

1. Calculate the exact value of $\int_1^e x^2 \ln x dx$.

$$\int_1^e (x^2) \ln(x) dx$$

$$u = \ln(x) \quad v = \frac{x^3}{3}$$

$$u' = \frac{1}{x} \quad v' = x^2$$

$$\frac{d}{dx} \left(\frac{x^3}{3} \ln(x) - \frac{x^3}{9} \right)$$

$$\frac{x^3}{3} \left(\frac{1}{x} \right) + \ln(x) (x^2) - \frac{x^3}{3}$$

$$\frac{x^2}{3} + x^2 \ln(x) - \frac{x^3}{3}$$

$$x^2 \ln(x)$$

$$\int_1^e (x^2) \ln(x) dx = \left[\frac{x^3}{3} \ln(x) \right]_1^e - \int_1^e \left(\frac{1}{x} \right) \left(\frac{x^3}{3} \right) dx$$

$$\log_e e = 1 \quad e^1 = e$$

$$\log_e 1 = 0 \quad e^0 = 1$$

$$= \left[\frac{x^3}{3} \ln(x) \right]_1^e - \int_1^e \frac{x^2}{3} dx$$

$$= \left[\frac{x^3}{3} \ln(x) \right]_1^e - \left[\frac{x^3}{9} \right]_1^e$$

$$= \left(\left(\frac{e^3}{3} \ln(e) \right) - \left(\frac{1^3}{3} \ln(1) \right) \right) - \left(\left(\frac{e^3}{9} \right) - \left(\frac{1^3}{9} \right) \right)$$

$$= \left(\left(\frac{e^3}{3} \right) - \frac{1}{3}(0) \right) - \left(\frac{e^3}{9} - \frac{1}{9} \right)$$

$$= \left(\frac{e^3}{3} \right) - \frac{1}{9} (e^3 - 1)$$

$$= \frac{3e^3}{9} - \frac{e^3 - 1}{9}$$

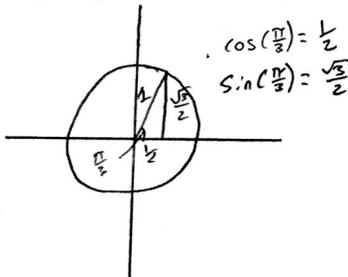
$$= \frac{2e^3 + 1}{9}$$

2. Show that $\int_0^{\frac{\pi}{6}} x \sin 2x dx = \frac{\sqrt{3}}{8} - \frac{\pi}{24}$.

$$\int_0^{\frac{\pi}{6}} (x) (\sin(2x)) dx$$

$$u = x \quad v = -\frac{1}{2} \cos(2x)$$

$$u' = 1 \quad v' = \sin(2x)$$



$$\int_0^{\frac{\pi}{6}} (x) (\sin(2x)) dx = \left[-\frac{x}{2} \cos(2x) \right]_0^{\frac{\pi}{6}} - \int_0^{\frac{\pi}{6}} -\frac{1}{2} \cos(2x) dx$$

$$= \left[-\frac{x}{2} \cos(2x) \right]_0^{\frac{\pi}{6}} - \left[-\frac{1}{4} \sin(2x) \right]_0^{\frac{\pi}{6}}$$

$$= \left(\left(-\frac{\pi}{12} \right) \left(\cos\left(\frac{\pi}{3}\right) \right) \right) - \left(-\frac{0}{2} \cos(0) \right) - \left(-\frac{1}{4} \sin\left(\frac{\pi}{3}\right) \right) - \left(-\frac{1}{4} \sin(0) \right)$$

$$= \left(\left(-\frac{\pi}{12} \right) \left(\frac{1}{2} \right) - 0 \right) - \left(-\frac{1}{4} \right) \left(\frac{\sqrt{3}}{2} \right) - 0$$

$$= -\frac{\pi}{24} + \frac{\sqrt{3}}{8}$$

$$= \frac{\sqrt{3}}{8} - \frac{\pi}{24}$$

3. Find the exact value of the integral $\int_0^{\sqrt{2}} \sqrt{4-x^2} dx$.

$$\int_0^{\sqrt{2}} \sqrt{4-x^2} dx$$

$$x = 2 \sin \theta$$

$$dx = 2 \cos \theta d\theta$$

$$\int_0^{\sqrt{2}} \sqrt{4 - (2 \cos \theta)^2} \cdot 2 \cos \theta d\theta$$

$$\int_0^{\sqrt{2}} \sqrt{4 - 4 \cos^2 \theta} d\theta$$

$$\int_0^{\sqrt{2}} \sqrt{4(1 - \cos^2 \theta)} d\theta$$

$$2 \int_0^{\sqrt{2}} \sqrt{\sin^2 \theta} d\theta$$

$$\sqrt{2} = 2 \cos \theta$$

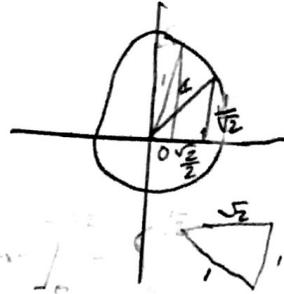
$$\frac{\sqrt{2}}{2} = \cos \theta$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\frac{\pi}{4}$$

$$0 = 2 \cos \theta \Rightarrow \theta = \frac{\pi}{2}$$

$$2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin \theta d\theta$$



$$2 \left[-\cos \theta d\theta \right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$2 \left(-\cos\left(\frac{\pi}{2}\right) - \left(-\cos\left(\frac{\pi}{4}\right)\right) \right)$$

$$2 \left(-\frac{\sqrt{2}}{2} - 0 \right)$$

$$\boxed{-\sqrt{2}}$$

4. Find the exact value of the integral $\int_0^{0.5} \arcsin x dx$.

$$\int_0^{0.5} \arcsin(x) dx$$

$$\int_0^{0.5} \arcsin(x) dx = \left[x \arcsin(x) \right]_0^{0.5} - \int_0^{0.5} \frac{x}{\sqrt{1-x^2}} dx$$

$$u = \arcsin(x) \quad v = x$$

$$u' = \frac{1}{\sqrt{1-x^2}} \quad v' = 1$$

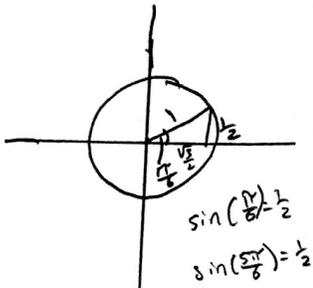
$$u = 1 - x^2$$

$$du = -2x dx$$

$$dx = \frac{du}{-2x}$$

$$x = 0.75 \Rightarrow u = 1 - (0.75)^2 = 0.4375$$

$$x = 1 \Rightarrow u = 0$$



$$\left[-\sqrt{1-x^2} \right]_0^{0.5}$$

$$-\sqrt{1-0.5^2} - (-\sqrt{1-0^2})$$

$$-\sqrt{0.75} - (-1)$$

$$-\frac{\sqrt{3}}{2} + 1$$

$$= \left[x \arcsin(x) \right]_0^{0.5} - \left[-u^{1/2} \right]_0^{0.4375} - \sqrt{u}$$

$$= (0.5 \arcsin(1/2) - 0) - \left((-\frac{3}{4})^{1/2} - (-1)^{1/2} \right)$$

$$= (0.5) \left(\frac{\pi}{6} \right) - \left(-\frac{\sqrt{3}}{2} + 1 \right)$$

4/4

$$= \left[\frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 \right]$$

or

$$\left[\frac{5\pi}{12} + \frac{\sqrt{3}}{2} - 1 \right]$$

5. Using the substitution $t = \tan \theta$, find the value of the integral

$$\int_0^{\frac{\pi}{4}} \frac{d\theta}{3\cos^2\theta + \sin^2\theta}$$

$$\int_0^{\frac{\pi}{4}} \frac{d\theta}{3\cos^2\theta + (1 - \cos^2\theta)}$$

$$\int_0^{\frac{\pi}{4}} \frac{d\theta}{2\cos^2\theta + 1}$$

$$\int_0^{\frac{\pi}{4}}$$

divide
top &
bottom
by $\cos^2\theta$

$$t = \tan \theta$$

$$dt = \sec^2\theta d\theta$$

$$d\theta = \frac{dt}{\sec^2\theta}$$

Didn't finish

I see that ~

	Test Item Number: (Check any boxes that apply.)	1	2	3	4	5														
Written Work	My work is ORGANIZED and easy to follow.	x	x	x	x	x														
	My work is correct and complete; includes support, multiple approaches, graph/table, whenever possible.	x	x		x															
	I used MATH NOTATION flawlessly.	x	x	x	x	x														
Mathematical Methods	I used the appropriate mathematical method to answer this question. (I applied what was taught for this section.)	x	x	x	x															
	I SUCCESSFULLY used the appropriate mathematical method to answer this question. (I correctly applied what was taught for this section.)	x	x		x															
Evidence of Comprehension	I <u>attempted</u> to support my work.	x	x	x	x															
	I <u>correctly</u> supported my work. If my answer was wrong, I may not have had a chance to change my answer, but I made it clear I knew I was wrong.	x	x		x															
Preparation	I did work just like this for homework.	x	x	x	x	x														
	I did work like this for homework, and I checked it to make sure I was right.	x	x	x	x	x														
	I did work like this for homework, I checked it, AND I posted it to my homework file.																			

Copy and paste your goals from the previous assessment (unless this is the first assessment of the year) below:

My goal for this year is to continue to get much better at showing my work (especially graphically) and finding the simplest way to solve the problem. I would also like to continue to understand why the things I am doing work mathematically, not just memorizing things. In addition, I would like to clean up my work and make as few little mistakes as possible.

Answer each question with “yes” or “not really”:

I did what I said I would do in my previous goals. yes

Following through with my goals helped me improve. yes

distraction

- Test Score 7-9 NDA Grade C
 - Minimally achieving the standard/“In the ballpark” on some topics.
 - Work is often unsupported
 - Some mathematical methods applied, but it is clear that some of the methods being assessed were not learned or mastered at this point
 - reasonable evidence of understanding in some places, gaps in comprehension exist, many errors went unnoticed
 - Communication is clear in some places, incomplete in others
- Test Score 4-6 NDA Grade D
 - Routine/basic problems are not done correctly
 - Major gaps in logic and/or comprehension are clearly evident
 - Communication is lacking
 - Notation errors minimal/no attempt to apply conventions to written work
- Test Score 1-3 NDA Grade F
 - Forget to do it. Leave blank. Don't try.

Student perception of score on a 1-21 scale 16
 (No partial points, please; 15.5 is not an acceptable score.)

Rubric rating submitted on: 9/5/2018, 2:30:32 PM by dguyette@notredameacademy.com

	100	99	98	97	96	95	94	93	91	89	87	85	83	81	79	76	73	69	65	59	52
S c o r e Y o u r s c o r e: 52	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1

Comments: