

## A Routine for Reasoning to Ensure ALL Students are Modeling with Mathematics



NATIONAL COUNCIL OF  
TEACHERS OF MATHEMATICS  
CELEBRATING 100 YEARS

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PRACTICES

#NCTM100

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## Agenda

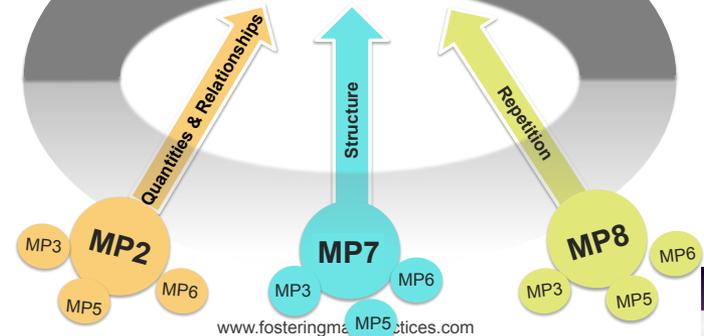
- Framing and Agenda
- What does it mean to Model with Mathematics?
- Overview of *Analyzing Contexts and Models* Routine
- Unpack *Analyzing Contexts and Models* Routine
- Q & A

## Standards for Mathematical Practice

- MP1 Make Sense of Problems and Persevere in Solving Them
- MP2 Reason Abstractly and Quantitatively
- MP3 Construct Viable Arguments and Critique the Reasoning of Others
- MP4 Model with Mathematics
- MP5 Use Appropriate of Tools Strategically
- MP6 Precision in Mathematics
- MP7 Look for and Make Use of Structure
- MP8 Look for and Express Regularity in Repeated Reasoning



## MP4 Model with Mathematics



## What does it mean to Model WITH Mathematics?



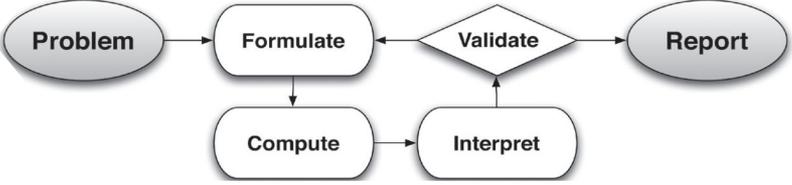

Model WITH Mathematics



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## Model with Mathematics



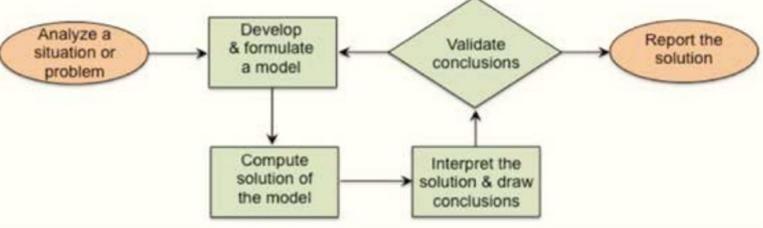
```

    graph LR
      Problem([Problem]) --> Formulate([Formulate])
      Formulate --> Validate{Validate}
      Validate --> Report([Report])
      Formulate --> Compute([Compute])
      Compute --> Interpret([Interpret])
      Interpret --> Validate
  
```



**COMMON CORE  
STATE STANDARDS**

## Model with Mathematics



```

    graph LR
      Analyze([Analyze a situation or problem]) --> Develop[Develop & formulate a model]
      Develop --> Compute[Compute solution of the model]
      Compute --> Interpret[Interpret the solution & draw conclusions]
      Interpret --> Validate{Validate conclusions}
      Validate --> Report([Report the solution])
      Validate --> Develop
  
```

## How do we develop mathematical modelers?



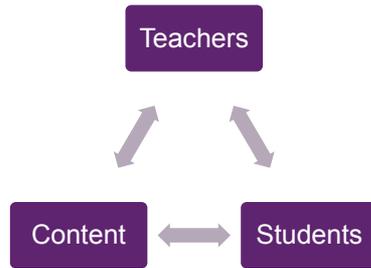

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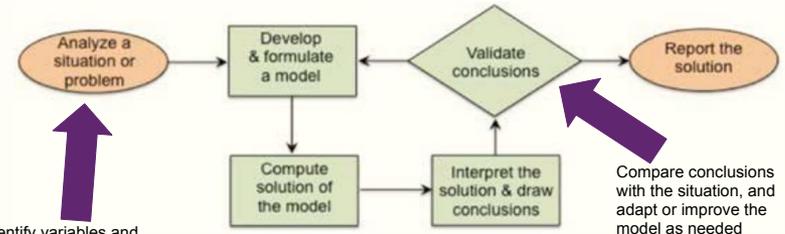
## Instructional Routines: Repeatable & Predictable

"Designs for interaction that organize classroom activities"

— Magdalene Lampert, NCSM 2015

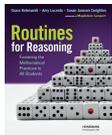


## Model with Mathematics



Identify variables and constraints in the situation and select those that represent essential features

Compare conclusions with the situation, and adapt or improve the model as needed



The designs for interaction stay the same, the tasks change.

### Analyzing Contexts & Models

<b>1</b>	Launch Routine		THINKING GOAL: Model with mathematics
<b>2</b>	Analyze the Situation	Individual Think Time, Pairs, Tour, Pairs	
<b>3</b>	Interpret a Model	Individual Think Time, Pairs, Annotate	
<b>4</b>	Analyze and Adapt Models	Partner Share, Discuss & Annotate	
<b>5</b>	Reflect on Your Thinking	Individual Write Time, Pairs, Share & Record	

### Analyzing Contexts and Models

**WHAT:** Consider the mathematics of a real world situation, and analyze a model that represents the situation

**WHY:** To interpret and engage in the real world with a mathematician's eye. To develop a bank of questions to ask yourself that are critical in the mathematical modeling process.

## Analyzing Contexts and Models



Make Sense



Analyze the Situation



Interpret the Model



Analyze and Adapt Models



Reflect on Learning



## Make Sense of the Situation

Ask Yourself:

- What's the question I'm exploring?
- What about the context do I need to consider?

*How many square inches of pizza will everyone in this class eat in their lifetime?*

## Analyze the Situation

Pair



Tour



Pair



## Share Interpretations & Analyze the Context

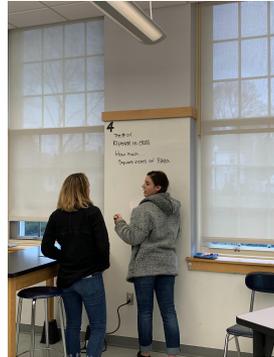
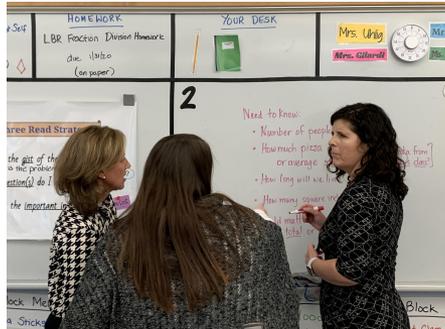
Standing with your partner, share your interpretations of the context and together create a list:

- Important quantities are...
- It will be helpful to know...

*How many square inches of pizza will everyone in this class eat in their lifetime?*



## Share Interpretations & Analyze the Context



## Consider Classmates' Interpretations

Tour the room and read others' lists. Consider what you'd add to your own.

Ask yourself:

- Have they considered something we should also consider?
- Have they (or we) made assumptions?

*How many square inches of pizza will everyone in this class eat in their lifetime?*



## Consider Classmates' Interpretations

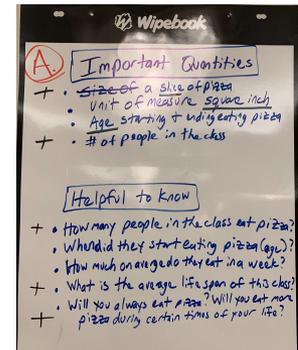
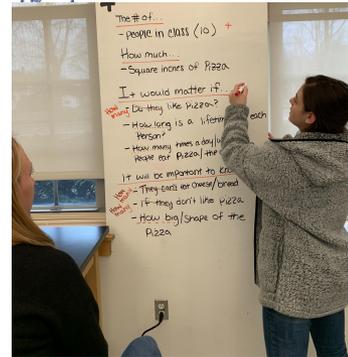
Return to your chart, reflect and refine

- Place a + next to key ideas
- Place a - next to ideas less relevant
- Describe quantities as 'The number/amount of...'
- Articulate questions as quantities 'How much/many...?'

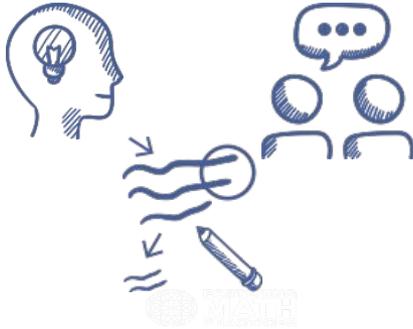
*How many square inches of pizza will everyone in this class eat in their lifetime?*



## Consider Classmates' Interpretations



## Interpret a Model



## Interpret a Model



Ask yourself:

- Where/how does the model represent quantities?

- What is the average amount of pizza someone eats in their life time?  
 CICI's Pizza poll estimated the average person eats 6,000 slices. (Reported by South West News Service on Jan. 12, 2013)
- How many square inches are in one slice of pizza?  
 12 inch pizza:  $\pi r^2 \approx 113 \text{ in}^2$   
 $113 \div 8 \text{ slices} \approx 14 \text{ in}^2$   
 18 inch pizza:  $\pi r^2 \approx 254 \text{ in}^2$   
 $254 \div 12 \text{ slices} \approx 21 \text{ in}^2$
- How many people in this class? 23  
 $6,000 \times 23 = \frac{138,000}{132,000}$   
 $132,000 \times 14 = 1,848,000 \text{ in}^2$       $132,000 \times 21 = 2,772,000 \text{ in}^2$   
 $18.2 \times x \geq 12 \text{ in}$   
 $254 \text{ in}^2 \geq x \geq 113 \text{ in}^2$   
 $21 \text{ in}^2 \geq x \geq 14 \text{ in}^2$   
 $2,772,000 \text{ in}^2 \geq x \geq 1,848,000 \text{ in}^2$

## Interpret a Model



Share the quantities with your partner, together identify questions you have and assumptions the model makes.

They considered the number/amount of...

They found the number/amount of... by...

A question I have about the model is...

An assumption the model makes is....

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## Analyze and Adapt the Model



- We think the model predicts ... because...
- We aren't sure the model is precise because...
- The estimation impacts the outcome because...

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## Reflect on learning



- Next time I consider a situation and try to mathematize it I will ask myself....
- When thinking about constraints, it's important to....
- When analyzing models, I learned to pay attention to....
- A critical feature of modeling is....



## Reflect on learning

A. When making sense of a situation with my mathematician's eye. I

learned to pay attention to all the variables that can change the situation

B. Next time I consider a situation and try to mathematize it I will ask myself

what is important in the problem and what I have to pay attention to.

C. When analyzing models, I learned to pay attention to

details that could make a big impact on the solution or the starting base numbers

## Reflect



I notice...

I wonder...

### Analyzing Contexts & Models

1	Launch Routine	THINKING GOAL Model with mathematics
2	Analyze the Situation	Individual Think Time, Pairs, Tour, Pairs
3	Interpret a Model	Individual Think Time, Pairs, Annotate
4	Analyze and Adapt Models	Partner Share, Discuss & Annotate
5	Reflect on Your Thinking	Individual Write Time, Pairs, Share & Record

## Designs for Interaction

### Analyzing Contexts & Models

1	Launch Routine	THINKING GOAL Model with mathematics
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### Routine Designs

- Think-Pair-Share
- Ask Yourself Questions
- Sentence Starters and Frames
- Turn and Talks
- Four Rs

## 5 Essential Instructional Strategies that keep the focus on mathematical thinking while providing access for ALL learners...routinely



Ask-yourself questions



Annotation



Sentence frames and starters



The Four Rs – repeat, rephrase, reword,  
record



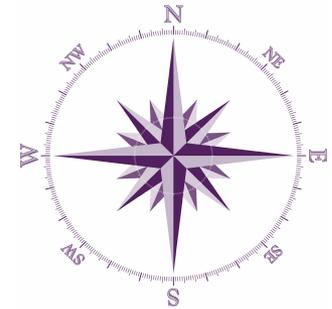
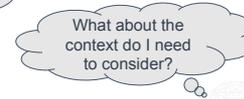
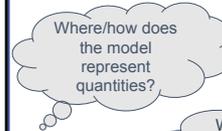
Turn-and-Talk

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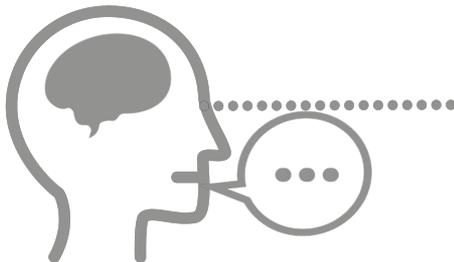


## Ask-Yourself Questions...

- Combat learned helplessness
- Promote student agency



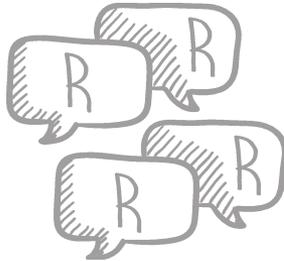
## Annotation Connects the Verbal to the Visual



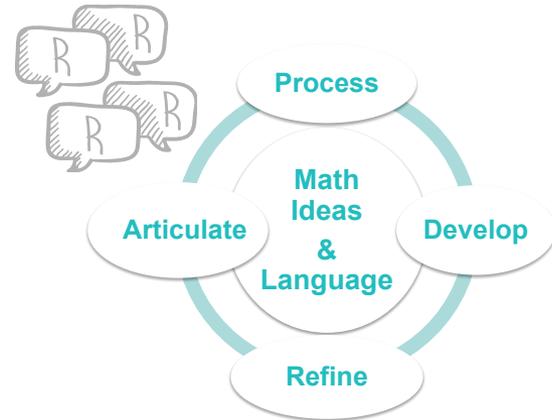
## Sentence Frames and Starters

- Important quantities are...
- It will be helpful to know...
- An assumption the model makes is....
- When analyzing models, I learned to pay attention to....

## The Four Rs

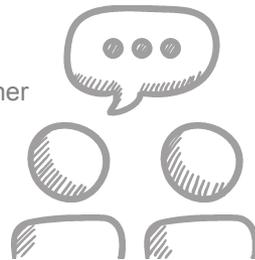


Repeat Rephrase Reword Record



## Turn and Talks

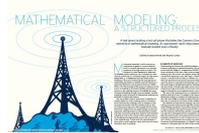
- Engage ALL students
- Work out ideas and language with a partner



## Designing remote interactions Ask yourself...?

- How can I ensure students have the space they need to think and talk mathematically? (e.g. pregnant pause, press pause, turn and talk, stop and jot, etc.)
- Based on my goal, where should I provide the most processing time/space? (e.g. analyze situation, interpret model, analyze and adapting the model, etc.)
- How are students working with rough draft talk and rough draft thinking to refine both? (Individual: Think it through, Jot it down, Talk it out; Partner: talk together, write together)
- How can I see/hear student thinking?

# Q & A



## Modeling Resources

Anhalt and Cortez, NCTM Mathematics Teacher Vol. 108, No. 6 • February 2015

Blum and Ferrie, Mathematical Modeling: Can it be taught and Learnt?, 2009

[www.corestandards.org](http://www.corestandards.org)

Tasks: 3 Act Tasks, Fermi Problems, Achieve the Core  
[www.fosteringmathpractices.com](http://www.fosteringmathpractices.com)