MESSAGE

12 Upside-Down Teaching

STARTING WITH A GOOD PROBLEM AND SHIFTING TO *YOU-WE-I*

> The hardest part of teaching by challenging is to keep your mouth shut, to hold back. Don't *say;* ask! … Keep asking "Is that right? Are you sure?" Don't say "no"; ask "why?"
>
> —Paul Halmos in *I Want to Be a Mathematician*

In 2013 the United States had almost 4 million job openings, and yet more than 7 percent of those looking for work were unemployed (U.S. Department of Labor 2013). The problem with this continuing disconnect is not geography—where the jobs are compared to where the workers live—but rather that workers simply do not have the skills required for today's available jobs at any level, whether blue-collar or white-collar or requiring a high school education, technical certificate, two- or four-year degree, or graduate study. All workers in jobs today need to be able to think, reason, and solve problems that haven't been solved before, often working in a team or with a small group of individuals contributing different areas of expertise.

Preparing for this kind of future demands a different kind of education, especially in mathematics, than most schools have offered in the past. Throughout most of the twentieth century, it was enough for the educational system to focus on helping students acquire knowledge. But as the century drew to a close, it was becoming obvious that knowledge alone was not enough to secure future employment. Nor was knowledge enough to help communities and the nation address their challenges and thrive. As we made the transition into the twenty-first century, report after report called for ramping up our academic expectations and incorporating significant attention to reasoning, thinking, creativity, and high-level problem solving (see, for example, Friedman 2007; National Center on Education and the Economy 2008).

Those calls continue today, both from within the ranks of mathematics educators and from outside. In particular, the Common Core Standards for Mathematical Practice (NGA Center and CCSSO 2010) put a strong emphasis on reasoning, thinking, problem solving, and communication, and the few states not adopting these standards reflect the same priorities in their own.

So how can we create the classrooms we've been calling for during more than two decades? Maybe we need to turn our traditional teaching model upside down if we're going to prepare students to thrive in their future, rather than our past.

The Traditional Right-Side Up Model

Many mathematics classrooms today reflect the teaching model I experienced years ago as a student, a model that would become my basis for teaching in the early years of my career. That model—what I call the "right-side up" model—involved preparing a lesson thoroughly so that I could clearly explain to my students the specific procedure or concept to be covered next. I was encouraged to fill my explanation with enthusiasm and energy so that my students would stay with me and absorb what I was telling them. Then, after we practiced the procedure together, I would give students exercises to first practice the procedure and then eventually to apply the procedure to solve a few word problems. One way to characterize this teaching approach is *I-We-You*. In other words, *I* (the teacher) will present the mathematical concepts and rules for the lesson; then *We* (students and teacher) will do some guided practice, where we walk through some examples of those concepts and rules, perhaps including word problems involving these same concepts and rules; finally *You* (students) will practice on your own and later do homework on what you have learned.

This method is too often accompanied by several hurdles. First, some students don't learn well from a teacher-delivered explanation; many become bored and, thus, disengage from what's going on. Some students also see an error-free teacher explanation as further proof that they (the students) simply don't have the "math gene." They believe that mathematics is something only some people can do, as demonstrated by their teacher's explanation and by the few students who seem to be able to master the particular concept or procedure being demonstrated. Most of all, when we primarily present students with problems for which they come to expect that they will apply the procedure they have just learned, we withhold perhaps the most important experience students need. We deny them the opportunity to dig into a problem, get a sense of what mathematics might be involved, constructively grapple with the underlying mathematical ideas, try out possible solution approaches, and learn from mistakes they make in the process of coming to actual solutions. That opportunity represents the heart of *upside-down teaching*.

The Upside-Down Model

Teaching upside down involves choosing to first present to students a problem they are expected to mess around with for a while, without having first taught them the particular rules or procedures they could use to solve the problem. Engaging students in this way helps them interact with the mathematics and sets them up to learn the mathematical content the teacher intends.

Rather than the *I-We-You* structure used in many mathematics classrooms today, this model could be characterized as *You-We-I*: *You* (students) will mess around with a task for a while, ideally engaging in some thinking, trying things out, and generally wrestling with or constructively struggling with mathematics arising from the problem; then *We* (students and teacher) will discuss the different approaches students tried, with students explaining, questioning, clarifying, and further grappling with the mathematics; finally, *I* (the teacher) will connect this work and the class's productive discourse around the problem and related mathematical ideas, facilitating the whole process and ensuring that students come away with the intended mathematics learning. Sometimes, students' learning may emerge naturally from their engagement with the task. Other times, it may involve the teacher directly telling students a key point or working through an explicit example. Even when such direct instruction may be called for, students' engagement with the task and participation in the resulting discourse sets them up to also take in what the teacher presents.

The way that I learned to teach—clear explanations, shared practice, application of what was just learned—represented a very teacher-centered approach. The upside-down model I'm advocating here is more difficult to implement well, calling for considerable time and teacher skill in orchestrating and managing the classroom—a teacher-*structured* approach focused on student engagement, rather than a teacher-*centered* approach with students playing a more passive role. Teaching in this way allows students the opportunity to push their thinking as they constructively struggle with problems that may go beyond more predictable one- or two-step word problems typically found at the end of a lesson or chapter in a textbook. And by drawing students into thinking about the problem, students are more likely to attend to the intended mathematics than they would be if listening more passively to a teacher explanation.

Choosing Problems to Turn Upside Down

Several years ago, in *Professional Standards for Teaching Mathematics* (1991), the National Council of Teachers of Mathematics suggested organizing mathematics teaching around three key elements: worthwhile tasks, productive discourse, and a safe and supportive learning environment. These basic elements offer timeless recommendations for helping students learn to think and reason on their own and make sense of the mathematics they are learning. The process is centered on *worthwhile tasks*, described in Standard 1 of that document. This standard offers such a clear and beautiful description of the importance of and nature of the tasks we select, I'm inserting the direct text of the standard here.

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' intellect;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions; and
- promote the development of all students' dispositions to do mathematics. (NCTM 1991, 25)

Finding such tasks is not always easy. However, the increasing availability of online resources, especially those addressing common standards, makes it more likely than in the past that a teacher will be able to organize a lesson around a rich, deep, challenging, and engaging task. Another place to look for good, worthwhile tasks may be the supplementary materials that come with many textbooks; often, good problems are included as project suggestions or extensions to textbook lessons. And, of course, some curriculum materials themselves are organized around rich tasks. (See Appendices A, B, and C for resources for selecting and evaluating tasks for upside-down teaching.)

Not all tasks offer the same level of opportunity for student engagement and thinking; individually evaluating tasks can be a time-consuming job. But finding and considering such tasks for classroom use can provide an excellent opportunity for collaboration and discussion within a professional learning community, grade-level team, department, or any group of colleagues. And sometimes, a potentially good task can be made even better with the addition of a question or a slight modification, something that might arise in such a collaborative discussion.

Considering Contexts

Problems need not always to be in real-world contexts in order to be effective in upside-down teaching. Some straightforward problems posed in a purely mathematical context can offer nice opportunities for discussing, struggling, thinking, and learning. In *Fostering Geometric Thinking* (Driscoll, DiMatteo, Nikula, and Egan 2007) the authors present the following geometric problem:

> *Two vertices of a triangle are located at (0,6) and (0,12). The triangle has area 12.* (2007, 47)

The authors then describe the kinds of questions that can engage students in deep thinking and discussion:

> *What are all the possible positions for the third vertex?*
> *How do you know you have them all?*
> *How many of the triangles you form are isosceles?* (2007, 47)

It can also be useful to organize a lesson around a task presented in a context outside of mathematics. Choosing contexts should be done carefully so as not to distract students from the mathematics, but rather draw them into it. When we look for problems in contexts outside of mathematics, it simply is not possible to find tasks in which a context will resonate with all students. Students come from different backgrounds with different experiences and interests, and every student will find different tasks engaging or interesting. It's unrealistic and frustrating to eliminate any context that might be unfamiliar to one or more students in a class. Rather, the teacher can help optimize the use of a context by discussing that context with the class in setting up the problem at the beginning of the lesson. For example, a lesson about numerical reasoning based on tire sizes might start out with a discussion of the numbers on tires, perhaps even bringing a tire into class for students to see. Even nondriving students or students who have never looked at the tires on a car can see the numbers printed on the tire and deal with a real-world context from which to explore the mathematics. In the process of doing so, they not only deal with the mathematical content, they also expand their familiarity with that context just a little.

Overall, perhaps the best outcome in terms of teachers' choices of relevant, interesting tasks is that students will engage with enough problems in a wide enough variety of contexts that they come to see mathematics as something actually used in the world outside of school. And, if we're lucky, we can hope that every student's particular interests are piqued by enough problems over time that they come to develop a personal identity with mathematics as relevant to that student's life. Samuel Otten, in a rich discussion of cautions related to real-world contexts, suggests that the most important thing for students to notice about problems posed in contexts is that the mathematical processes they use—the thinking and reasoning skills they develop—are what carry over and apply to a multiplicity of situations (2011, 20–25).

What Can We Do?

Shifting to a problem-centered You-We-I teaching approach, described here as upside-down teaching, involves both instructional time and planning time. Some students, as well as their parents, may complain that "You're not teaching us!," meaning that you aren't telling them every step they should take in solving a problem. Students, parents, teachers, and administrators need to understand the benefits of organizing instruction around good problems that students don't know in advance how to solve. Taking the extra time called for with this teaching approach is an investment in student learning with tremendous potential for positive returns. If we are successful, students not only learn the content they need, they also develop mathematical habits of mind like perseverance, thinking, reasoning, discussing, justifying their point of view, considering variations of a problem, and developing a positive disposition toward mathematics. These habits of mind pay off over and over again—students not only build on their understanding with new content connected to what they have learned, they will also have learned exactly the kinds of skills employers are looking for in filling millions of open positions in the twenty-first-century marketplace.

When I taught mathematics in Burkina Faso I used an upside-down, problem-centered approach. About two-thirds of the way through my first year there, one young man came to me after class. He said to me, "Madame, I know you like these problems of yours. But, you know, we have a program to cover." Although he would never have spoken to a Burkinabé teacher this way, he had seen a few American television shows, and so he believed that Americans were more open to such conversation. He continued, "Perhaps you could do your problems on Fridays and the rest of the time we could cover the program." Looking ahead to the major test students took at the end of high school, he was concerned that the class would not cover all of the material. I thanked him for his suggestion, and continued teaching around problems. Needless to say, he was not happy to find out that I would also be his teacher for the following year. Nevertheless, he came to my house with a group of students on the day that I was leaving to return to the United States at the end of my two years of service. He took me aside, and with a sheepish grin on his face, he said to me, “Madame, I think I learned more mathematics with your problems than I would have learned otherwise.” He went on to complete a university degree and became a teacher.

Upside-down teaching seems to have worked in turning this one student’s thinking upside down, and I’m sure I was a better teacher by using that approach. Maybe it’s time for upside-down teaching to become the new right-side up model for mathematics classrooms.

Reflections and Discussion

FOR TEACHERS

- What issues or challenges does this message raise for you? In what ways do you agree with or disagree with the main points of the message?
- In what ways does your current teaching approach compare to upside-down teaching?
- Do you believe there are certain groups of students for whom upside-down teaching might not be effective or certain topics or courses for which you don’t think this kind of teaching would be possible? Why or why not?
- If you don’t already teach primarily using a problem-centered approach, what challenges do you see in trying to move closer to upside-down teaching? How might you (and your colleagues) address those challenges?

FOR FAMILIES

- What questions or issues does this message raise for you to discuss with your daughter or son, the teacher, or school leaders?
- How open are you to your son or daughter not being shown all the steps necessary to solve a problem before he or she is asked to deal with the problem? What might be the benefits of such an approach? What might be the drawbacks?
- How can you best support your daughter or son if she or he complains that the teacher isn’t “teaching,” but rather is expecting students to figure things out?

FOR LEADERS AND POLICY MAKERS

- How does this message reinforce or challenge policies and decisions you have made or are considering?
- How can you best support teachers in developing student thinking using an upside-down model, even if students and parents complain that the teacher isn't "teaching"—meaning that the teacher isn't telling students everything they need to know before giving them a good problem?

RELATED MESSAGES

Smarter Than We Think

Many of the messages in this book advocate an upside-down teaching model; below are a few examples.

- Message 32, "Problems Worth Solving," considers the nature of problems and what is called for from students to solve them.
- Message 31, "Developing Mathematical Habits of Mind," addresses the mathematical habits of mind that characterize real understanding and proficiency.
- Message 16, "Let It Go," offers thoughts on focusing the curriculum through instructional decisions.
- Message 4, "They're Just Not Motivated!," considers motivating students with engaging problems and opportunities for discourse.

Faster Isn't Smarter

- Message 17, "Constructive Struggling," emphasizes the importance of students being challenged to solve mathematically worthwhile problems.
- Message 1, "Math for a Flattening World," makes a case for the kind of thinking and reasoning workers of the future will need.
- Message 33, "Engaged in What?," considers the importance of students engaging in meaningful mathematics while participating in engaging activities.

MORE TO CONSIDER

- *What's Your Math Problem? Getting to the Heart of Teaching Problem Solving* (Gojak 2011) considers the importance of giving students rich, nonroutine problems without having first taught students exactly how to solve them and offers classroom strategies for helping students learn mathematics meaningfully through their work with such problems.

- *Strength in Numbers: Collaborative Learning in Secondary Mathematics* (Horn 2012) presents recommendations for organizing classrooms around student engagement and discourse, as described in this message.
- *Mathematics for Equity* (Nasir et al. 2014) uses experiences in one high school as a basis for offering strategies for implementing a teaching model like that advocated in this message.
- "Delving Deeper: In-Depth Mathematical Analysis of Ordinary High School Problems" (Stanley and Walukiewicz 2004) suggests how to consider high school problems from a deep mathematical perspective.
- "Takeaways from Math Methods: How Will You Teach Effectively?" (Bay-Williams 2014) offers three big ideas for teaching toward student thinking.
- *Teaching with Your Mouth Shut* (Finkel 2000) advocates a variety of ways to teach without telling (not specific to mathematics).
- "Student-Centered Learning Approaches Are Effective in Closing the Opportunity Gap" is a series of four case studies supporting student-centered learning. https://edpolicy.stanford.edu/news/articles/1137.
- "Connecting Research to Teaching: Shifting Mathematical Authority from Teacher to Community" (Webel 2010) advocates rich, engaging teaching practice based on research about what works with students.
- "The Role of Contexts in the Mathematics Classroom: Do They Make Mathematics More 'Real'?" (Boaler 1993) discusses the use and limitations of real-world contexts in problem solving.
- "Cornered by the Real World: A Defense of Mathematics" (Otten 2011) offers a thought-provoking perspective using real-world contexts in problem solving.
- *Motivation Matters and Interest Counts* (Middleton and Jansen 2011) discusses building on students' interests, including a discussion on the use of real-world contexts in selecting tasks.
- *The World Is Flat 3.0: A Brief History of the Twenty-First Century* (Friedman 2007) emphasizes the importance of educating twenty-first–century workers for creativity, innovation, and the ability to work together to solve problems.
- *That Used to Be Us: How America Fell Behind in the World It Invented and How We Can Come Back* (Friedman 2011) makes a renewed call for investing in education that prepares workers of the future to think, analyze, and create.

- *Tough Choices or Tough Times: The Report of the New Commission on the Skills of the American Workforce* (National Center on Education and the Economy 2008) lays out the needs for citizens to be educated in more powerful higher-level skills, including creativity, communication, and problem-solving.
- See also Appendices A, B, and C on selecting and evaluating in-depth tasks and Appendix D for several resources on teaching around problem solving listed as part of the Essential Library.

Related Research Briefs from the National Council of Teachers of Mathematics

- "Why Is Teaching with Problem Solving Important to Student Learning?" (Lester and Cai 2010) summarizes research findings about the role of problem solving in the mathematics classroom.
- "What Does Research Say the Benefits of Discussion in Mathematics Class Are?" (Cirillo 2013b) describes how research findings support the importance of offering students opportunities to discuss their work on mathematical tasks.
- "What Are Some Strategies for Facilitating Productive Classroom Discussions?" (Cirillo 2013a) offers research-based techniques in support of student discourse around mathematical tasks.

Resources Related to Specific Problem-Based Curricula

- "A Designer Speaks: Challenges in U.S. Mathematics Education Through a Curriculum Developer Lens" (Lappan and Phillips 2009) offers insights into effective mathematics teaching through the eyes of the developers of the Connected Mathematics Project.
- "The Consequences of a Problem-Based Mathematics Curriculum" (Clarke, Breed, and Fraser 2004) describes results of research on the effectiveness of IMP.
- "Teaching Sensible Mathematics in Sense-Making Ways with the CPMP" (Hirsch, Coxford, Fey, and Schoen 1995) describes results of research on the effectiveness of the Core-Plus Mathematics Project.
- *Advanced Mathematical Decision Making* (Student and Teacher Materials) (Charles A. Dana Center at the University of Texas at Austin 2010b) includes video of the lesson referenced in this message. This resource provides materials and resources for teaching this innovative twelfth-grade capstone mathematics course.

WWW This message is also available in printable format at mathsolutions.com/smarterthanwethink.