The Mathematics of Game Shows NCTM100 Edition

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PRIZES!

Want to win?

We'll need some volunteers for games.

You may leave here with *a TI calculator*!

(Seriously, we're giving stuff away.)

PRIZES!

Speaking of which…

Who wants to win?

Let’s all play a game together!

Best of Ten

I'm going to start calling out numbers from 1 to 10. But before I do, you pick three.

What'll it be?

Just pick three.

The first player who gets all their numbers is the winner.

1 6

2 7

3 8

4 9

5 10

Best of Ten

Hopefully you picked three numbers.

How many people do you think picked this number?

1	6
2	7
3	8
4	9
5	10

Best of Ten

Hopefully you picked three numbers.

How many people do you think picked *both* of the first two numbers?

1	6
2	7
3	8
4	9
5	10

Best of Ten

But did you pick *these* three numbers?

What is the probability of picking these three numbers?

1	6
2	7
3	8
4	9
5	10

Best of Ten

I won’t need this, but just in case…

What is the probability of picking three of these four numbers?

1	6
2	7
3	8
4	9
5	10

Analyzing Best of Ten

10 numbers: pick 3.

What is the probability of picking 3 correct numbers on 3 turns?

Take it turn by turn: on the first turn there is a 3 in 10 chance at picking correctly…

Analyzing Best of Ten

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$$\frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} = \frac{6}{720} = \frac{1}{120}$$

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Take it all at once: pick 3 from among a group of 10, using Pascal's Triangle.

Analyzing Best of Ten

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$$\frac{3\ choose\ 3}{10\ choose\ 3} = \frac{1}{120}$$

Analyzing Best of Ten

10 numbers: pick 3.

What is the probability of picking 3 correct numbers on no more than 4 turns?

You can still use Pascal's Triangle!

$$\frac{3\ choose\ 3 \cdot 7\ choose\ 1}{10\ choose\ 4} = \frac{7}{210} = \frac{1}{30}$$

(What's the probability of picking within 9 turns? 10 turns?)

Analyzing Best of Ten

10 numbers: pick 3.

What is the probability of not one person in the entire audience winning in 3 turns? Exactly one winner?

Each player had a *119/120* chance of losing. If we knew the number of players…

(119/120)^300 is approximately *8%*

Best of Twenty

20 numbers. Each player picks 10 numbers.

100,000 people are playing. What is the probability of the game ending on Turn 10 with a single winner?

What is the probability that there is no winner until Turn 11? On average, how many players would win on this turn?

Things get difficult quickly!

Best of Seventy-Five

75 numbers. Each player picks 25 numbers: 5 each chosen in these ranges: 1-15, 16-30, 31-45, 46-60, 61-75. Except also there's a free space, some blue hair, maybe a 5:00 buffet…

If 200 people are playing bingo, what is the probability of a player getting bingo by Turn 10? Turn 15? Turn 20? What is the probability of there being a tie?

Computer simulation helps…

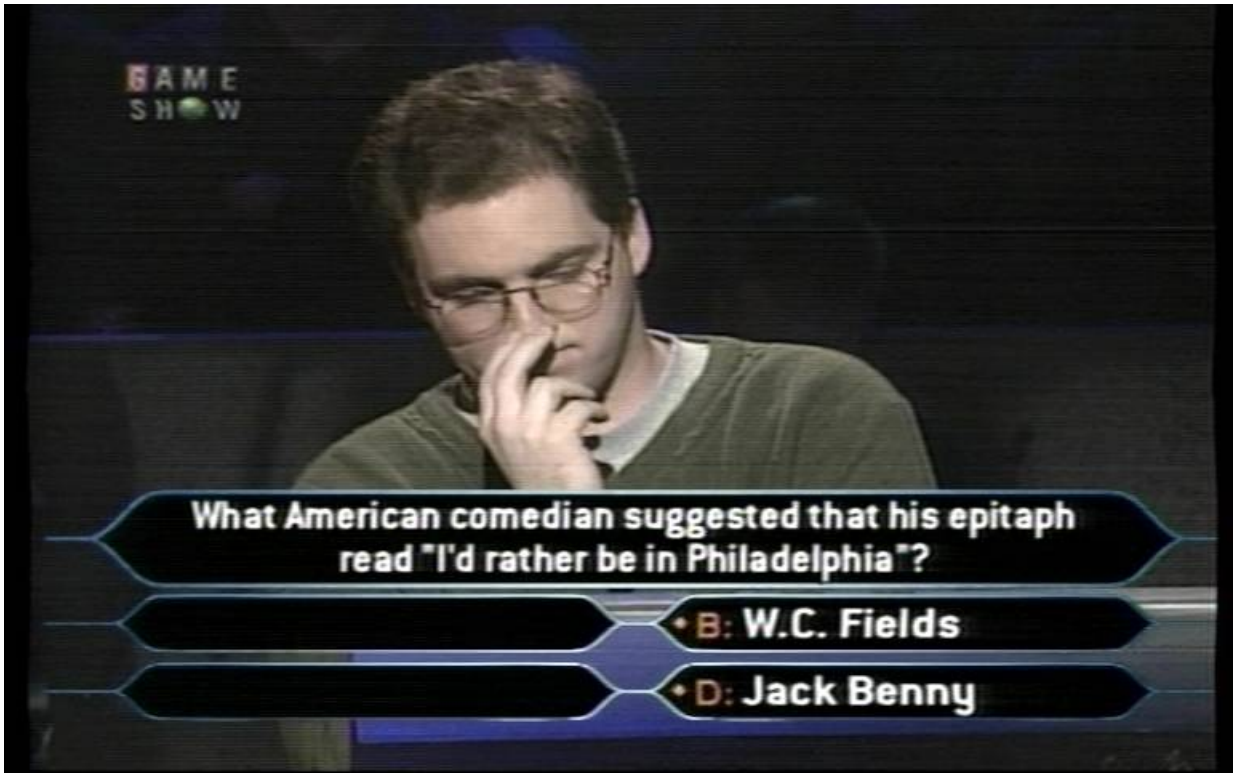
Math in Game Shows

Game shows are full of math problems…

- Contestants
  - How do I play best?
  - How much risk should I take?
- Producers
  - How do I build a fun game to watch?
  - How will contestants behave?
  - What happens with N live players?
  - *How much money are we giving out?*

Personal Encounters

February 2000: *Millionaire* (episode #49)



(for $1000: How many degrees in a right angle?)

Personal Encounters

February 2000: *Millionaire* (episode #49)

(Got the next one wrong. 30 million people saw me insult Hawaii.)

Personal Encounters

April 2004: *The Price Is Right*

(Double overbid on the showcase! Bummer.)

Personal Encounters

May 2007: *National Bingo Night (ABC)*

(I worked on this show a lot longer than it lasted.)

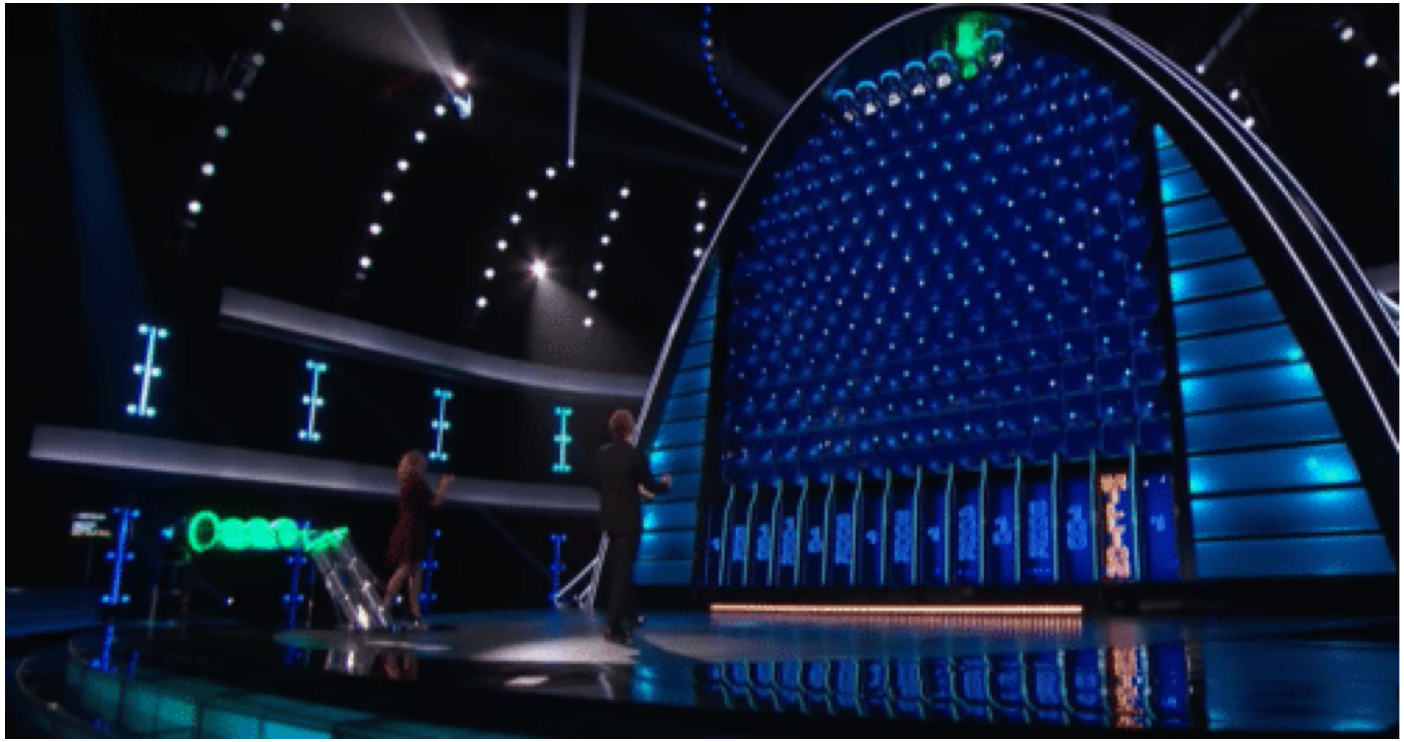
Personal Encounters

August 2012: *Oh Sit! (CW)*

(Wipeout + musical chairs + Jamie Kennedy = ???)

Personal Encounters

December 2016: *The Wall (NBC)*



(also known as *Million Dollar Plinko*)

Personal Encounters

2017: *The Joker's Wild (TBS)*

(My resume says I am Snoop's mathematical advisor.)

The Price Is Right

- Now in its 48th year
- Lots of good math problems!
- Huge sample size of repeated play

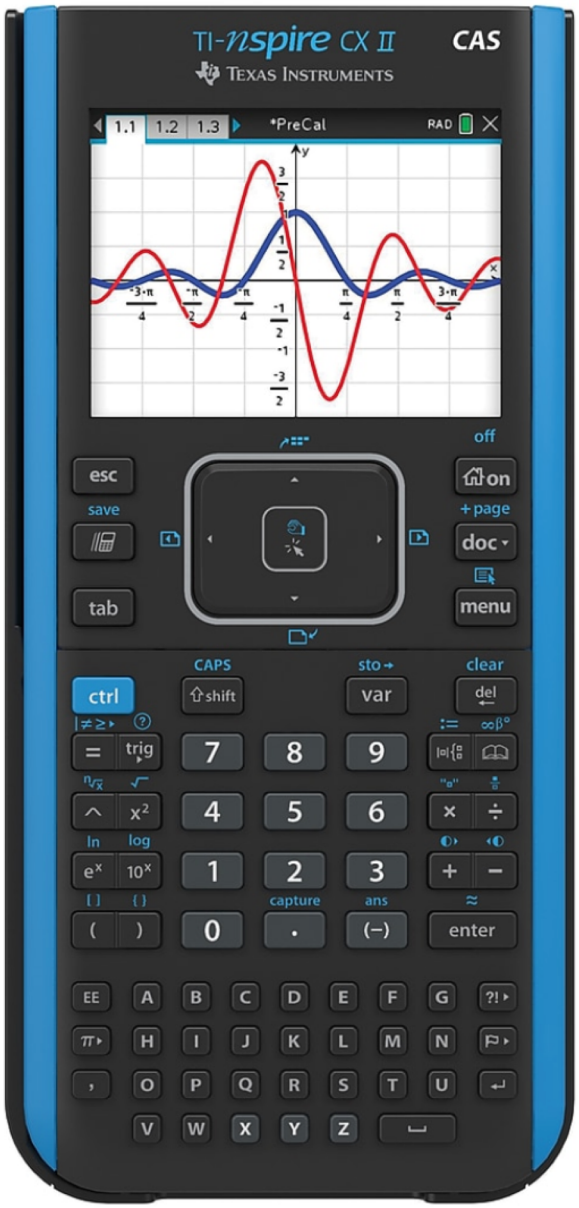
tpirstats.com

Sponsored by… Texas Instruments!

Surely you know us!

What's next in the sequence 81, 82, 83, 84…?

We are!



Dice Game

There are five digits to guess.

Every digit is from 1 to 6, only.

You will roll a die. If it's incorrect, you'll have to tell me if the real digit is higher or lower than the roll.

So, who's got a die, uh, number cube?

Dice Game

How many registrations have there been for NCTM 100 seminars?

We'll learn this by rolling dice.

If you win, NCTM will send you a *free TI calculator*!

If you lose, you still get a *free webinar*.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
CELEBRATING 100 YEARS

,

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The Player's Question

Based on how I roll…

how likely am I to win the game?

An Unlikely Event

The Producers' Questions

If we keep offering this game repeatedly,
how much will we have to pay for it?

How likely is a win?

(and the most important question…)

The Producers' Questions

If we keep offering this game repeatedly,
how much will we have to pay for it?

How likely is a win?

Is this game fun to watch??

Analysis: Dice Game

The probability of winning is heavily influenced by the correct number in the price.

Digit	P(correct)
1	
2	
3	
4	
5	
6	

Take a moment and try to fill in the table.

Analysis: Dice Game

The probability of winning is heavily influenced by the correct number in the price.

Digit	P(correct)
1	4/6
2	5/6
3	6/6
4	6/6
5	5/6
6	4/6

What can we do with this?

Analysis: Dice Game

For any prize, we can compute the probability of winning by using this chart.

Digit	P(correct)
1	4/6
2	5/6
3	6/6
4	6/6
5	5/6
6	4/6

What's P(32,631)?

What's P(3455)?

Analysis: Dice Game

This is an especially good TPIR game because the show can control its win rate.

Digit	P(correct)
1	4/6
2	5/6
3	6/6
4	6/6
5	5/6
6	4/6

This car costs $26,165. What do you think happened?

Historical Data

Dice Game has been played 381 times since 2000, fully detailed on tpirstats.com.

<u>2000-2020</u>

Win: 48.8% (186 times)

Lose: 51.2% (195 times)

All but one right: 74.9% of losses (146 times)

Every number wrong: NEVER

An Unintended Consequence

The restrictions on prizes for Dice Game bleed into other games that award cars.

Another Game!

We promise this game will not involve rolling one die five times.

Instead it will involve rolling five dice at least once!

Thanks again to Texas Instruments and NCTM for their generous support.

Who wants to play?

Mathematics Immersion for Secondary Teachers (MIST)
mist.edc.org

Mathematics like you've never done before!

Join me for a virtual immersion experience

- A low-cost immersive mathematics experience in number theory
- Live collaborative learning with other teachers - sharing information, ideas, and results
- Certificate for 18 hours of PD completion
- Potential stipend for contributing to research data collection

Flexibility — Multiple date options in July

Nine 2-hour sessions are scheduled throughout July. Sessions 2 through 9 offered twice for your convenience.

Registration is limited to 60 participants. Send contact info to mist@edc.org.

Visit mist.edc.org or email mist@edc.org for more information.

Let ‘Em Roll

In this game you… uh, I will *roll five dice*.

To win the big prize, *roll a 4, 5, or 6 on each die*.

You can earn *3 rolls* but the first is free.

Earning Roll #2

You'll earn a roll if you can tell me whether the actual price is higher or lower.

100-Ounce Slurpee

$\$e$

Higher or Lower?

(Slurpees are especially tasty in mid-July.)

Earning Roll #2

You'll earn a roll if you can tell me whether the actual price is higher or lower.

100-Ounce Slurpee

$1.79

Lower

(Disclaimer: we do not recommend drinking this much.)

Earning Roll #3

You'll earn a roll if you can tell me whether the actual price is higher or lower.

10-Dollar Bill

$$\$\,\pi^2$$

Higher or Lower?

(Hamilton ticket prices may be higher than $10.)

Earning Roll #3

You'll earn a roll if you can tell me whether the actual price is higher or lower.

10-Dollar Bill

$10

Higher

(Confused? 10 is more than pi squared.)

The Producers' Questions

If I keep offering this game repeatedly, *how much will we have to pay for it?*

How likely is a win *if we give the player*…

1 roll? 2 rolls? 3 rolls?

(and the most important question…)

The Producers' Questions

If I keep offering this game repeatedly, *how much will we have to pay for it?*

How likely is a win *if we give the player*…

1 roll? 2 rolls? 3 rolls?

Is this game fun to watch??

Analysis: 1 roll

There's not much to say here. Each die has a 1/2 chance of hitting. You must go 5 for 5.

The probability of winning in 1 roll is

$$(1/2)^5 = 1/32 \approx 3.1\%$$

(It's a terrible game when this happens.)

Analysis: 2 rolls

The first roll determines how likely it is to win on the second roll.

First Roll	P(win)
5 hits: 1/32	1
4 hits: 5/32	1/2
3 hits: 10/32	1/4
2 hits: 10/32	1/8
1 hit: 5/32	1/16
0 hits: 1/32	1/32

Where did those numbers for the first roll come from?

What do we do from here?

Analysis: 2 rolls

Use expected value or a weighted average to determine the probability.

First Roll	P(win)
5 hits: 1/32	1
4 hits: 5/32	1/2
3 hits: 10/32	1/4
2 hits: 10/32	1/8
1 hit: 5/32	1/16
0 hits: 1/32	1/32

It's…

1/32 • 1

+ 5/32 • 1/2

+ 10/32 • 1/4

+ …

Analysis: 2 rolls

More complicated, but piecing together all the ways you can win makes it work.

The probability of winning in 2 rolls is

$$243/1024 \approx 23.7\%$$

(Much more interesting to watch. 243 and 1024, hmm.)

Analysis: 3 rolls

The first roll determines your situation for the second and third rolls.

First Roll	P(win)
5 hits: 1/32	1
4 hits: 5/32	3/4
3 hits: 10/32	
2 hits: 10/32	
1 hit: 5/32	
0 hits: 1/32	243/1024

Why is the last probability 243/1024?

What are the other probabilities?

Analysis: 3 rolls

The first roll determines your situation for the second and third rolls.

First Roll	P(win)
5 hits: 1/32	1
4 hits: 5/32	3/4
3 hits: 10/32	9/16
2 hits: 10/32	27/64
1 hit: 5/32	81/256
0 hits: 1/32	243/1024

Some might be inclined to use Σ here.

Analysis: 3 rolls

Use expected value or a weighted average to determine the probability.

First Roll	P(win)
5 hits: 1/32	1
4 hits: 5/32	3/4
3 hits: 10/32	9/16
2 hits: 10/32	27/64
1 hit: 5/32	81/256
0 hits: 1/32	243/1024

It's…

1/32 • 1

+ 5/32 • 3/4

+ 10/32 • 9/16

+ …

Analysis: 3 rolls

It's more difficult than 2 rolls, but it works!

The probability of winning in 3 rolls is

$$16807/32768 \approx 51.3\%$$

(Cool.)

A Second Perspective

But there's another way. Look at the game from the perspective of one of the dice.

"Hey, I could help the player win on any of the three rolls. It's pretty likely! Also I can talk!"

How likely is it?

What about five dice?

A Second Perspective

The probability that the yellow die hits in 3 rolls is 7/8.

The probability of hitting all 5 dice in 3 rolls is

$(7/8)^5 = 16807/32768 \approx 51.3\%$

We could extend to any number of dice or rolls now!

(Wicked awesome.)

Historical Data

Let ‘Em Roll has been played 239 times since 2000, fully detailed on tpirstats.com.

<u>2000-2019</u>

3 rolls, play to end: 45.4% (54 / 119)

2 rolls, play to end: 38.0% (27 / 71)

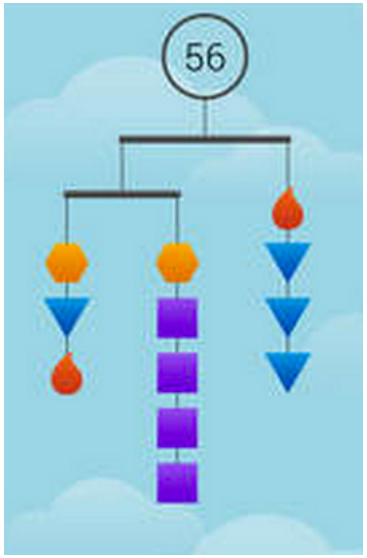
1 roll, uh oh: 0.0% (0 / 8)

Walked with money: 45

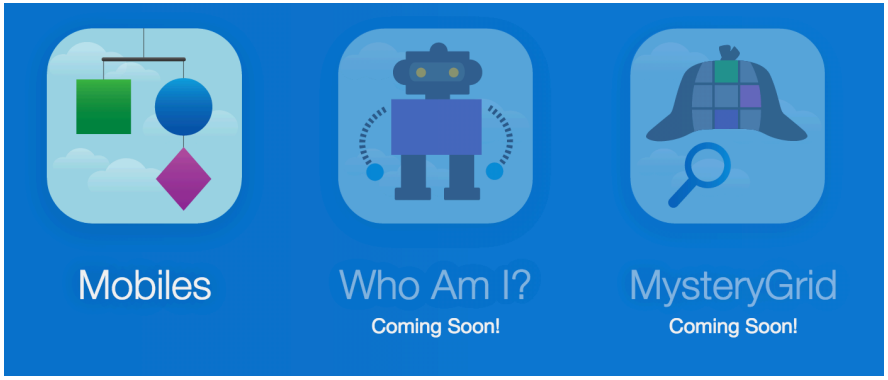
Won car on first roll: 7 / 239

Sponsored by… SolveMe!

Hundreds of puzzles to play … or make your own!



It's fun and teaches equation solving! Oh, and it's *FREE* for iPad.



solveme.edc.org

Classroom Interlude

Here are a few potential projects to try.

- Make a game with P(win) ≈ 1/3.
- What are good wagers in Final Jeopardy?
- What other Price Is Right games could be played better through strategy? (Slate)
- What's the probability of winning $1 million on Wheel of Fortune?

Classroom Interlude

In my teaching, I found some game shows worked better than others. Games are great test review! Good as openers / wrap-ups.

Good	*Bad*
Press Your Luck	Jeopardy! *(yes, bad)*
Card Sharks	Deal or No Deal
Millionaire	Wheel of Fortune
High Rollers	Are You The One?

What's In The Bag?!

This bag contains ten green chips and seven red chips and you will win or lose by chips.

Pull out a chip.

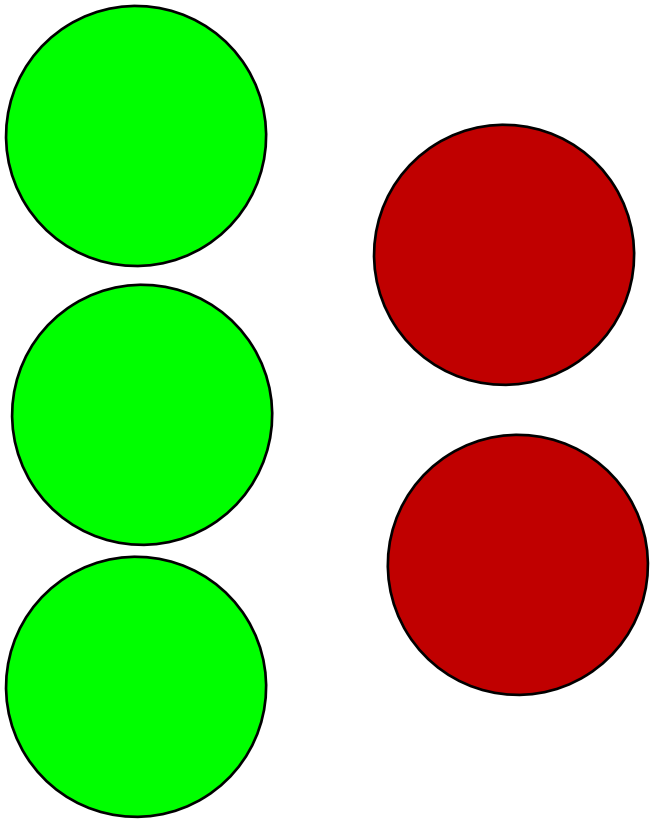
Track them…

Pull three green chips:

WIN.

Pull two red chips:

LOSE.



Analyzing The Bag

17 chips: 10 green, 7 red.

What is the probability of pulling out 3 green chips before pulling out 2 red chips?

There are lots of ways this might be done…

Analyzing The Bag

17 chips: 10 green, 7 red.

What is the probability of pulling out 3 green chips before pulling out 2 red chips?

One way to solve the problem is to list all the ways one could win and compute the probability of each…

GGG

GRGG

RGGG

GGRG

Analyzing The Bag

17 chips: 10 green, 7 red.

What is the probability of pulling out 3 green chips before pulling out 2 red chips?

You could solve the problem by listing all the ways to win and computing probabilities…

GGG = 10 • 9 • 8 / 17 • 16 • 15 = 3/17

RGGG = 7 • 10 • 9 • 8 / 17 • 16 • 15 • 14 =3/34

GRGG = ? GGRG = ? Total = ??

Analyzing The Bag

17 chips: 10 green, 7 red.

What is the probability of pulling out 3 green chips before pulling out 2 red chips?

You could write a computer program to simulate the game and run it 10,000 times.

Win: 4,384 (43.84%)

Lose: 5,616

Analyzing The Bag

17 chips: 10 green, 7 red.

What is the probability of pulling out 3 green chips before pulling out 2 red chips?

You can get sneaky.

What happens if you just reach in and pull four chips, right from the beginning?

Analyzing The Bag

17 chips: 10 green, 7 red.

What is the probability of pulling out 3 green chips before pulling out 2 red chips?

Pull 4 of 17 chips and see if you get 3 or more green, using combinatorics.

$$\frac{\binom{10}{4}}{\binom{17}{4}} + \frac{\binom{10}{3}\binom{7}{1}}{\binom{17}{4}} = \frac{15}{34}$$

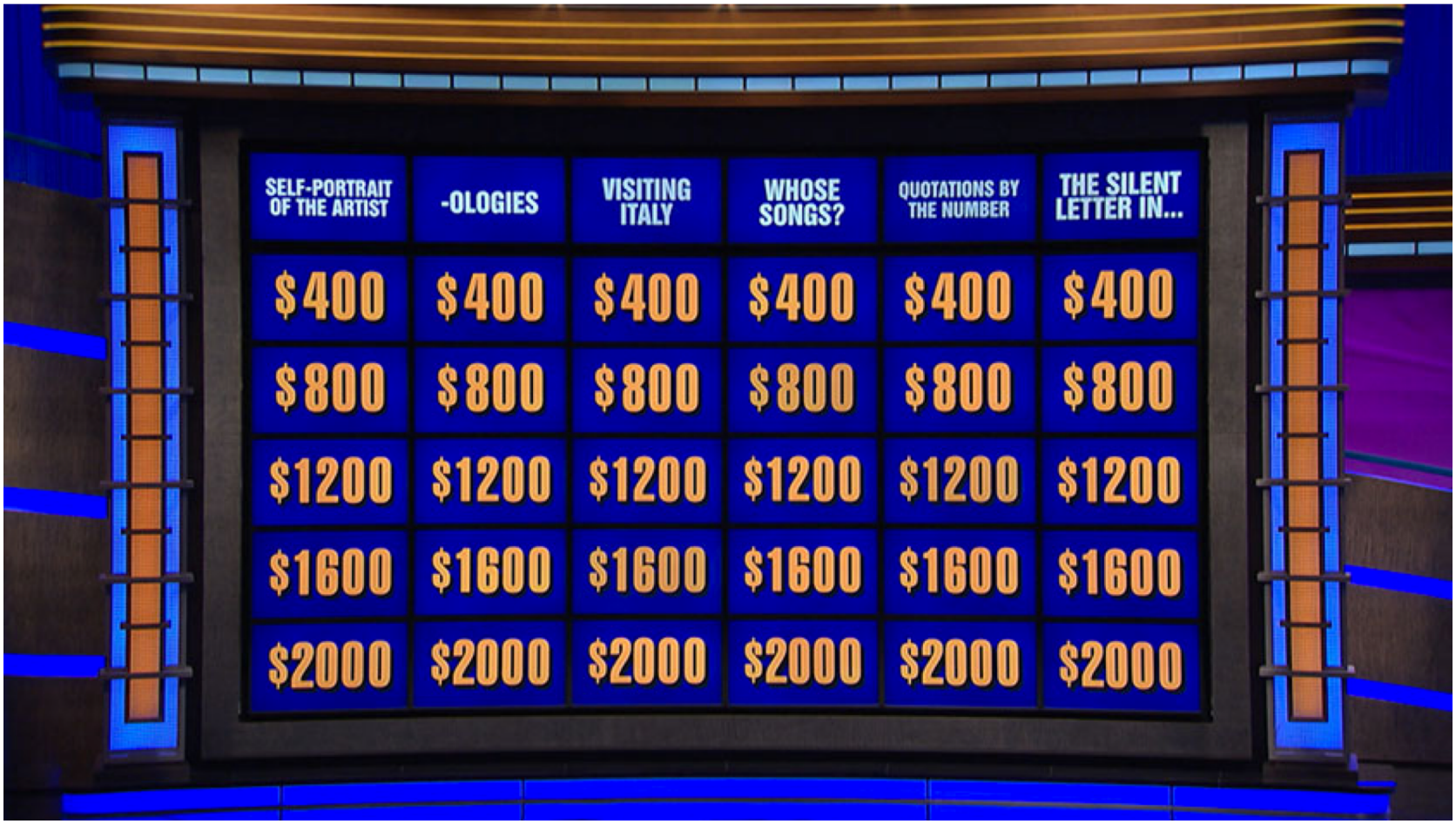
The Real Show

You've got balls: A green, B red.

What is the probability of pulling out 4 green balls before pulling out 3 red balls?

Daily Doubles

Where are the Daily Doubles?



Daily Doubles

This “heat map” is based on 13,663 actual Daily Double locations.

0%		0%	0%	0%	0%
2%	1%	2%	1%	2%	1%
6%	3%	5%	5%	5%	3%
8%	5%	7%	7%	7%	5%
6%	3%	5%	5%	4%	3%

More to Explore

Many related topics are asked about in *CME Project*, and in the Park City Math Institute materials at

projects.ias.edu/pcmi/hstp/sum2013/morning

- How can spinners or dice be represented by polynomials?
- What makes cards different from dice, and what impact might that have on gameplay?
- What's the best possible total in an episode of Jeopardy?

Thanks and good luck!

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mist.edc.org
patternsinpractice.wordpress.com