Please work on the problem below while we wait to begin:

**Do Now**

When the midpoint M of one side of a square paper is folded over to a vertex as shown on the right, a 37° angle is created. Find the measures of all the angles in the diagram.

NCTM 2020
Presented by Joy Hsiao
Brooklyn Technical High School
Today’s goals

- Share my experiences
- Share my students’ work
- Build your experiences
- Q&A (last 5-10 minutes)
Review Problem

(Answers can be found on the handout.)
Q: What are the side lengths of the right triangles?

Solution on the next page
Two extension problems:
Prove
1. All three triangles are similar.
2. The triangle side lengths are multiples of 3-4-5.
My origami journey with my students
Menger Sponge  (a 14” fractal cube)
Bard HS Early College (2009)

Menger Sponge (a fractal)
Bard HS Early College (2009)

Math Horizons, April 2011
Bridges Conference  Towson University (2012)

Photo by Christopher Bartlett, *Bridges Conference Art Exhibition*, 2012
http://bridgesmathart.org/bridges-2012/month-long-art-exhibition
Brooklyn Tech HS (2012) 90th Anniversary

Holiday tree

Photo by Joy Hsiao, Brooklyn Tech, 2012

Math investigations (2015)

Math investigations (2015)

Instructions:
1. Use a square paper
2. Fold in half
3. Fold along the diagonal of the rectangle
4. Open the back flap
5. Fold the bottom edge to point P
6. Open the right side and fold the bottom cross all the way through

Prove that this white edge is 1/2 of the original edge of the square.

Photos and origami by Joy Hsiao,
Stars and origami diagram, 2020
Math investigations (2015)

- Algebra
- Geometry
- Trigonometry

Hsiao J., Finding Fifths in Origami, Mathematics Teacher, Vol. 109, August 2015

Fig. 5 The angle measure of interest is approximately 70.52°, not the ideal 72°.
Sample student work 1

By Anna Tao, Brooklyn Tech 2012
Origami Exercise

• Two-fold angle – conjecture, proof, construction of crease pattern

1. Pinch a midline

2. Bring the lower right corner to the midline and create a crease that ends at the lower left corner.

3. Find $x^\circ$.
Stuyvesant High School

Holiday display

Lunar New Year Decorations

Photos by Joy Hsiao, Stuyvesant High School, 2017
2017 Origami USA Annual Convention

Photos by Joy Hsiao, Origami USA Convention, 2017
Sample student work 2

Trigonometric Application

After folding the modular piece, I outlined the crease patterns on the paper with different colors. It was helpful to also fold and indicate the 8x8 grid so that I could notice patterns in the folds. What I noticed was that vertex A of the triangle was exactly 1 unit away from the edge of the paper and that vertex C was exactly 2 units away from the edge of the paper. From this, I was able to determine that length AC is exactly 5 units long. I also noticed that the perpendicular dropped from vertex B was exactly 2 units long and the point at which the perpendicular intersects with side AC was 1 unit away from vertex A. Using this information, I used the Pythagorean theorem to find the lengths of sides AB and BC.

“I used the Pythagorean theorem to find the lengths of side AB and BC.”

After, I used the law of cosines to find the angles of the triangle.

\[ AB = \sqrt{BO^2 + AO^2} = \sqrt{3^2 + 1^2} = \sqrt{5} = 2.236 \ldots \]
\[ BC = \sqrt{BO^2 + CO^2} = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5} = 4.472 \ldots \]

After, I used the law of cosines to find the angles of the triangle:

\[ BC^2 = AB^2 + AC^2 - 2(AB \cdot AC \cdot \cos A) \]
\[ BC^2 = 5^2 + 5^2 - 2(\sqrt{5} \cdot 5 \cdot \cos A) \]
\[ 0.4472 \ldots = \cos A \]
\[ A = 63.4349 \ldots \]

“I confirmed these calculations by drawing all the crease patterns on GeoGebra.”

“I confirmed the calculations by drawing all the crease patterns on GeoGebra.”

Pre-calculus student, Stuyvesant High School, 2017
Question:
If the square paper used to fold this model is 1x1, what is the edge length of the finished cube?
Hidden Beauty –
the folding process, logic, and changing patterns

Photos and origami by Joy Hsiao, *Polyhedron*, 2017
Four Fish (2017)
(A 30 cm x 30 cm tessellation)

Photos and origami by Joy Hsiao, *Four Fish*, 2017
Bridges Conference
University of Waterloo (2017)
Bridges Conference
University of Waterloo (2017)

Nämnaren. Tidskrift för Matematikundervisning, January 2018

Photo by Joy Hsiao, Bridges Conference, 2017
Queens Metropolitan High School
(Arts and Mathematics 2018)

Mini books (one sheet)  Boxes with lids

Photos by Joy Hsiao, *Mini books and boxes with Lids*, 2018
Brooklyn Technical HS (Valentine’s Day 2020)

Students’ observations:
• Perpendicular and parallel lines
• Angle bisectors
• Diagonals of a square are congruent and perpendicular
• 8 congruent isosceles right triangles
• Rotational symmetry of 90°
• Line symmetries
• Dilation of a square with a constant of dilation equal to 3 (estimated)
• Similar triangles
• 45-45-90 triangles and so on...

Photos and origami by Joy Hsiao, *Hearts and crease pattern*, 2020
Sample student work 3 (origami heart crease pattern)

This is my interpretation of the heart crease pattern, with all the creases marked in red.

First attempt by Lang Ni
Brooklyn Tech 2019
Origami and Math
Polyhedra — regular, semi-regular, truncated, stellated

Polyhedra — regular, semi-regular, truncated, stellated

<table>
<thead>
<tr>
<th>Platonic Solids</th>
<th># vertices</th>
<th># edges</th>
<th># faces</th>
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<tr>
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<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Cube</td>
<td>8</td>
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<td>Octahedron</td>
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</tr>
<tr>
<td>Icosahedron</td>
<td>12</td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

Tessellations (made with one piece of paper)

Photos and origami by Joy Hsiao, *Tessellations*, 2016-2019
Fractals (self-similar property)

Hydrangea (Shuzo Fujimoto)

Fractal Flower (Roman Diaz)

Hydrangea (Shuzo Fujimoto)

Menger Sponge (Karl Menger)

Photos and origami by Joy Hsiao, *Flowers and Menger sponges*, 2009-2020
Recycled Materials

- Placemat
- Gift wraps
- Tea bags
- Money
- Metrocards
- Ribbons
- Wallpaper
- Chopstick sleeve
- Wallets made with subway maps, book cover, and shopping bag

Photos and origami by Joy Hsiao, *Origami made with recycled materials*, 2009-2020
Interactive origami

Magic ball  Clic-clac  Flexicubes  Double flexicubes

Photos and origami by Joy Hsiao, *Interactive origami*, 2009-2020
Interactive origami  Fireworks

Photos and origami by Joy Hsiao, *Fireworks*, 2020
Origami Exercise

- Two-fold angle – conjecture, proof, construction of crease pattern

1. Pinch a midline

2. Bring the lower right corner to the midline and create a crease that ends at the lower left corner.

3. Find $x$. 

![Diagram of origami exercise]
Proof without words
Crease pattern constructions

- square
- midpoint
- circle
- intersection
- auxiliary line
- angle bisector
- perpendicular bisector
When I **fold origami**, I
1. Try to understand the logic
2. Pose questions
3. Think about extensions (variations)
4. Prove observations mathematically
When I **solve math problems**, I
1. Try to understand the logic
2. Pose questions
3. Think about extensions (variations)
4. Prove observations mathematically
Origami as a research topic

- Easy to pose original questions
- Hands-on, multidimensional
- High school math, applications of prior knowledge in new situations
- Accessible to students on all levels
- Rich in math connections
Thank you!

JHsiao@schools.nyc.gov
Origami Dog

Photo and diagram by Joy Hsiao, *Dog*, 2020