(Anti)racist ideas around Thinking

Frequency (interactions per 4 lesson)

Acknowledging Student Contributions

- High Negative
- Low Negative
- Low Positive
- High Positive

White Suburban Teachers
White Urban Teachers
Black Urban Teachers
(Anti)racist ideas around Culture

Frequency (interactions per 4 lesson)

- **White Suburban Teachers**
- **White Urban Teachers**
- **Black Urban Teachers**

Attending to Culture and Language:
- High Negative
- Low Negative
- Low Positive
- High Positive
(Anti)racist ideas around Emotions

Frequency (interactions per 4 lesson)

- High Negative
- Low Negative
- Low Positive
- High Positive

Setting the Emotional Tone

- White Suburban Teachers
- White Urban Teachers
- Black Urban Teachers
Relational Interactions: Findings

- 96% of behavioral interactions are negative
- 63% of acknowledging student contributions are positive
- 55% of all Relational Interactions are negative
- Negative interactions are typically of higher intensity
- RIs account for between 11-13% of change in mathematics achievement
How do you build students’ mathematical competence?

• How do you credit students for their ideas?
• How do you support students in taking ownership of the mathematics?
• How do you explicitly note students’ mathematical competence?
• How do you find the valid mathematics in students’ ideas?
• How do you normalize student difficulties?
• How do you challenge existing gendered and racial stereotypes?
4 + 9 = 5 \times 3 - 2
Finding Valid Mathematics

[Jonathan presents a solution for $10+10-10=5+\Box$]

**Students** [in unison]: I don’t get it.

**Ms. Moore**: Hold on a second. Let’s try and understand this, okay? Let’s try to understand this.

**Ms. Moore**: So, ten plus ten, and you added another ten [Jonathan incorrectly added another ten on the left side to get $30-10$], so you had ten plus ten plus ten equals thirty. And then you minused ten and you got twenty. And then you took this side and five plus fifteen equals twenty. Ok, that makes sense.

**Nigel**: We’re going to be on this problem for a month, until we turn old… we’ll be on that problem. I’m taking my shoes off!

What would you do here?
Finding Valid Mathematics

Jonathan presents a solution for $10+10-10=5+\square$

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Nigel: We’re going to be on this problem for a month until we turn old… we’ll be on that problem. I’m taking my shoes off!

Ms. Moore: But that makes sense, what he did. He added ten plus ten plus ten minus ten, that equals twenty.

Sandra: But, teacher, you said it was a takeaway sign.

Ms. Moore: But what Jonathan did was he added another ten. So that makes sense.

[Ms. Moore then poses $10+10+10-10=5+\square$ to the whole class]
Ms. Townsend: What is a net?
Leticia: It’s like a shape laid out.
Ms. Townsend: Ok, that's good, so you said a net is a shape that's laid out. Ok, Leonard, can you add to that?
Daron: You can fold it into a solid figure.
Ms. Townsend: OK! A net that’s laid out (points to Leticia) that you can fold into a solid figure (points to Daron). Before you can fold it into a solid figure, what do you have to do? What you gotta do? (students mumble) Ok, let’s, let’s do it again. Leticia said a net was, give it to me again.
Leticia: A shape that's laid out
Ms. Townsend: So a shape that’s laid out, flat, on paper, what dimension is that?
Brenda: 2-dimensional.
Ms. Townsend: And then Daron said when you fold it together you get what?
Daron: A solid figure.
Ms. Townsend: A solid figure, which is three dimensional.
Teachers Building Students’ Mathematical Competence

Task Design:
• Supporting students in learning mathematics (even the basics) through high cognitive demand tasks
• Positioning students’ everyday lives as the source for mathematics

Building on Student ideas:
• Normalizing student difficulties
• Finding value in students’ incorrect answers
• Drawing on students’ multilingual resources

Affirming Students’ Mathematics Ability:
• Clearly noting students’ competence
• Building collective responsibility for the mathematics

Behavior:
• Noting model behavior
• Handling behavioral issues privately to avoid escalating interactions
• First approaching students with care
Shifting Instruction to Antiracist Ideas

Watching our own interactional patterns –

- How do you react when students struggle with the “basics”?
- How do you tend to react to misbehavior?
- How do you react to students’ thinking when they give an incorrect answer?
- How do you note ALL students’ competence?
- How do you react when students’ bring their informal lives and language into the classroom?
- How do you provide space for a variety of emotions in mathematics?
Strategies for Attending to RIs

• Begin to *notice* your relational interactions.
• *Monitor* your own interactions.
  – Or better yet, have a colleague do it for you!
  – Start with one dimension
  – Ask yourself: What intentional and *unintentional* messages might my interactions be sending to students?
• Strategize how to *modify* your interactions in areas where you find yourself possibly sending negative messages.

Challenge the consumption of racist ideas!
“I think first and foremost I wanted to convey that this sort of striving to be antiracist is an ongoing journey. I realized that in many ways my sort of story or journey, tracks the journey that many people have gone through or are going through... I wanted to model for people what each and every one of us can do and should do in terms of how deeply critical we need to be of ourselves.”

Ibram X. Kendi
Thank you for attending!

Many thanks to those who’ve contributed to this work - Rebecca Neal, Luis Leyva, Karlyn Adams-Wiggins, Immanuel Williams, Victoria Belizario, Rachel Greco, Roshni Shah, and all of the TEACHERS who’ve opened their classrooms!

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