Anyone who studies mathematics should not be afraid of the difficulty of multiplication and division, but should be afraid of the mysteries of manipulating fractions.”

Chinese mathematician Zhang Qiujian (circa 450 AD)
What is a fraction?

**a number**
“a fraction is any rational number that is not an integer” (*Collins English Dictionary*)

**an expression**
“a fraction is an expression that indicates the quotient of two quantities” (*The American Heritage Dictionary*)

**a number that is an expression**
“a fraction is a number (such as 1/2 or 3/4) which indicates that one number is being divided by another” (*Merriam-Webster Dictionary*)
The word *fraction* comes from the Latin “*fractio*” which means to break, to fracture.

Horus, an ancient Egyptian deity

The sacred unit fractions attributed to the six parts of the eye of the Horus.
3/4 - three-fourths

“out of four parts, take three”
An Egyptian ceremonial cubit rod in the Louvre Museum.
Pictorial representation of fractions in Japanese textbooks

Textbooks include a diagram such as this to show how non-unit fractions are composed of unit fractions.

Pictorial representation of $\frac{4}{5}$ liter

### How to Express the Size of Fractions

1. How many meters are two, three, and four \( \frac{1}{5} \) m?

2. What is the length of six \( \frac{1}{5} \) m, seven \( \frac{1}{5} \) m, and so on?

Let's express fractions on a number line.

Let's think about how to express fractions greater than 1.

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Figure 2. Typical representations used in the Japanese textbooks

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Positioning fractions on the number line in Japanese textbooks

Connecting fraction models to the number line

➢ A fraction is always a fraction of something.

➢ If this something is not indicated then a fraction is of the number 1.

➢ To find out the value of a fraction \( \frac{a}{b} \) we divide the something the fraction is of into \( b \) equal parts and take \( a \) number of resulting parts or as Chinese would say “out of \( b \) parts take \( a \)”.
Simon Stevin who introduced decimal fractions to the Europeans in 1585

\[
15.739 \quad 15739 \overline{3}
\]

\[
0.056 \times 1.2
\]

\[
56\overline{3} \times 12\overline{1} = 56 \times 12 (\overline{3} + \overline{1}) = 672\overline{4}
\]

\[
0.056 \div 1.2
\]

\[
56\overline{3} \div 12\overline{1} = 56 \div 12 (\overline{3} - \overline{1}) \approx 47\overline{2}
\]

\[
\frac{2}{5} = 0.4 \quad \text{or} \quad \frac{2}{3} = 0.\overline{6}
\]
Operations on Fractions
Operations on Fractions the Chinese Way

\[
\frac{2}{3} + \frac{1}{5} = \frac{(2 \times 5)}{(3 \times 5)} + \frac{(1 \times 3)}{(5 \times 3)} = \frac{10 + 3}{15} = \frac{13}{15}
\]
\[
\frac{2}{3} \times \frac{2}{5} = \frac{(2 \times 2)}{(3 \times 5)} = \frac{4}{15}
\]

two third of two fifths
\[ \frac{2}{3} \div \frac{1}{5} = \left( \text{how many one fifths are in two thirds?} \right) \]

\[ \frac{2}{3} \div \frac{1}{5} = \frac{(2 \times 5)}{(3 \times 5)} \div \frac{(1 \times 3)}{(5 \times 3)} = \]

\[ = \frac{10}{15} \div \frac{3}{15} = 10 \div 3 = \frac{10}{3} \]

10 \div 3 = 3 \frac{1}{3} = \frac{10}{3}
Find the GCF of 51 and 136

136 - 51 = 85
85 - 51 = 34
51 - 34 = 17

➢ Subtract the smaller number from the greater number.

➢ Keep subtracting the smallest number from the middle number of the previous equation until you reach the equation with two equal numbers.

➢ This number is the GCF of two original numbers.
A fraction is always of something. If this something is not indicated then a fraction is of the number 1. \(a/b\) in the language of mathematics is a shortened form of the expression \(\div b \times a\). The fraction \(a/b\) being of something is found by dividing/partitioning this something into \(b\) parts and collecting together \(a\) number of these parts.

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<tr>
<th>Operations on Fractions</th>
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<td><strong>Multiplication</strong></td>
<td>Multiply the denominators to find a common denominator then multiply the numerators to find the numerator:</td>
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<td>((a/b) \times (c/d)=(ac)/(bd))</td>
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<tr>
<td><strong>Division</strong></td>
<td>Bring fractions to a common denominator by multiplying the denominators then divide the resulting numerators:</td>
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<td>(a/b \div c/d=ad/bd \div cb/db=(ad)/(cb))</td>
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<tr>
<td><strong>Addition/Subtraction</strong></td>
<td>Bring fractions to a common denominator by multiplying the denominators then add/subtract fractions by adding/subtracting their numerators:</td>
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<td></td>
<td>(a/b +or- c/d=ad/bd +or- cb/db=(ab +or- cb)/(bd))</td>
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