Mysterious Fractions

Anyone who studies mathematics should not be afraid of the difficulty of multiplication and division, but should be afraid of the mysteries of manipulating fractions."

Chinese mathematician *Zhang Qiujian* (circa 450 AD)

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What is a fraction?

a number

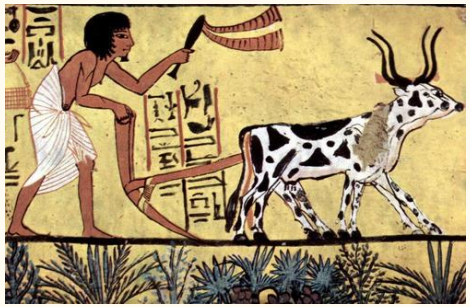
"a fraction is any rational number that is not an integer" (*Collins English Dictionary)*

an expression

"a fraction is an expression that indicates the quotient of two quantities" (*The American Heritage Dictionary*)

a number that is an expression

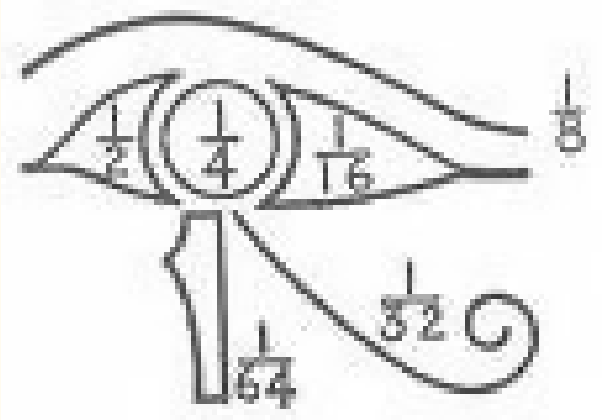
"a fraction is a number (such as 1/2 or 3/4) which indicates that one number is being divided by another" (*Merriam-Webster Dictionary*)



$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{5}$ $\frac{1}{23}$ $\frac{1}{12}$ $\frac{1}{6}$ $\frac{1}{8}$

Mark, J. J. (2016, March 28). Egyptian Afterlife - The Field of Reeds. *Ancient History Encyclopedia*. Retrieved from https://www.ancient.eu/article/877/

Horus, an ancient Egyptian deity



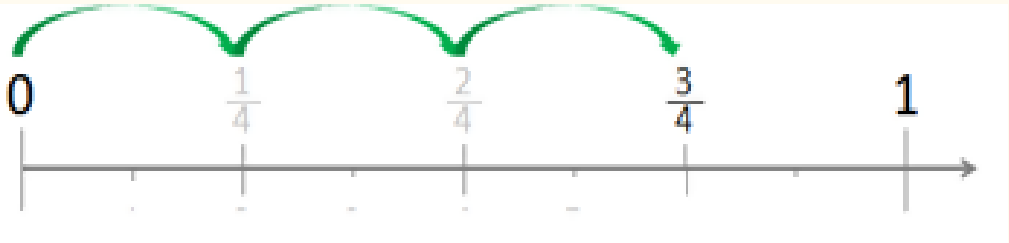
The sacred unit fractions attributed to the six parts of the eye of the Horus

The word *fraction* comes from the Latin "*fractio*" which means to break, to fracture

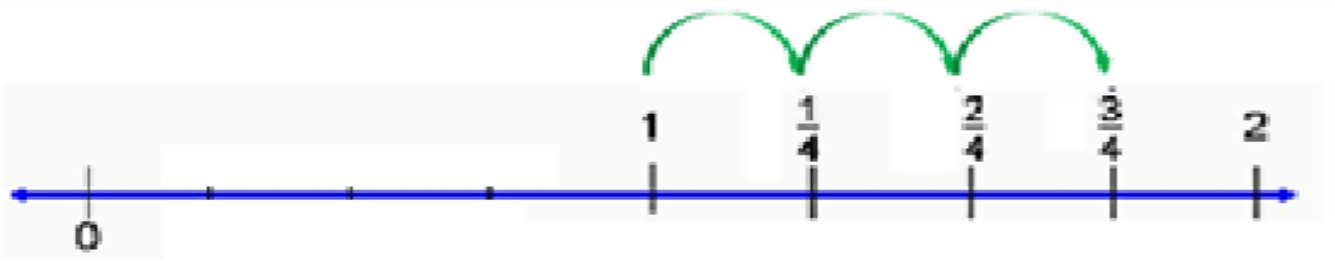
3/4 - three-fourths 四分之三

“out of four parts, take three”

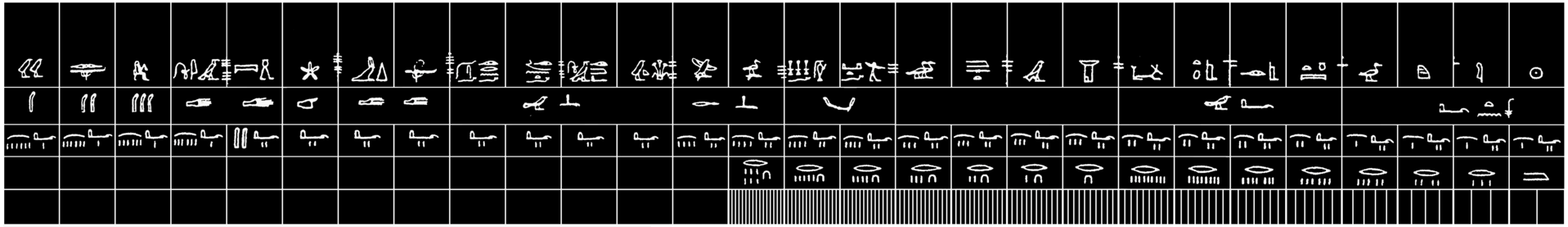
3/4



1 3/4

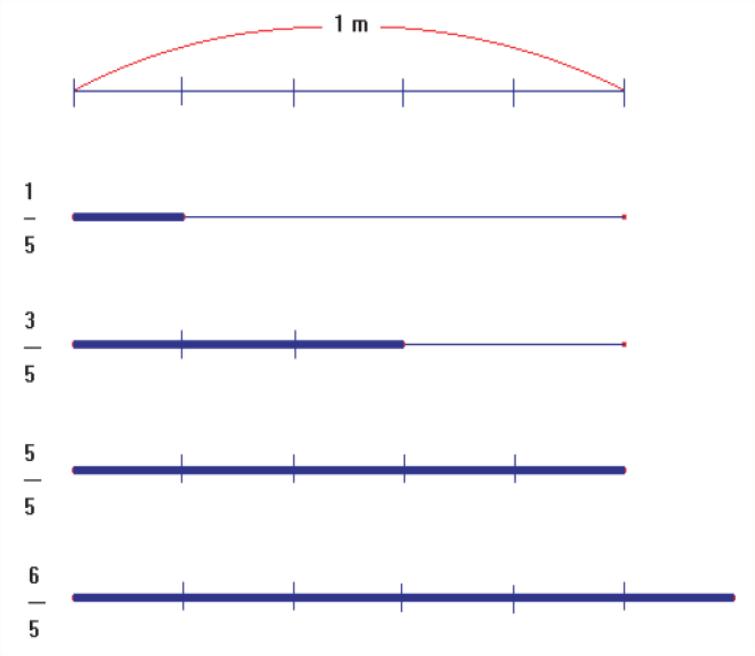


An Egyptian ceremonial cubit rod in the Louvre Museum.

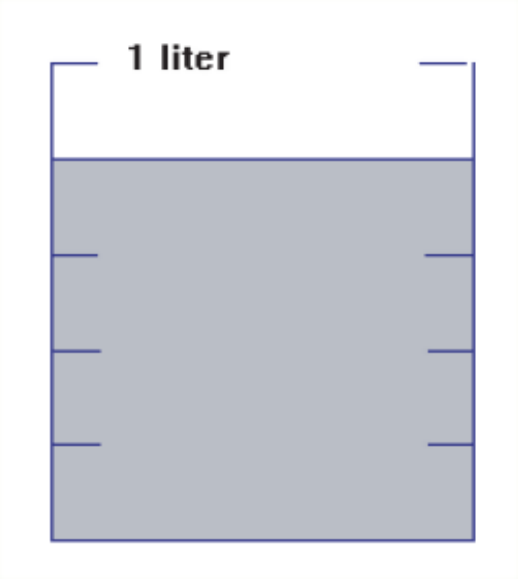


Pictorial representation of fractions in Japanese textbooks

Textbooks include a diagram such as this to show how non-unit fractions are composed of unit fractions.



Pictorial representation of 4/5 liter



Watanabe, T. (2006). The Teaching and Learning of Fractions: A Japanese Perspective. *Teaching Children Mathematics*,*12*(7), 368-374

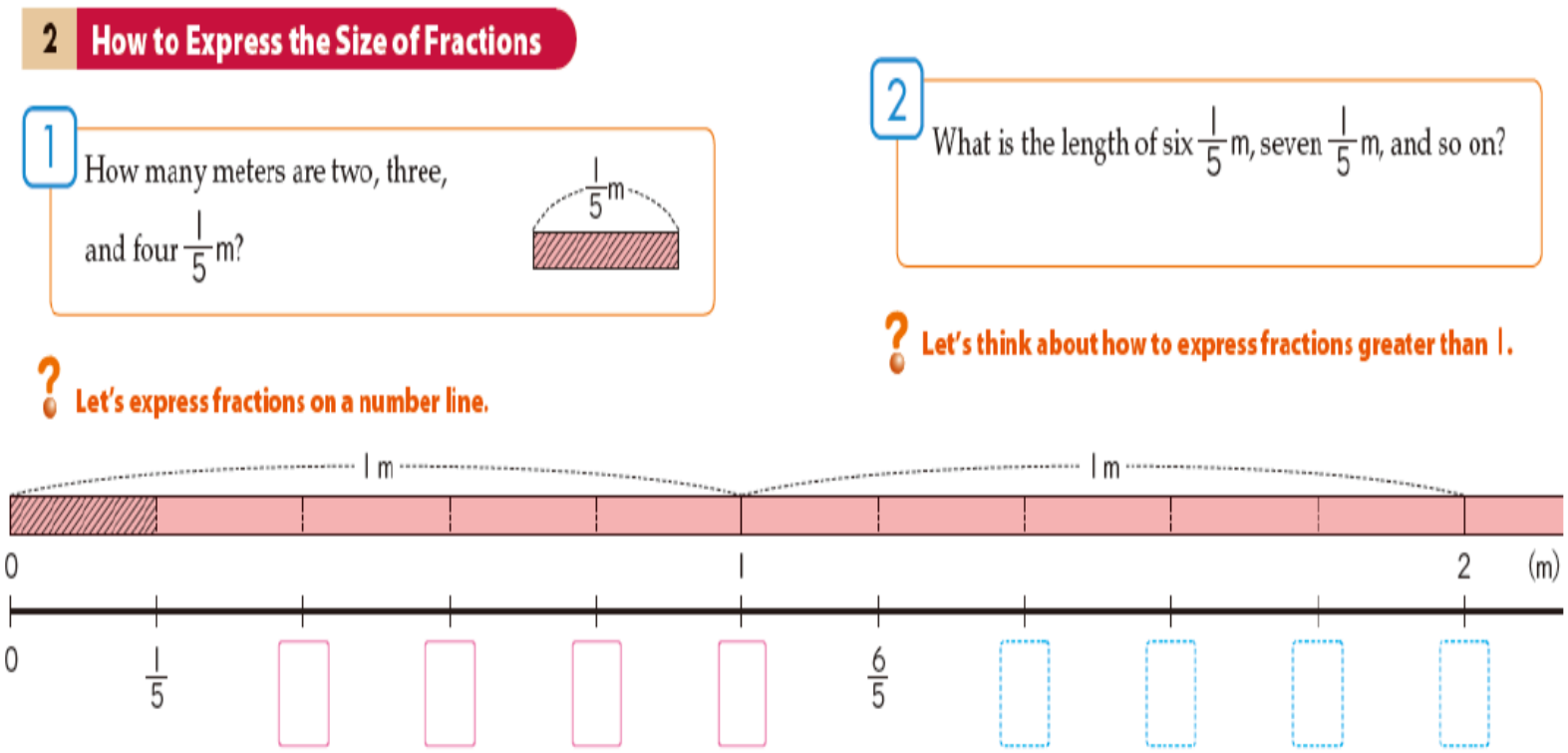
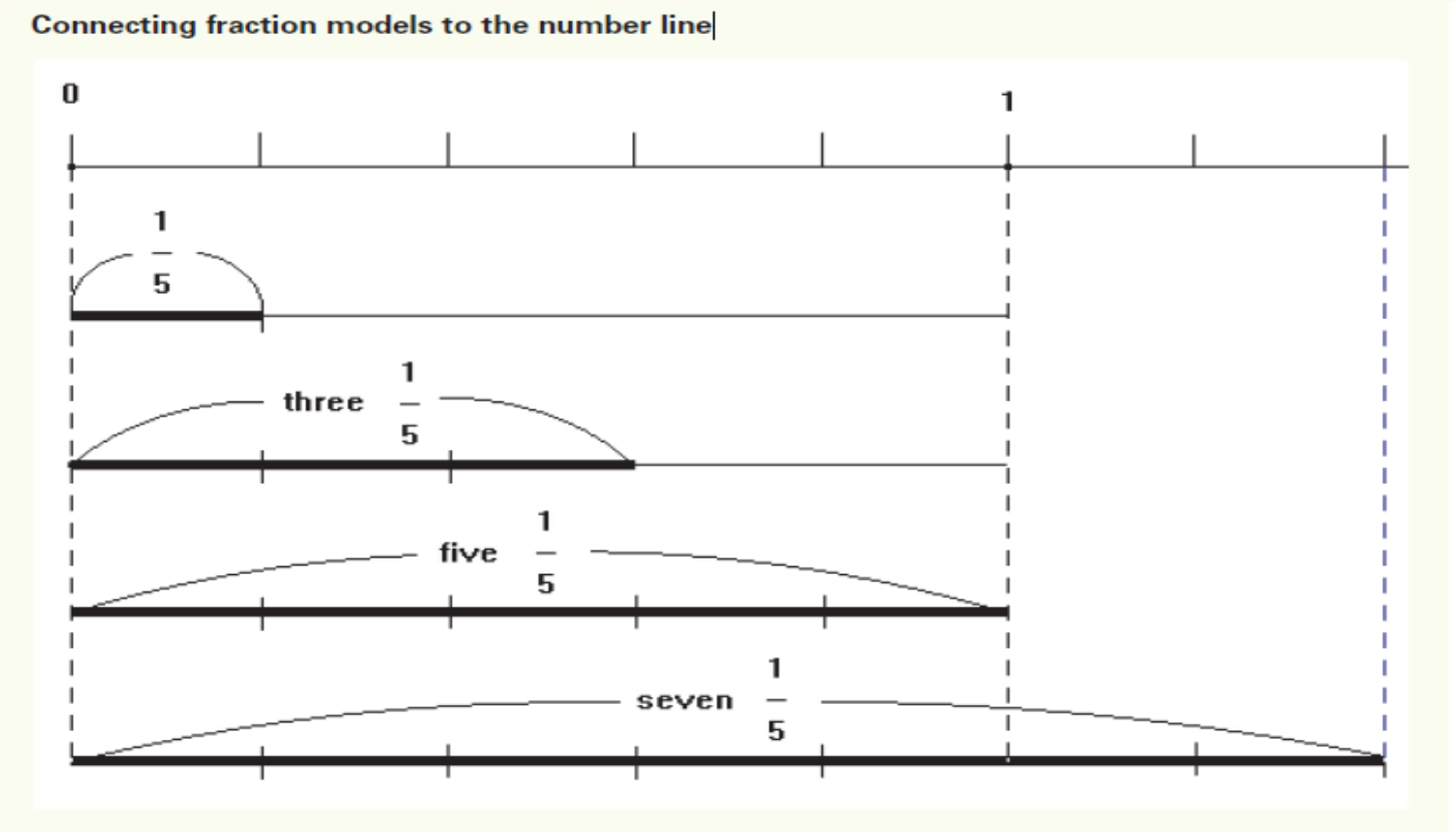


Figure 2. Typical representations used in the Japanese textbooks

Son, J., Lo, J., Watanabe, T. (2015). Curricular Treatment of Fractions in Japan, Korea, Taiwan, and the United States. *North American Chapter of the International Group for the Psychology of Mathematics Education*, Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (37th, East Lansing, MI, Nov 5-8, 2015)

Positioning fractions on the number line in Japanese textbooks



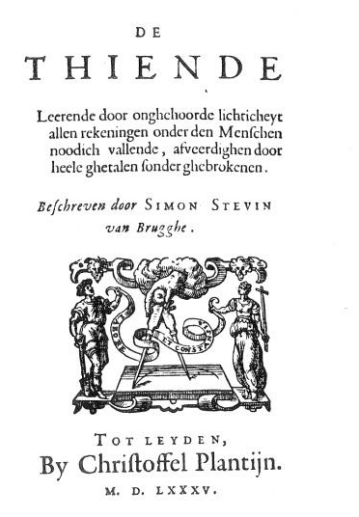
Watanabe, T. (2006). The Teaching and Learning of Fractions: A Japanese Perspective. *Teaching Children Mathematics*,*12*(7), 368-374

- A fraction is always a fraction of something.

- If this something is not indicated then a fraction is of the number 1.

- To find out the value of a fraction *a/b* we divide the something the fraction is of into *b* equal parts and take *a* number of resulting parts or as Chinese would say "out of *b* parts take *a*".

Simon Stevin who introduced decimal fractions to the Europeans in 1585



DE

THIENDE

Leerende door onghehoorde lichtheyt allen rekeningen onder den Menschen noodich vallende, afveerdighen door heele ghetalen sonder ghebrokenen.

Beschreven door SIMON STEVIN *van Brugghe.*

TOT LEYDEN,
By Christoffel Plantijn.
M. D. LXXXV.

15.739 15739③

0.056 x 1.2

56③ x 12① = 56 x 12 (③+①)= 672④

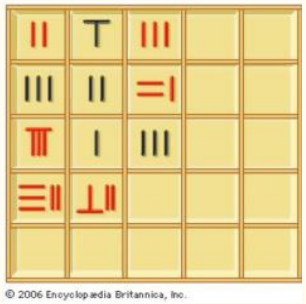
0.056 ÷ 1.2

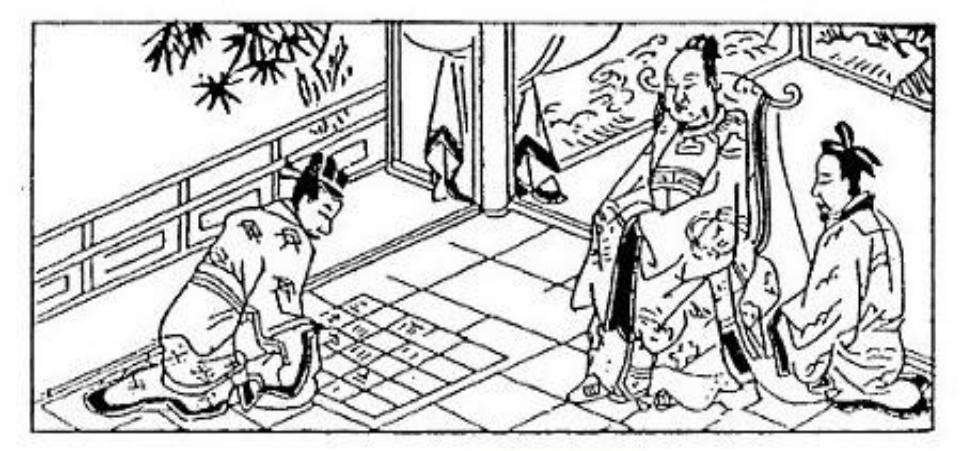
56③ ÷ 12① = 56 ÷ 12 (③-①) ≈ 47②

$$\frac{2}{5} = 0.4 \text{ or } \frac{2}{3} = 0.\overline{6}$$

Operations on Fractions

Operations on Fractions the Chinese Way

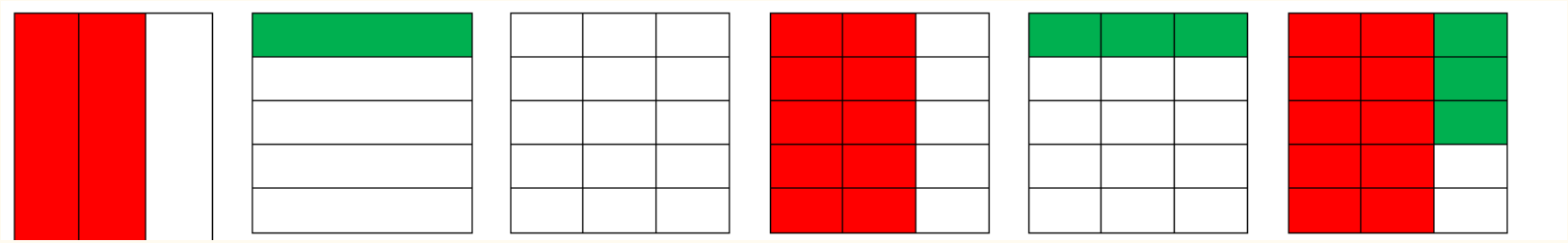




$$2/3+1/5 = (2\times5)/(3\times5)+(1\times3)/(5\times3) =$$

$$= (10+3)/15 = 13/15$$

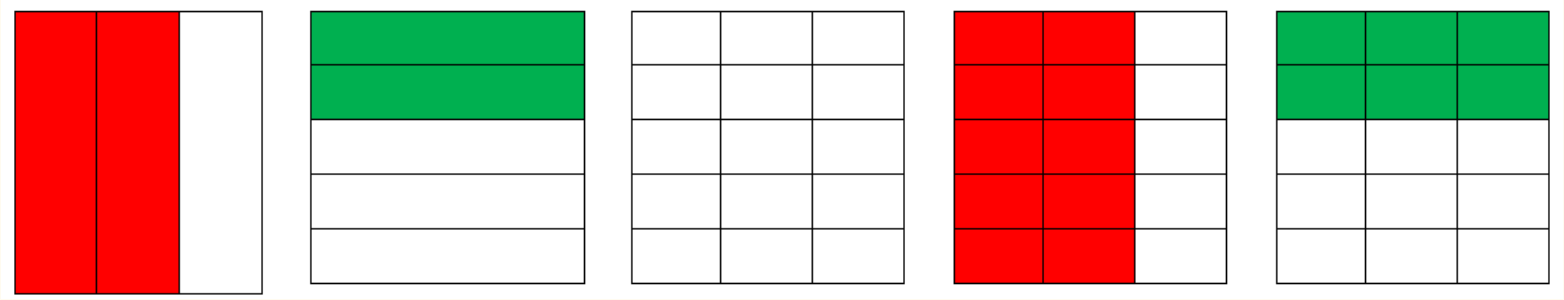
Addition

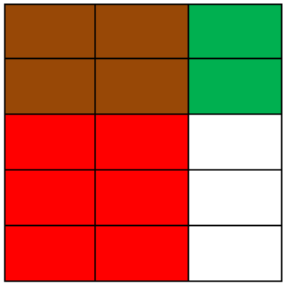


Multiplication

$$2/3 \times 2/5 = (2\times2)/(3\times5) = 4/15$$

two third of two fifths



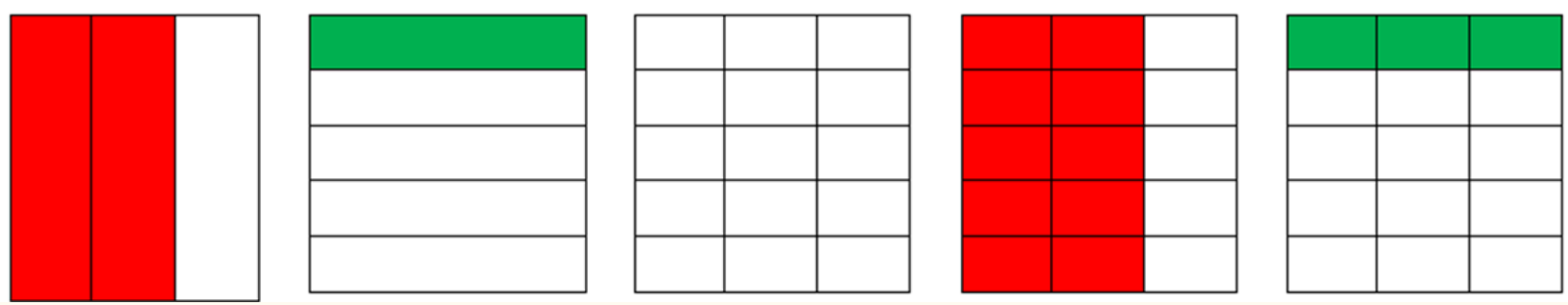


$2/3 \div 1/5 =$ (*how many one fifths are in two thirds?*)

$2/3 \div 1/5 = (2\times5)/(3\times5) \div (1\times3)/(5\times3) =$

$= 10/15 \div 3/15 = 10\div3 =$ **10/3**

Division



$10\div3 = 3\ 1/3 =$ **10/3**

Finding GCF

Find the GCF of 51 and 136

136 - 51 = 85

85 - 51 = 34

51 - 34 = 17

34 – 17 = 17

- Subtract the smaller number from the greater number
- Keep subtracting the smallest number from the middle number of the previous equation until you reach the equation with two equal numbers.
- This number is the GCF of two original numbers.

A ***fraction*** is always of something. If this something is not indicated then a fraction is of the number 1. ***a/b* in the language of mathematics is a shortened form of the expression ÷*b*×*a*.** The fraction ***a/b*** being of something is found by dividing/partitioning this something into *b* parts and collecting together *a* number of these parts.

Operations on Fractions	
Multiplication	Multiply the denominators to find a common denominator then multiply the numerators to find the numerator: $(a/b) \times (c/d)=(ac)/(bd)$
Division	Bring fractions to a common denominator by multiplying the denominators then divide the resulting numerators: $a/b \div c/d=ad/bd \div cb/db=(ad)/(cb)$
Addition/ Subtraction	Bring fractions to a common denominator by multiplying the denominators then add/subtract fractions by adding/subtracting their numerators: $a/b \text{ +or- } c/d=ad/bd \text{ +or- } cb/db=(ab \text{ +or- } cb)/(bd)$