It All Starts with Noticing and Wondering

Annie Fetter

21st Century Partnership for STEM Education
For NCTM's Noticing and Wondering Webinar Series, June 2021

anniefetter@gmail.com
Twitter: @MFAnnie, #NoticeWonder
annie.mathematicalthinking.org

(You'll get a PDF of the slides later)

Agenda

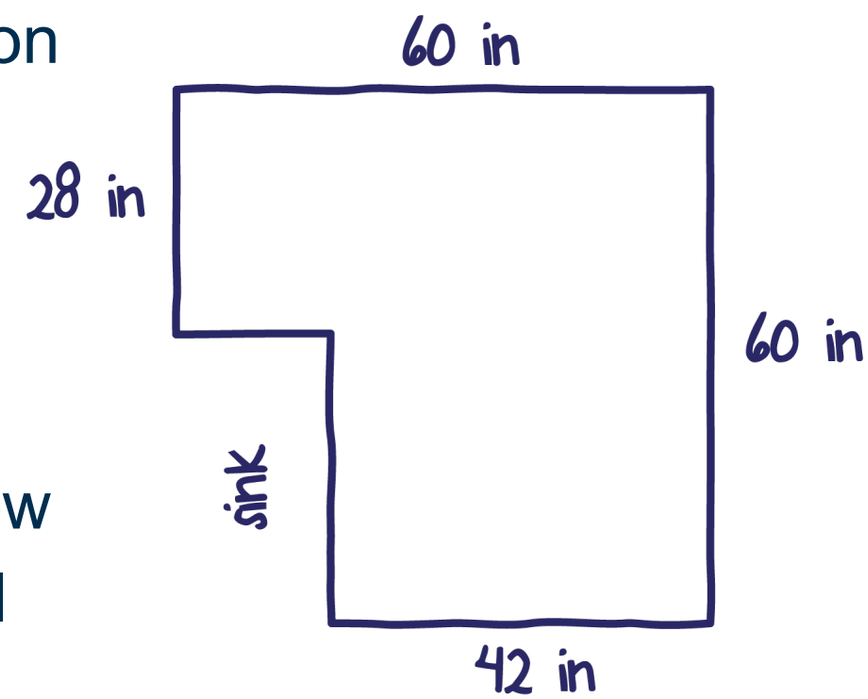
Access, Agency, Engagement, Leveraging, Looking, Spreading, Reflecting

Agenda Item 1

Access

Teresa's Tiles

Teresa is going to put down new ceramic tiles on her bathroom floor. She has selected square tiles that are 4 inches on each side. These are the kind of tiles that can be placed right next to each other without leaving additional space for grout. At The Home Station, she learned how to cut the tiles in case she needs any fractional pieces to cover her floor completely.

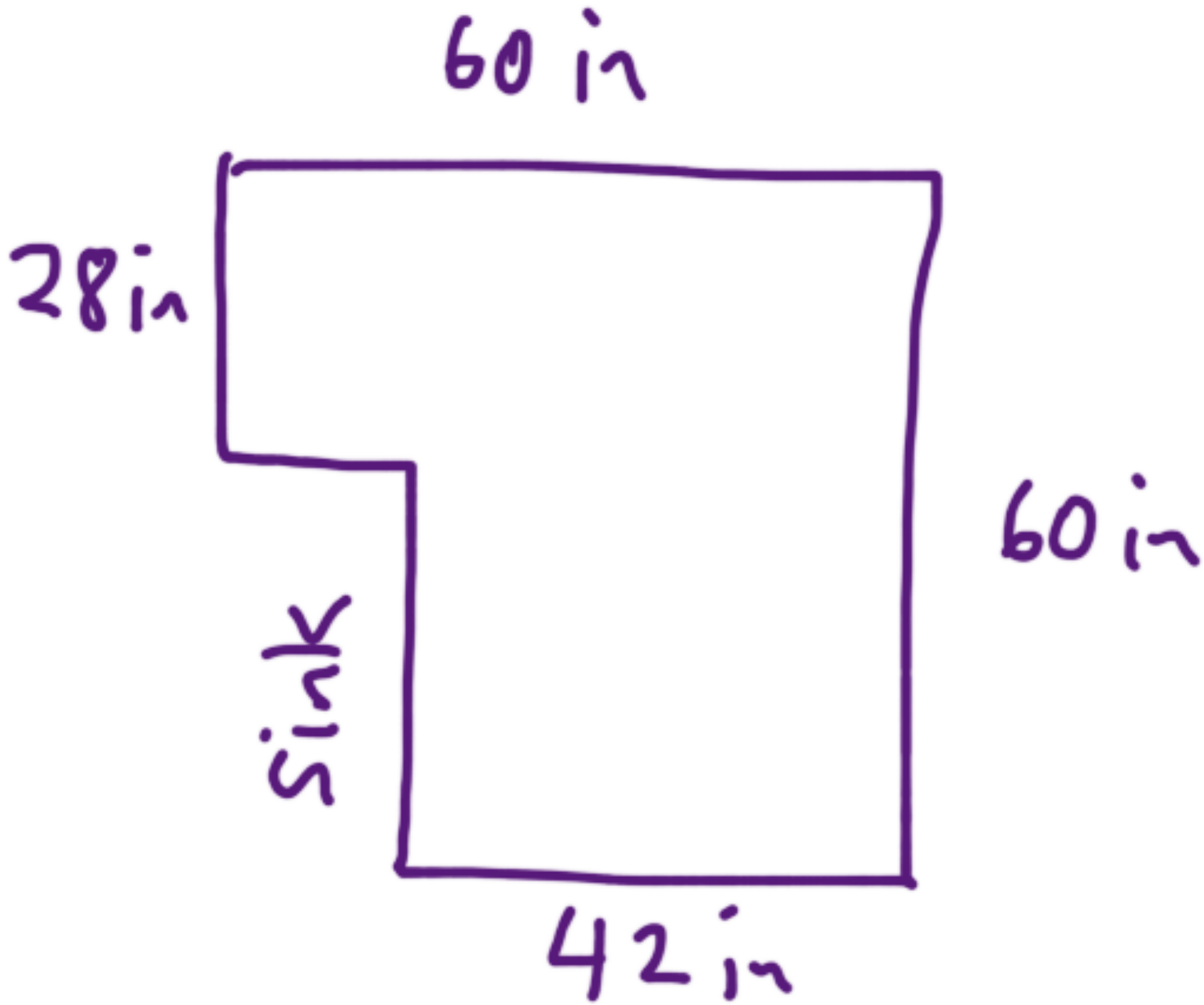


This diagram of the bathroom floor shows the dimensions of the floor space she needs to cover. The sink area does not get tiled.

Questions: How many tiles will she need to buy to cover her floor? How many tiles will she have to cut in order to cover the entire space?

Extra: What is the size, using whole numbers, of the largest square tile that could be used to tile the entire floor with no cut pieces?

Teresa's Tiles Scenario



Teresa's Tiles Student Ideas

Things that some "low-performing" 8th graders noticed about the picture:

- two sides are equal
- two sides are 60 inches
- one side is 28 inches
- they are longest
- one side is 42 inches
- it used to be a square
- your lines aren't very straight
- the short side of the sink is 18"
- the sink is a rectangle
- the long side of the sink is 32"
- can find the area of the whole thing by making it two pieces

Sample Grade 3 "Benchmark" Question

The corner deli sells roses in bunches of 6. If Dylan buys 3 bunches of roses, how many roses does he have?

A.6	18%
B.9	46%
C.18	31%
D.24	4%

Combined scores of the 160 third graders in a group of four "low-performing" schools I used to support.

Sample Grade 3 "Benchmark" Question REVISED

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches.

Sample Grade 3 "Benchmark" Question REVISED

The corner deli sells roses in bunches of 6. Dylan bought 3 bunches. Draw a picture of the story.

CCSS Mathematical Practice 1

Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution.

They analyze givens, constraints, relationships, and goals.

They make conjectures about the form and meaning of the solution and plan a solution pathway **rather than simply jumping into a solution attempt.**

CCSS Mathematical Practice 1

Beth Brandenburg
@Brandeli1974

Replying to @MFAnnie @MNmMath and 2 others

there is no better strategy than #NoticeWonder to tackle MP1! We use #noticewonder from PK-5!

10:46 AM · Apr 1, 2017 · Twitter for iPhone

Access

Melynee Naegele
@MNmMath

Replying to @MFAnnie @bkdidact and 2 others

#NoticeWonder is for everyone! Given real think time ALL can & do think critically It is life changing for everyone involved. POWERFULSTUFF!

10:22am · 1 Apr 2017 · Twitter for Android
Verdigris, OK, United States

Andrew Gael
@bkdidact

Replying to @MFAnnie @MNmMath and 2 others

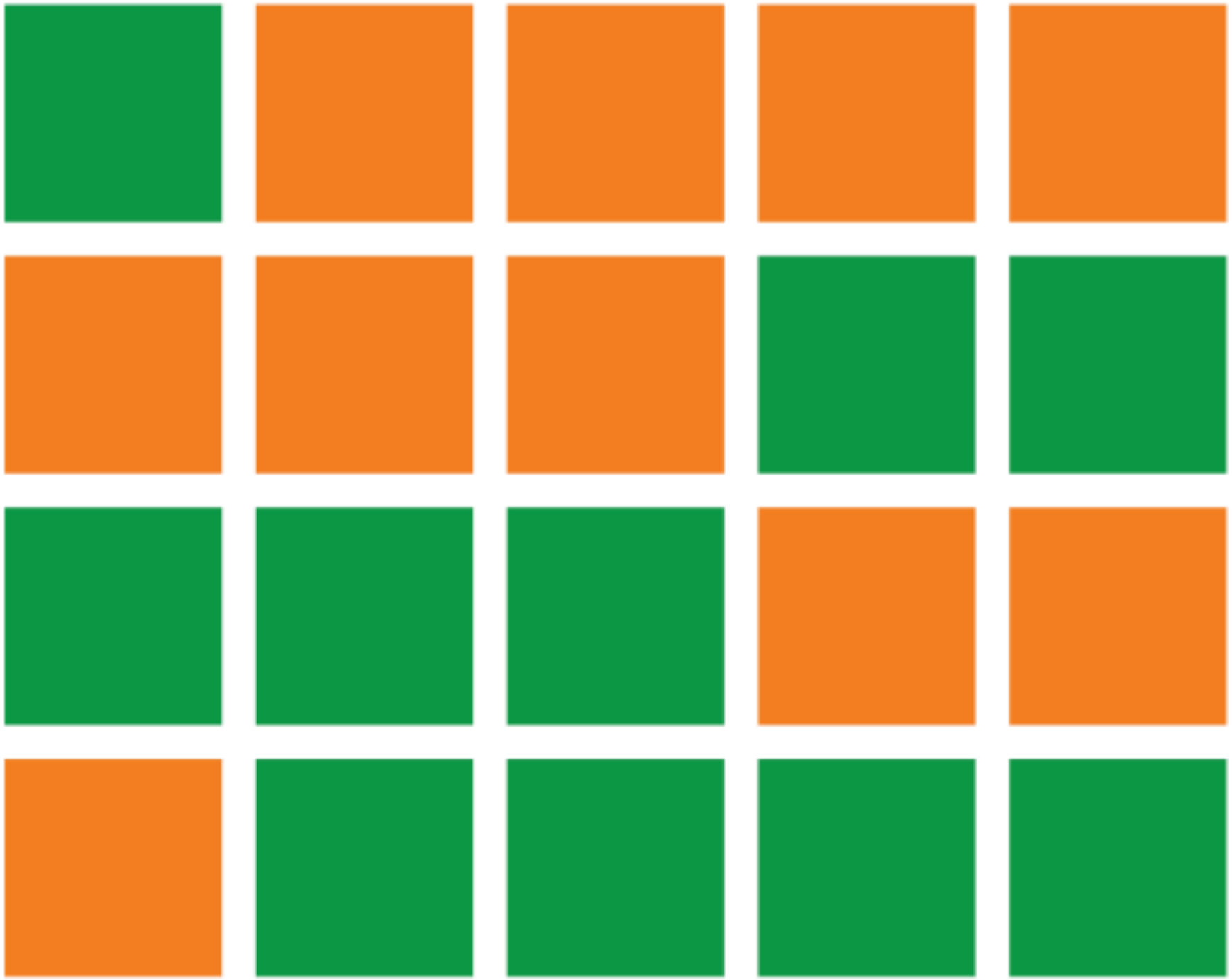
#noticewonder creates access for all Ss by focusing on sense-making and not answer-getting. Levels the playing field. Creates ownership!

11:05am · 1 Apr 2017 · Twitter for iPhone

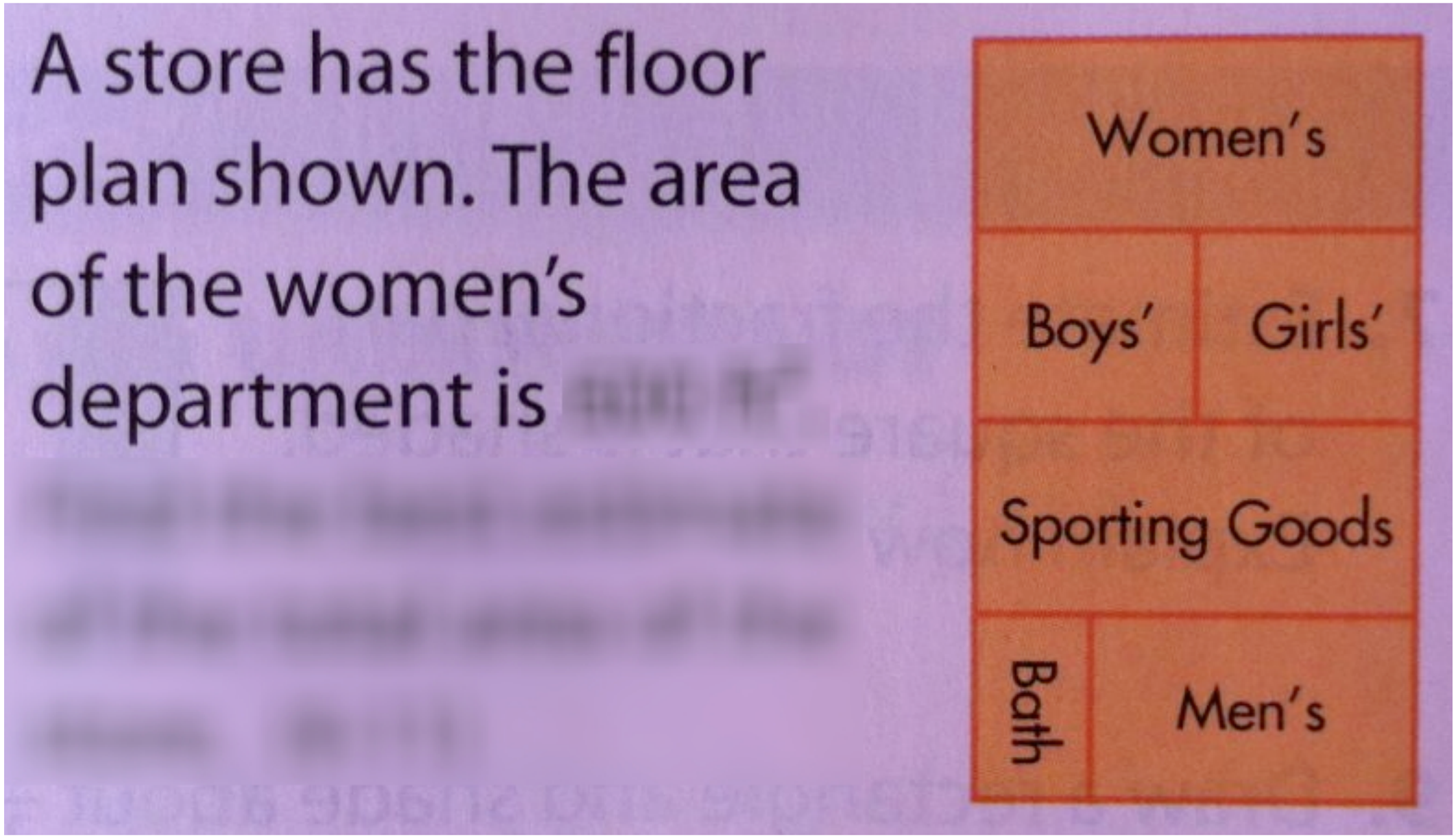
Shift Away from Answer-Getting

I Notice	I Wonder

Get Rid of the Question

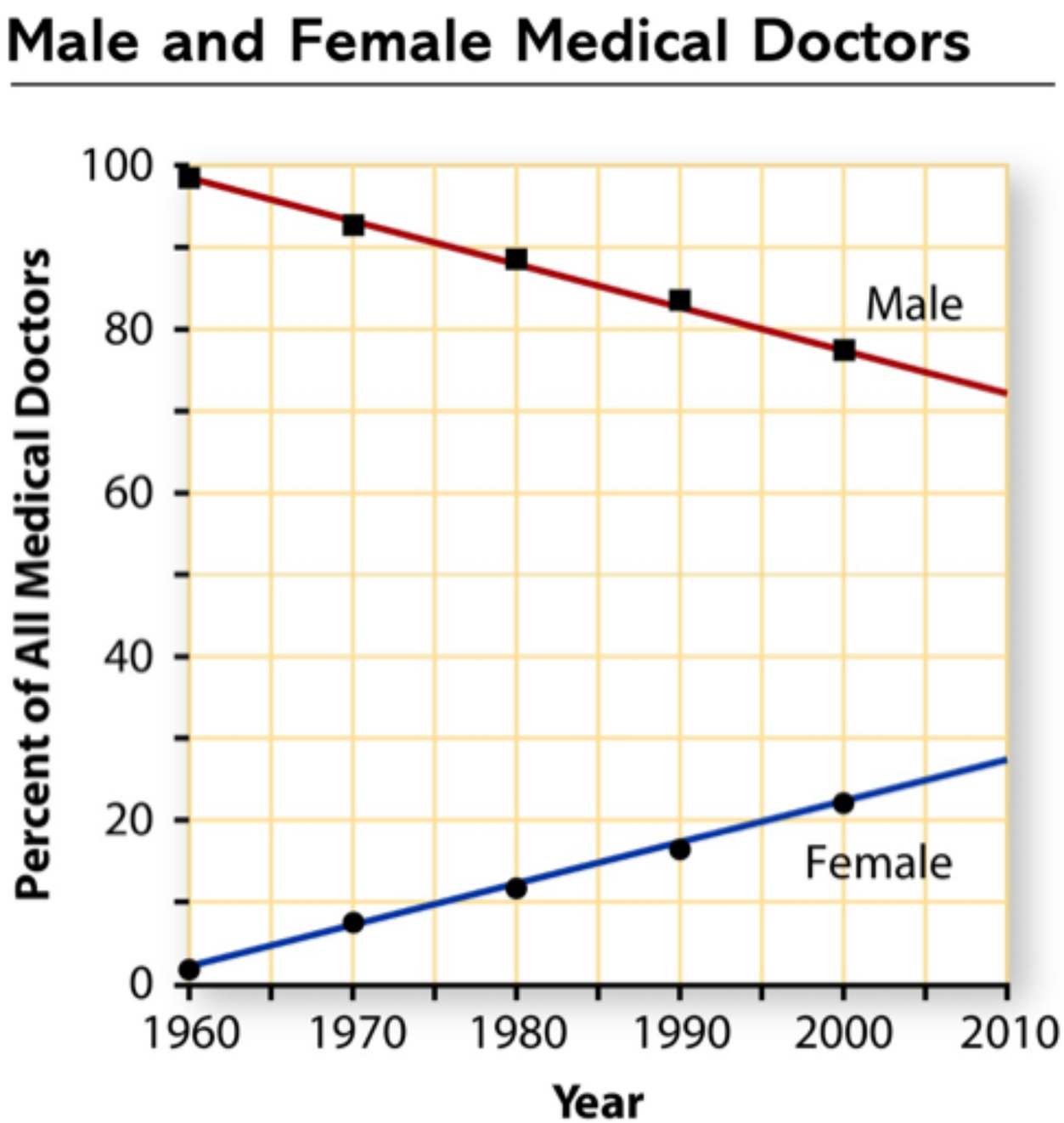


Get Rid of the Question and the Numbers



Get Rid of the Question and the Numbers

Launch Units with Less Information



Notice & Wonder About Naked Problems

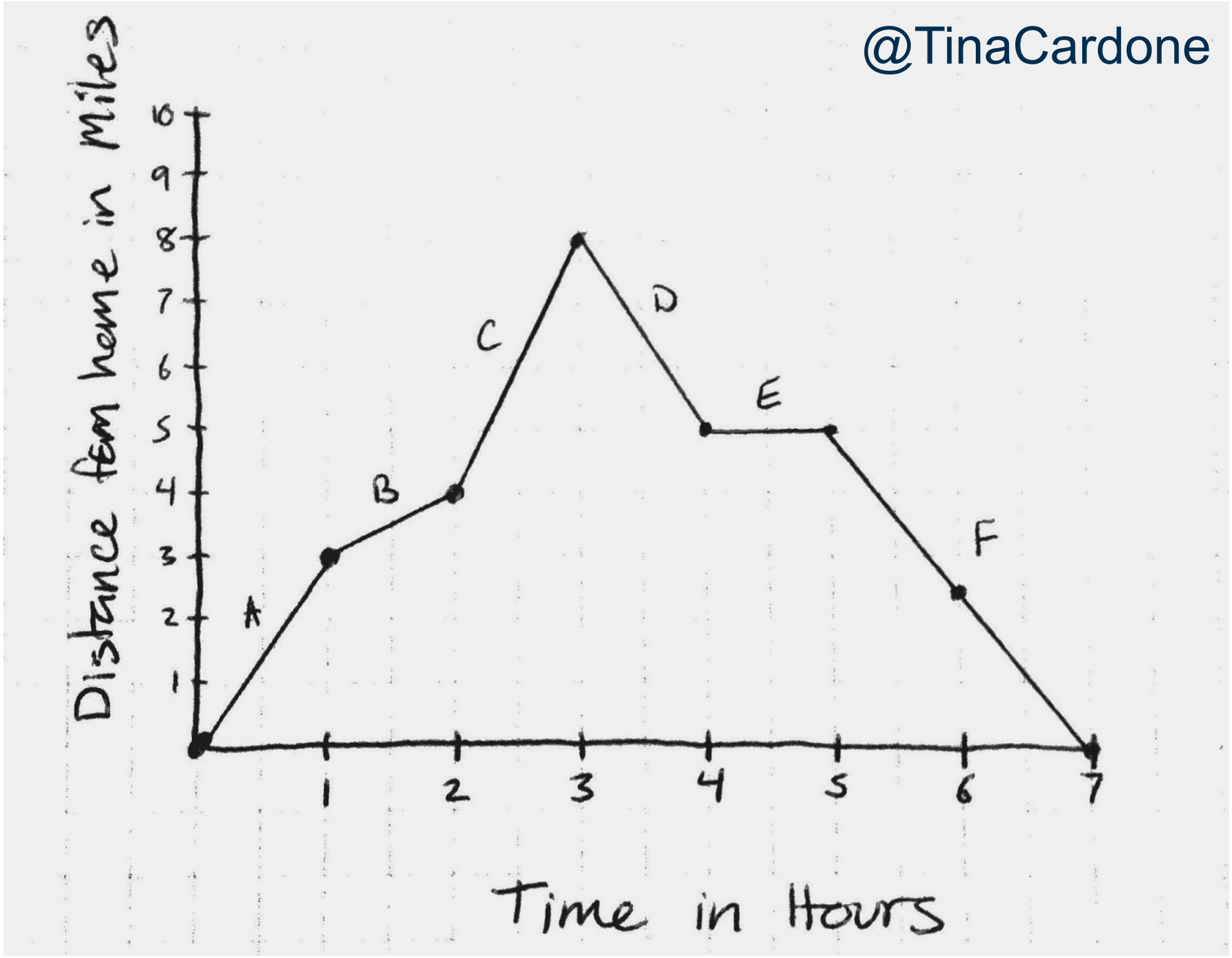
Write down everything you NOTICE and WONDER about these two pairs of equations.

23)

a) $$\left.\begin{aligned} x + 3y &= 20 \\ x^2 + y^2 + 5y &= 70 \end{aligned}\right\}$$

b) $$\left.\begin{aligned} x - 4y &= -1 \\ 3y^2 - 2x &= -3 \end{aligned}\right\}$$

Doesn't It Take a Lot of Time?



Doesn't It Take a Lot of Time?

Tina Cardone 🏳️‍🌈
@TinaCardone

Replying to @MFAnnie

@MFAnnie when I gave the graph and did notice/wonder first I didn't have to answer nearly so many questions when they did the handout

5:36 PM · Nov 24, 2014 · Tweetbot for iOS

Replying to @MFAnnie

@MFAnnie worth the few minutes it took and meant we skipped wrap up discussion (they already had it) drawingonmath.blogspot.com/2014/11/distan...

5:37 PM · Nov 24, 2014 · Tweetbot for iOS

http://drawingonmath.blogspot.com/2014/11/distance-graph.html

An Important Habit of Mind

Abigail Bates
@abbybates24

Replying to @MFAnnie and @NCTM

I feel like N&W really helped increase my students' confidence in their own math abilities and helped them know how to approach a problem, moving away from being "helpless handraisers."

5:38 PM · May 29, 2021 · Twitter for Android

An Important Habit of Mind

"I did what [my teacher] always tells me to do when I'm stuck. I noticed and wondered until I saw what to do."

—*a 4th grade student*

"Noticing and wondering is what you do until you know what to do."

—*Steve Weimar (@sweimar)*

"If nobody has any ideas to share, we take a Notice Wonder break."

—*Melynee Naegele (@MNmMath)*

An Important Habit of Mind

The One With The 8th Graders

"We're not done noticing and wondering yet!"

Things Fifth Graders Say

"…it helps me see new things I wouldn't have seen."

"…there are multiple answers so you can't really be wrong with it."

"…helps me look at a problem in a way I never thought of."

"…you get to think about the problem more and you realize more."

"…we don't have to do math at all, we just need to think on it without stress."

Continuing the Agenda

Agency

They're Humans

Everyone Has Ideas

"You get to share your own thinking and no one can ruin it."

—Aya, Grade 2

"We're getting really good at Noticing and Wondering. I mean, look at all that math!"

—Grade 9 Student

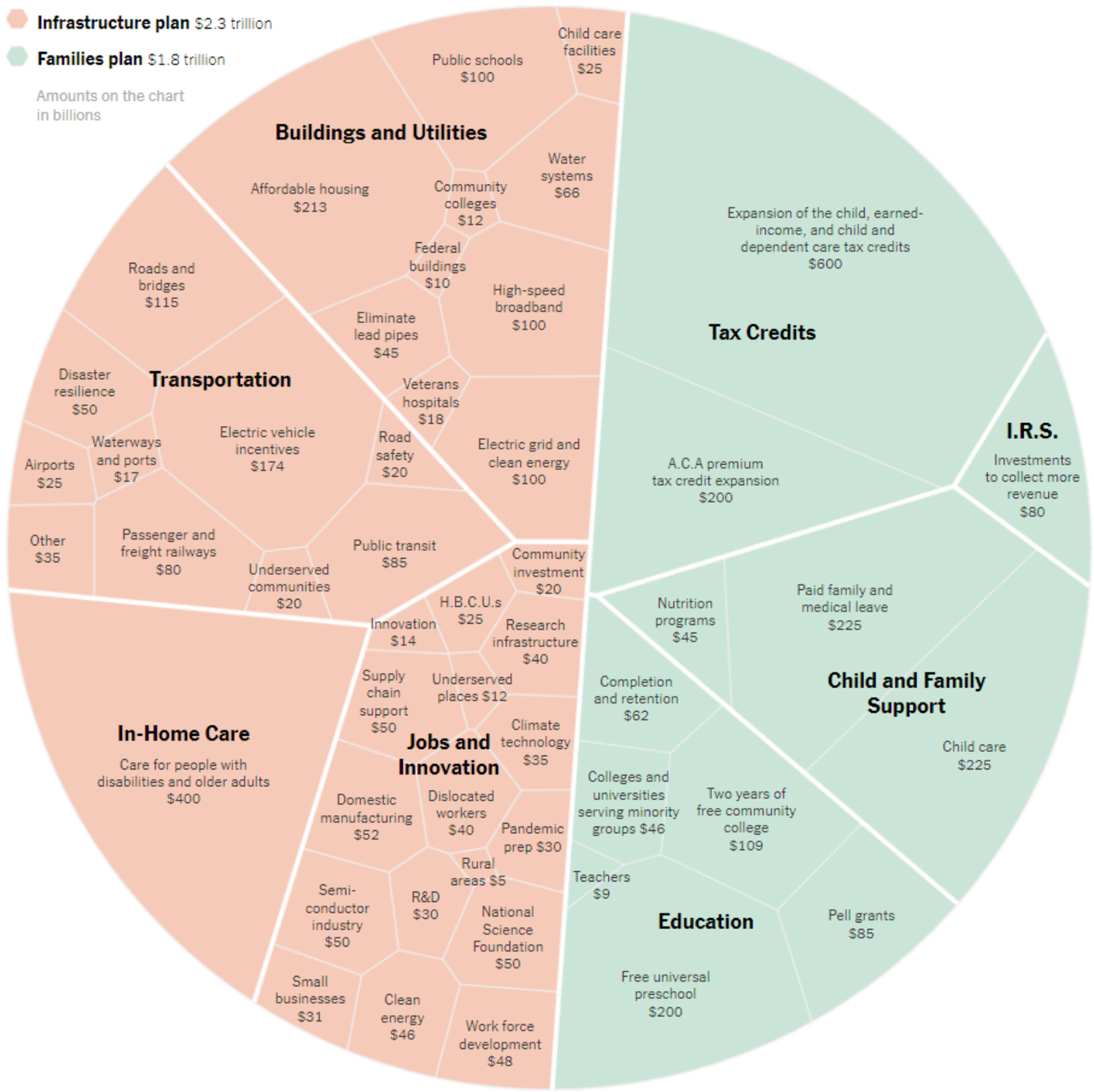
"If nobody has any ideas to share, we take a Notice Wonder break."

—Melynee Naegele (@MNmMath)

"Wait! We didn't NW yet!"

—a student

What's Going On in This Graph?, NYTimes Learning Network



Note: The plan released by the White House did not include estimated costs for the expansion of the child tax credit, earned-income tax credit, or child and dependent care tax credit. The $600 billion estimate is the difference between the total tax cuts mentioned in the plan ($800 billion) and the $200 billion included for Affordable Care Act premium tax credits. • Source: The White House

What's Going On in This Graph?, NYTimes Learning Network

What's Going On in This Graph? | President Biden's Economic Plan

Here's what's included in President Biden's $4.1 trillion infrastructure and families plan. What do you notice? What do you wonder?

What's Going On in This Graph?, NYTimes Learning Network

After looking closely at the graph above (or at this full-size image), answer these four questions:

- What do you notice?
- What do you wonder?
- What impact does this have on you and your community?
- What's going on in this graph? Write a catchy headline that captures the graph's main idea.

Ideas are Power

Math: Something you do, or something that's done to you?

Next Step on the Agenda

Engagement

Curious First Graders

Curious First Graders

Me and my buddy Cindy Cliche of Murfreesboro City Schools (@CindyCliche1)

Work with addition and subtraction equations.

CCSS.MATH.CONTENT.1.OA.D.7

Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Curious First Graders

$$2 + 3 = 4 + 1$$

Curious First Graders

$$2 + 3 = 4 + 1$$

Amy: "I noticed that 2 plus 3 is 5, and 4 plus 1 is 5, so in all it's 10."

Brandi: "They both equal 5."

Autumn: "Two plus three equals five, and four plus one equals five."

(Students used equations, number bonds, tally marks, dots, fingers, and number lines.)

Curious First Graders

$$2 + 3 = 4 + 1$$

Kirsten: "I noticed, at the back of the problem that equals 5, there is an equals sign, but there isn't a number at the back, there's just 4 + 1."

Sidney: "There's an equal but there's no answer, because it's the 4 + 1 is taking the 5 away from it. So it doesn't let the 5 go in the space that it has to go."

Curious First Graders

Is it always true?

$$2 + 3 = 4 + 1$$

Curious First Graders

Is it always true?

Gabriel: "I think it's true because 2 + 3 = 5 and 4 + 1 = 5. And if you smash those together, it will equal 10."

Aniyah: "I drew all the dots. I drew an equation but it was a little different. I actually used my fingers because my teacher last year taught us to count on. So I got 5, 6, 7, 8, 9, 10."

Curious First Graders

Is it always true?

Aniyah: "With the equal in front of that, I don't think that's true right there."

"Why is there equations like that, with equals in the middle? Like it's not, like, true, but why do some people do that?"

3-Act Tasks - Act 1

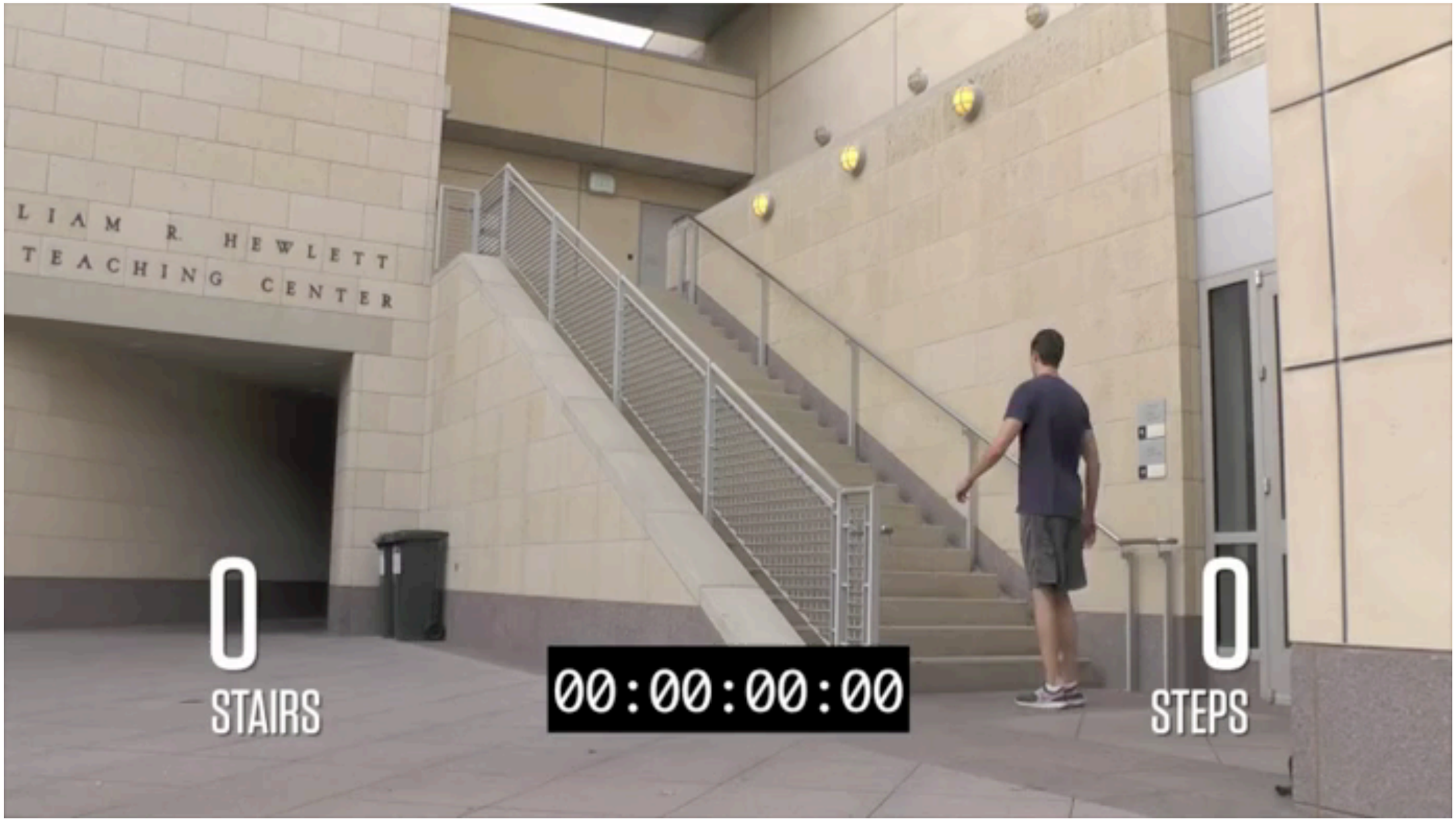
3-Act Tasks - Act 1

What do you notice? What do you wonder?

What do you want to know? What information do you think you need?

What's a guess that you know is too high?
What's a guess that you know is too low?
What's your best guess?

3-Act Tasks - Act 2



3-Act Tasks - Want More?

Next Week!

3-Act Tasks: Filling the Void of Mathematical Modeling in the Elementary Grades

with Graham Fletcher (@gfletchy)

Same bat time, same bat channel.

How Do You Encourage Curiosity?

People cannot answer questions they didn't ask.

"Math shouldn't be something to be over and done with."

—*Suzanne Alejandre (@SuMACzanne)*

"Notice and wonders make me curious, and makes me want to know more."

—*Anna, a 2nd grade student*

Moving Forward on the Agenda

Leveraging

Students Have a Lot of Ideas

Beth Brandenburg
@Brandeli1974

Stop telling Ss how to "do math" start asking them what they Notice & Wonder. You will be amaZed at what they know! @MFAnnie #wcpsmd

11:59 AM · Oct 11, 2014 · Twitter for iPhone

Students Have a Lot of Ideas

Jessica Strom
@strom_win

"We" dont give students enough credit! I had my Ss graph points for sinx & cosx, then #noticewonder about their graphs. They noticed EVERYTHING I wanted to teach them and the discussion was amazing! Thanks @saravdwerf & @MFAnnie for inspiring me! #MTBoS #iteachmath #NWMNmath

8:13 PM · Feb 15, 2019 · Twitter for Android

Students Have a Lot of Ideas

Notice

Boxes

Truck has three doors
There's the least amount of purple
There's no back door of the truck
There's a lot of brown boxes, then purple and green
The truck is full of boxes
They're using a board to walk through
Five boxes in the truck
They're all different sizes
The truck is multi-colored
The truck has two lines

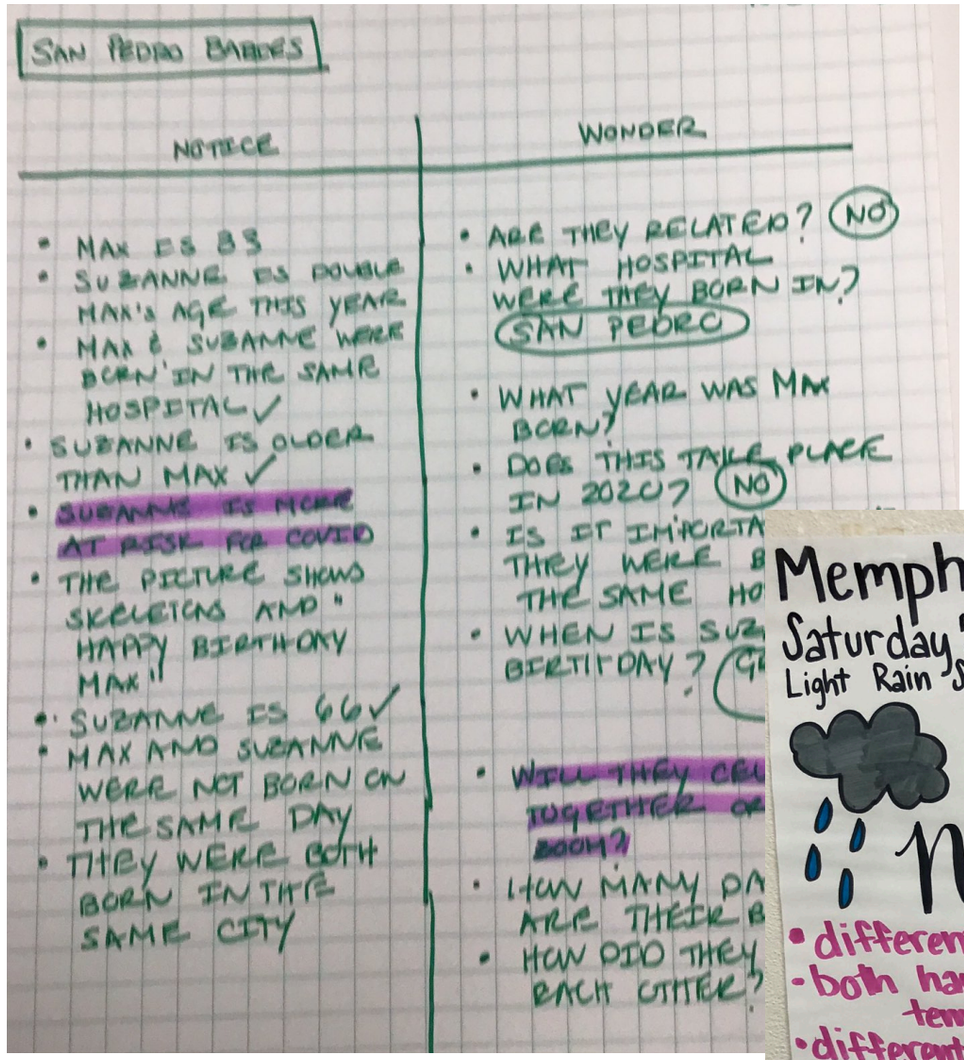
13 boxes

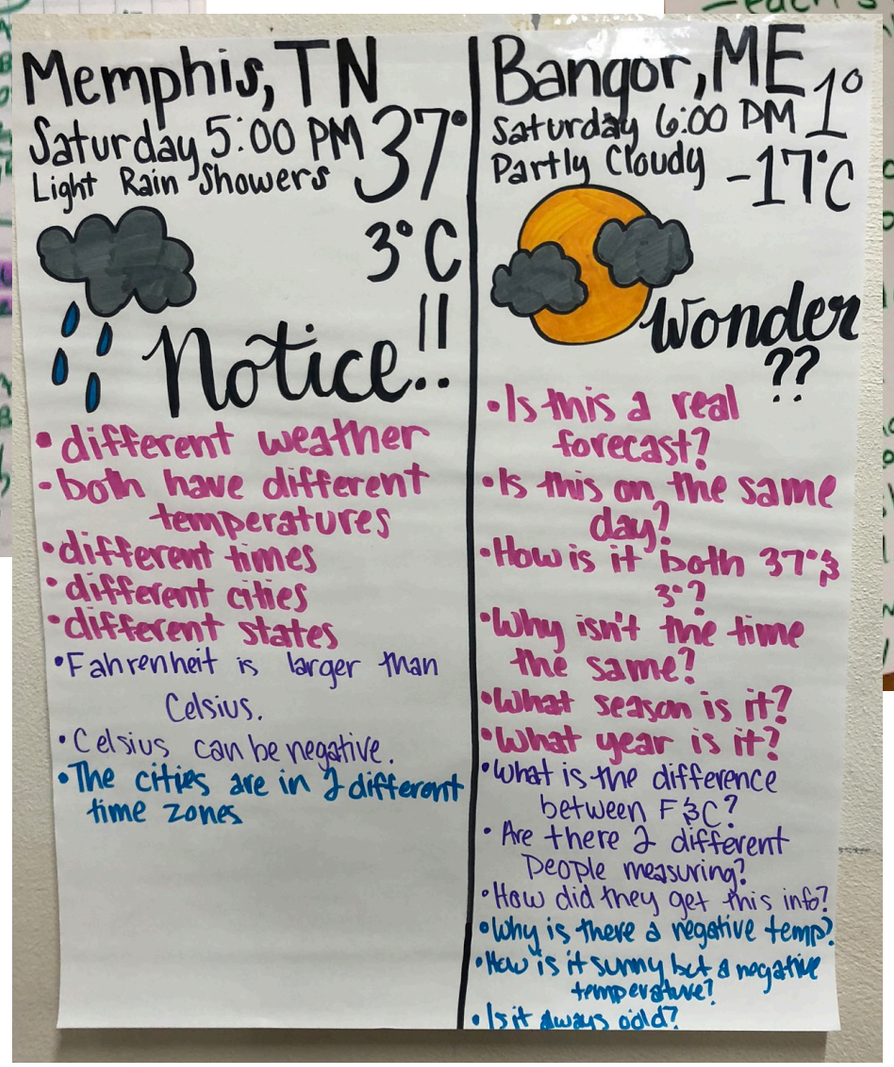
Wonder

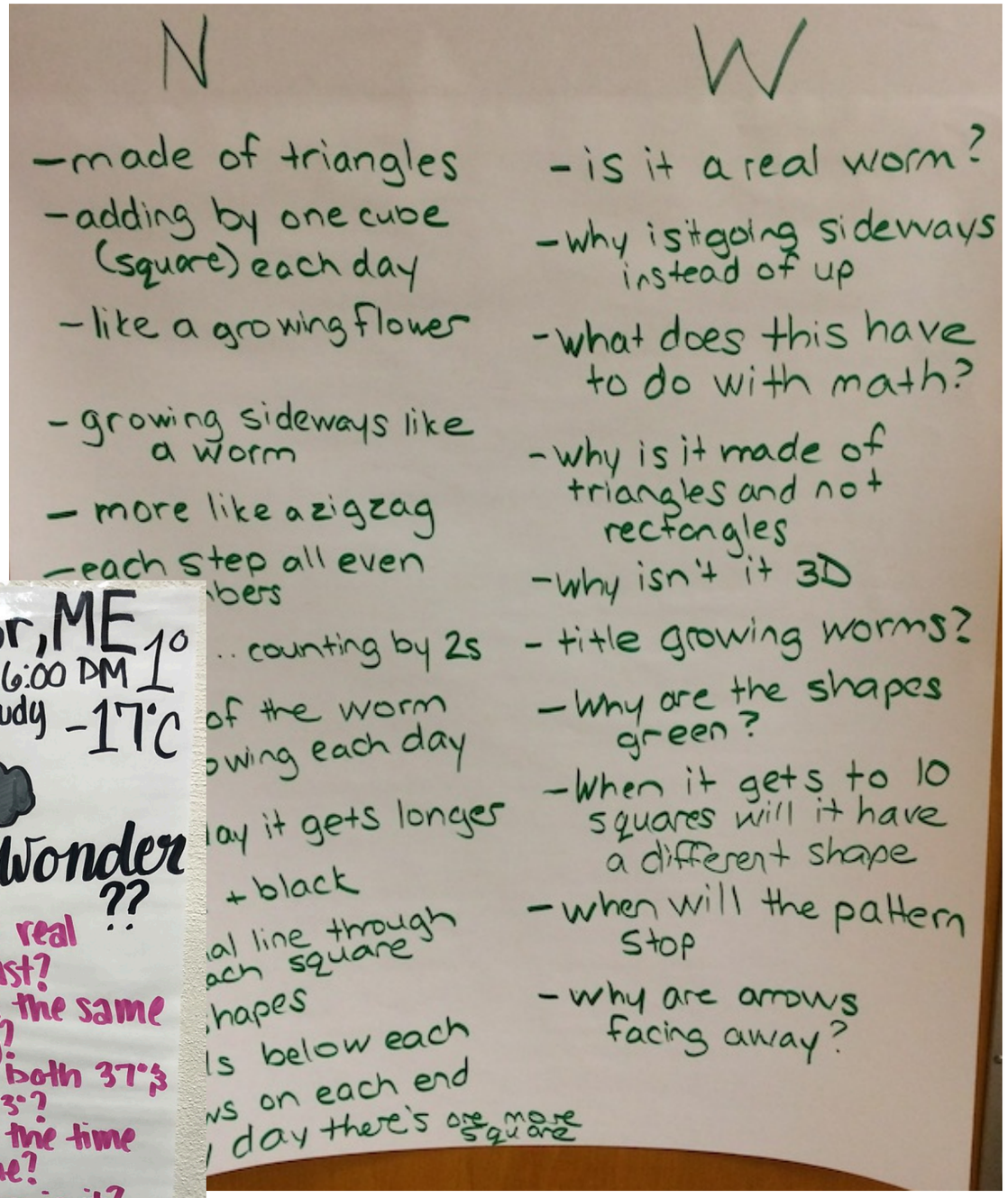
Why are they different colors
Why are there 8 boxes out of the truck?
What's in the boxes?
Why are they all different shapes and sizes:
Why is the truck all different colors
Why they're in the truck
How many boxes total
Why boxes outside? Did they fall out or not finished loading?
Where are they going?

Why only four green boxes?
Why aren't there more boxes?
Are they taking them out or in?
Is Someone moving?

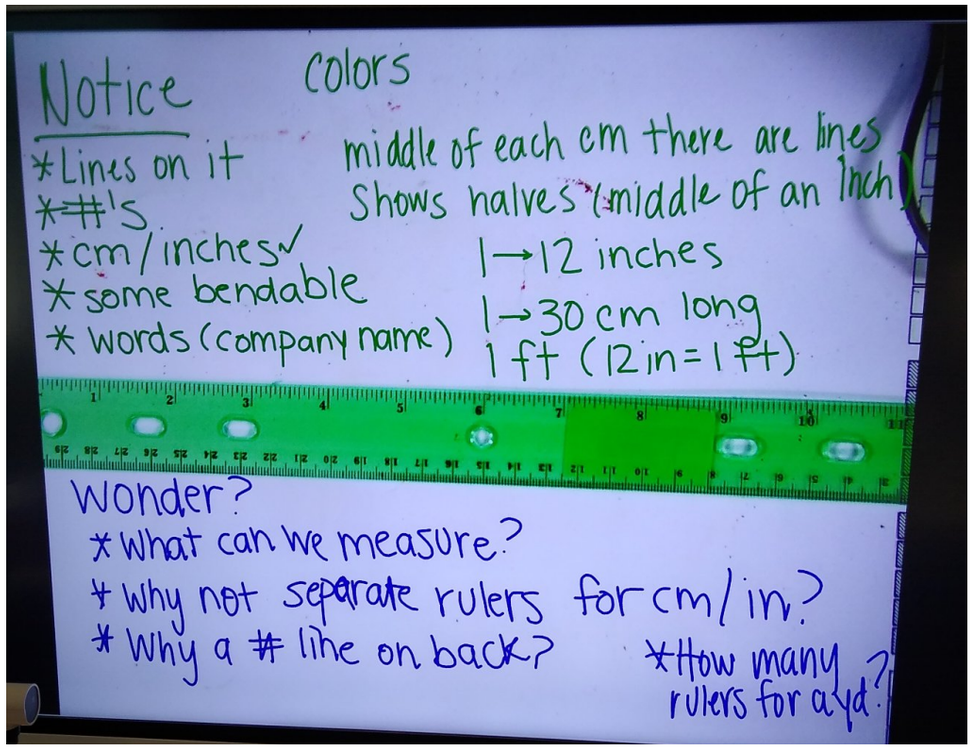
Students Have a Lot of Ideas

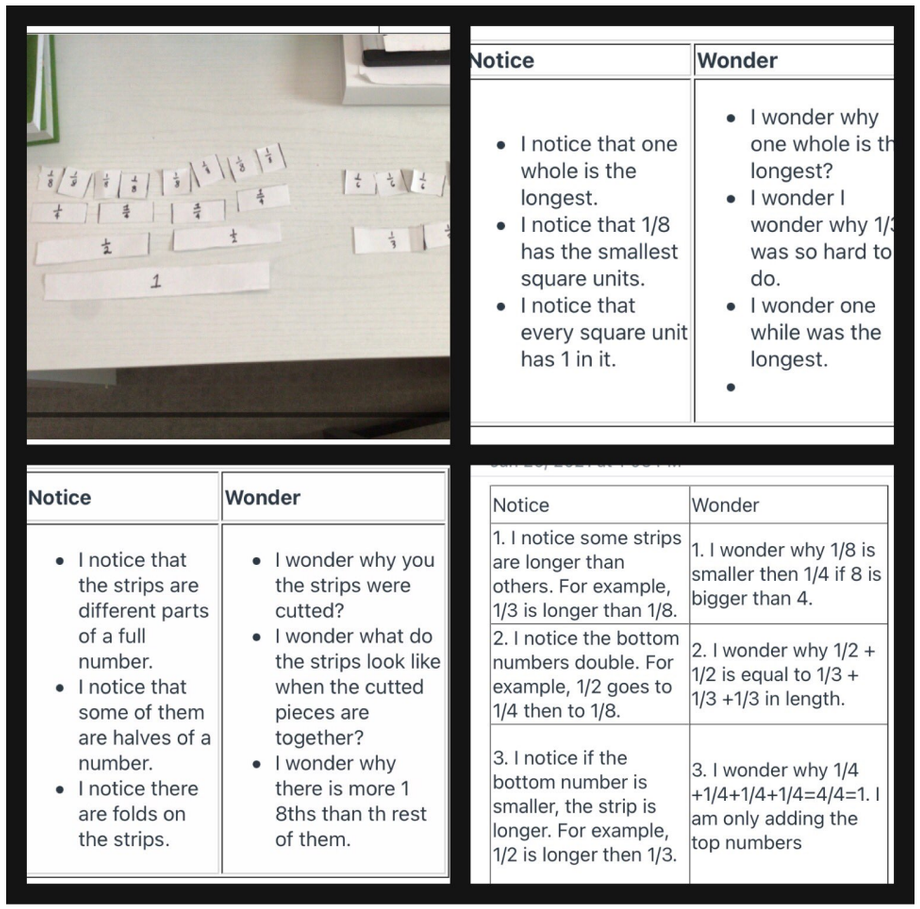






Students Have a Lot of Ideas

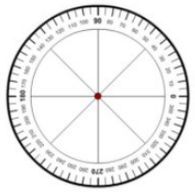




Notice	Wonder
• I notice that one whole is the longest. • I notice that 1/8 has the smallest square units. • I notice that every square unit has 1 in it.	• I wonder why one whole is th longest? • I wonder I wonder why 1/ was so hard to do. • I wonder one while was the longest. •

Notice	Wonder
• I notice that the strips are different parts of a full number. • I notice that some of them are halves of a number. • I notice there are folds on the strips.	• I wonder why you the strips were cutted? • I wonder what do the strips look like when the cutted pieces are together? • I wonder why there is more 1 8ths than th rest of them.

Notice	Wonder
1. I notice some strips are longer than others. For example, 1/3 is longer than 1/8.	1. I wonder why 1/8 is smaller then 1/4 if 8 is bigger than 4.
2. I notice the bottom numbers double. For example, 1/2 goes to 1/4 then to 1/8.	2. I wonder why 1/2 + 1/2 is equal to 1/3 + 1/3 +1/3 in length.
3. I notice if the bottom number is smaller, the strip is longer. For example, 1/2 is longer then 1/3.	3. I wonder why 1/4 +1/4+1/4+1/4=4/4=1. I am only adding the top numbers



I notice...

degrees
circle
unit circle
has lines
pointing to numbers
divisible by 45
quadrants
compass
it looks like circle drawing tool?

I wonder...

why does it have three legs
what are the three ring things?
if what we learned on the unit circle applies to this
how is it used?
what are the strings for?
why is this here?
why is it on a stool
why is it called a table
why is it called a force table?
how can we apply this to precal?

Students Have a Lot of Ideas

Claire Verti
@ClaireVerti

Replying to @MFAnnie and @NCTM

Notice & Wonder was my gateway strategy for learning to be comfortable with honoring all the different ideas students had. I heard you at CMCSOUTH 2015 and watched your ignite video 2,000 times. The more I used notice and wonder the more I grew comfortable with other strategies

12:34 AM · May 30, 2021 · Twitter for iPhone

Students Have a Lot of Ideas

Claire Verti
@ClaireVerti

Replying to @MFAnnie and @NCTM

Notice & Wonder was my gateway strategy for learning to be comfortable with honoring all the different ideas students had. I heard you at CMCSOUTH 2015 and watched your ignite video 2,000 times. The more I used notice and wonder the more I grew comfortable with other strategies

12:34 AM · May 30, 2021 · Twitter for iPhone

Replying to @ClaireVerti @MFAnnie and @NCTM

So as much as it benefited my students, it also helped the teacher.

12:35 AM · May 30, 2021 · Twitter for iPhone

NCTM's Principles to Actions

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Elicit It and USE It!

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

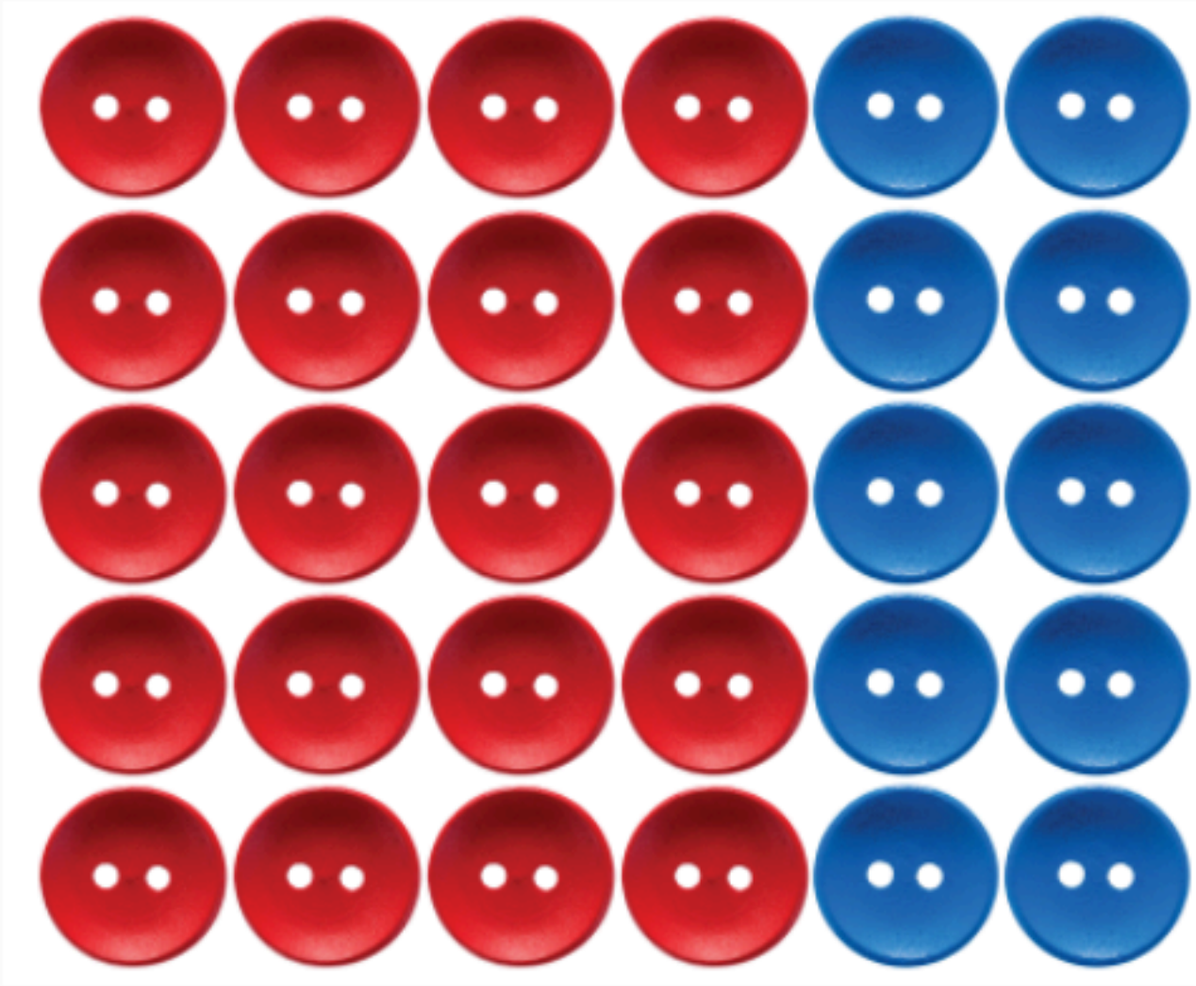
Elicit It and USE It!

The Arrival, Giorgio de Chirico, 1912-1913, from The Barnes Foundation

Sliding from Notice and Wonder into the Lesson

Is it always true?

$$5 \times 6 = 5 \times 4 + 5 \times 2$$

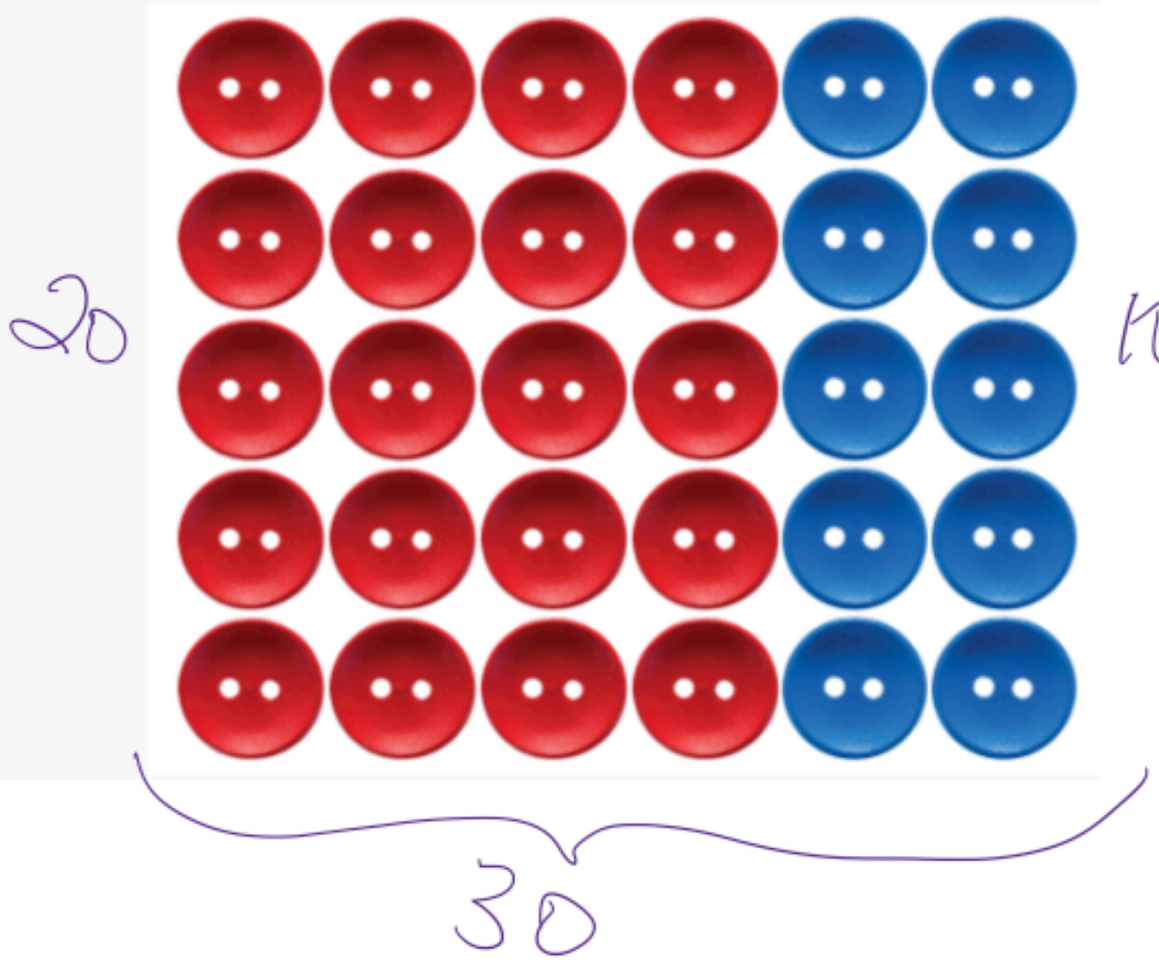


Sliding from Notice and Wonder into the Lesson

Is it always true?

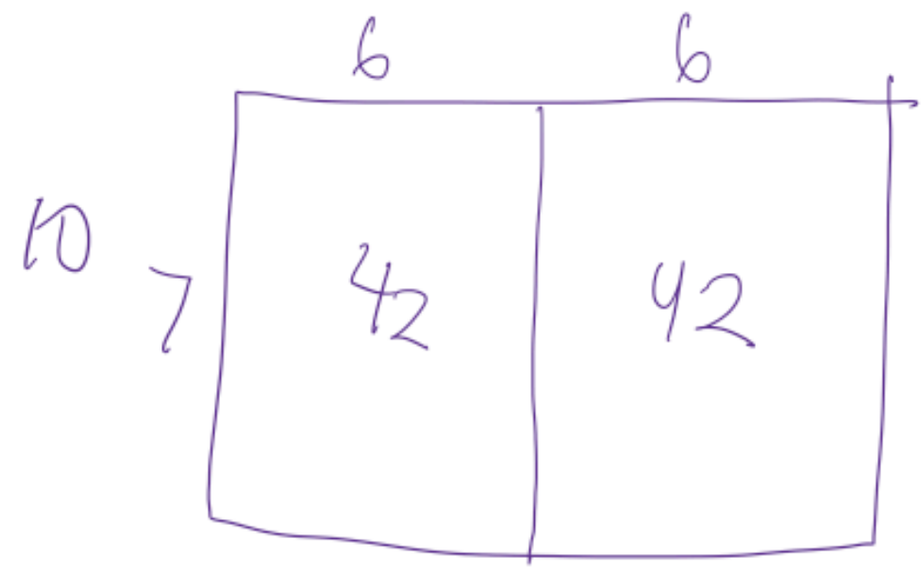
30 = 20 + 10

$5 \times 6 = 5 \times 4 + 5 \times 2$



7x12

$$\begin{array}{r} 12 \\ \times\ 7 \\ \hline \end{array}$$



42 + 42 = 84

12 = 10 + 2

7 x 10 = 70

7 x 2 = 14

70 + 14 = 84

Using Notice & Wonder with Textbook Problems

Looking

Looking at Student Work

Reflecting on Student Work
The Math Forum @ Drexel

1. Select the student work. Video clips of classroom work can be used for very young students. Usually plan on 1 piece (or set of pieces/drafts) for a 60-minute session. Pick a piece of work that represents a persistent or critical issue involving either a math concept or a student's approach to learning. Naturally the piece(s) selected should have enough material to support reflection on the student's thinking.

2. Do the problem/assignment that the students did. Solve the problem with as many different strategies as you can (e.g. estimation, with and without algebra, geometric reasoning, using data and probability tools, etc.).

3. Identify the main concepts and strategies involved, differentiating between those that have to be involved and those that are optional.

4. Use the following process to discuss what you see in the student work in order to understand what they did and were thinking.

   Work in groups of 3-5 people. Select a recorder to keep notes of what is said. Go around in a circle and each person shares one thing that they notice about the student work. Try to keep moving so that all ideas and voices are heard and keep discussion and judgment to a minimum for now. Keep going until there are no more noticings or time requires moving on. Example of types of noticing:

   - The way the work is organized
   - The understanding of the question or problem
   - The techniques and problem-solving strategies used: from random guessing to development of mathematical models.
   - Accuracy in calculations.
   - Which parts of their solution process do they communicate/make visible and which not?
   - What connections do they make to prior knowledge and experience?
   - ….

   Be careful not to impose your expectations on the students' work and not to judge it. Look closely at all aspects of a student's work and be prepared to have your view expanded or to be left with questions. If you find yourself making claims about what is going on in the student's head, turn those into questions or wonderings. Instead of "James doesn't know other factors", try "James did not write down any other factors. I wonder if James knows the other factors? I wonder if James thought about finding other factors."

5. Design a question or a task for the student that would help you understand more about what the student is doing and thinking. If the student appears to have made mistakes, do not try to teach the correct approach yet. Focusing on eliciting more information from the student. Use your wonderings from step 4 as good areas for investigation. Share your task with your colleagues in your group and have others describe how they think a student might approach such a task.

Looking at Student Work

Work in groups of 3-5 people. Select a recorder to keep notes of what is said. Go around in a circle and each person shares one thing that they notice about the student work. Try to keep moving so that all ideas and voices are heard and keep discussion and judgment to a minimum for now. Keep going until there are no more noticings or time requires moving on. Example of types of noticing:

- The way the work is organized
- The understanding of the question or problem
- The techniques and problem-solving strategies used: from random guessing to development of mathematical models.
- Accuracy in calculations.
- Which parts of their solution process do they communicate/make visible and which not?
- What connections do they make to prior knowledge and experience?
- ….

from Reflecting on Student Work, by The Math Forum, 2006

Looking at Student Work

Be careful not to impose your expectations on the students' work and not to judge it. Look closely at all aspects of a student's work and be prepared to have your view expanded or to be left with questions. If you find yourself making claims about what is going on in the student's head, turn those into questions or wonderings. Instead of "James doesn't know other factors", try "James did not write down any other factors. I wonder if James knows the other factors? I wonder if James thought about finding other factors."

from Reflecting on Student Work, by The Math Forum, 2006

Looking at Other Things

Scott Ellingson
@mremathteacher

Replying to @MFAnnie and @NCTM

I use it as an administrator with my staff to focus work to be done, help provide feedback, but most importantly provide ownership and voice. Whether looking at assessment/survey data, breaking down grade level standards, or after a presentation/something we read together.

11:16 AM · May 30, 2021 · Twitter for Android

Observing Each Other

I noticed...	I wondered...

from Chris Luzniak's school (@cluzniak)

February 2021 Learning & Teaching Journal

Noticing and Wondering

to Guide Professional Conversations

This article explores how teachers can use the sentence stems "I notice" and "I wonder" to deepen professional conversations with colleagues, both in person and in online spaces.

Tracy E. Dobie and Eleanor R. Anderson

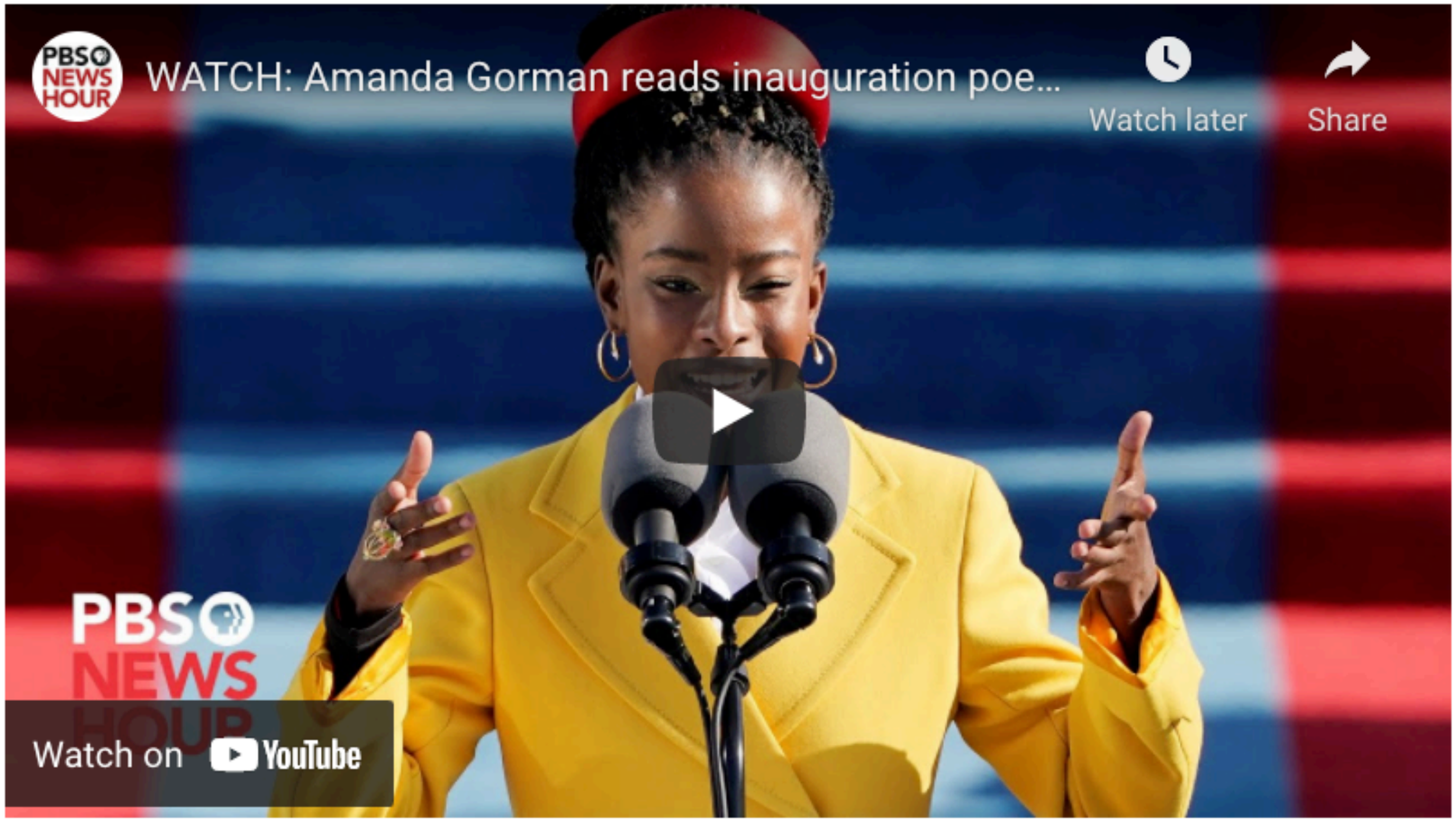
Getting Near the End of the Agenda

Spreading

It's Not Just Math, Folks!

PBS NEWSHOUR EXTRA

News for Students and Teacher Resources
6–12 Grade Level



1. As students watch, have them think about the following questions:
    - What do you notice?
    - What do you wonder?
    - What stands out to you?
    - Why might poets be tapped to read or speak at presidential inaugurations?

It's School Culture: Washington County PS, Maryland

Beth Brandenburg
@Brandeli1974

@MFAnnie visits @BesterWCPS! Thanks for stopping in!!! Come back when we have kids in classes.

7:48 PM · Aug 15, 2016 · Twitter for iPhone

Me and my buddy Beth Brandenburg, right, of Washington County Maryland Public Schools (and friends)

It's School Culture: Caesar Rodney SD, Delaware

It's School Culture: Caesar Rodney SD, Delaware

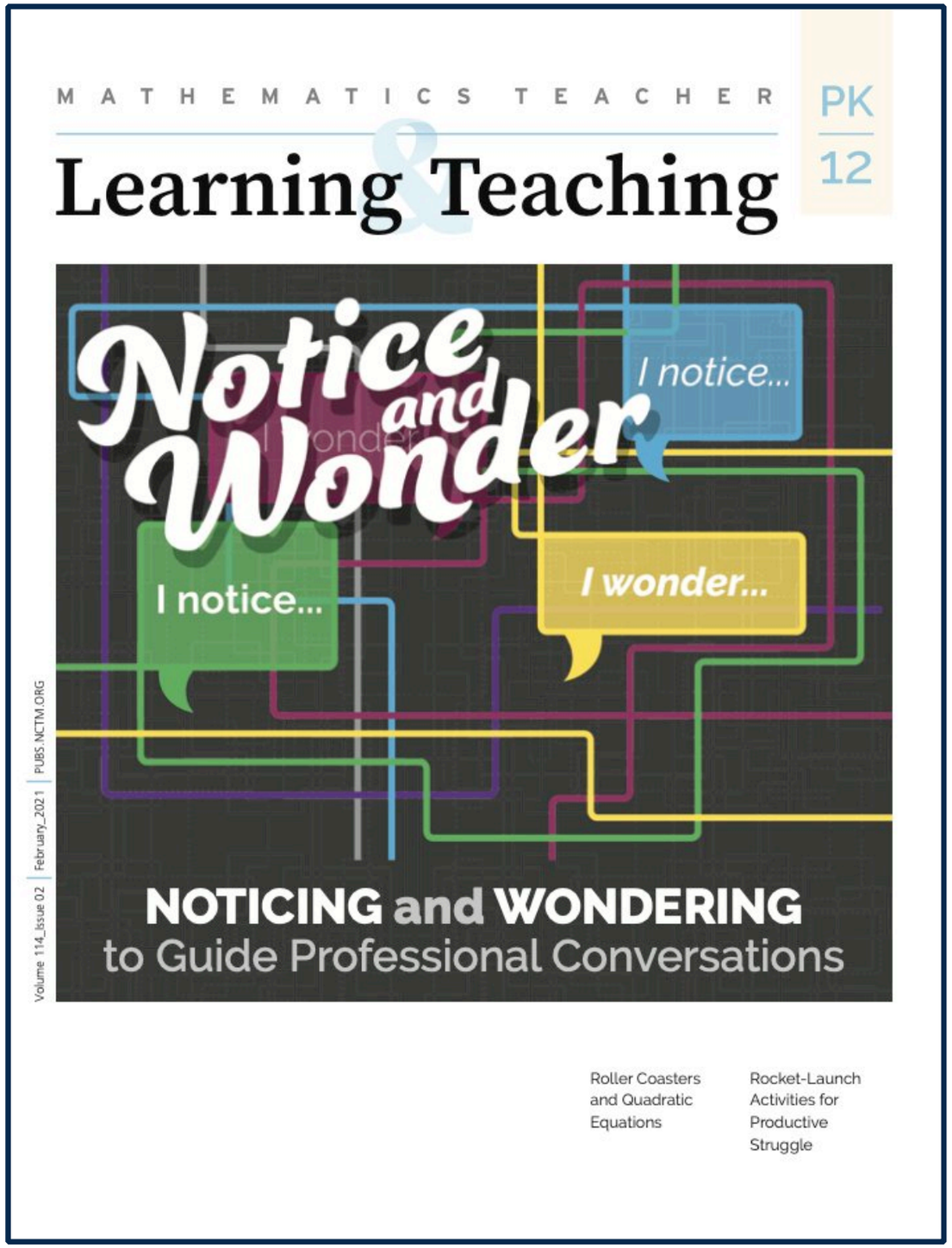
Annie Fetter
@MFAnnie

Want me to come do PD in your district, maybe even a keynote? Doesn't hurt to have #NoticeWonder as the theme and give me a shirt! @CaesarRodneySD

8:02 AM · Jan 26, 2018 · Twitter for iPhone

Want to Learn More?

nctm.org/noticeandwonder/



February 2021 Issue

It's a Big World!

- Follow #NoticeWonder on Twitter
- Search the web for Notice and Wonder - lots of blog posts out there
- Visit my blog (annie.mathematicalthinking.org) and start with
  - #NoticeWonder Love
  - Can Novices do #NoticeWonder?
  - Noticing and Wondering in Elementary School

Last Item on the Agenda

Reflecting

Always Leave Time for Reflection

So, what's something you're wondering?

Write it down in your journal, tweet it at me, blog about it, AND post in the chat!

(Reminder: You'll get a PDF of the slides, so no need to wonder that.)

Thanks!

Annie Fetter

anniefetter@gmail.com
Twitter: @MFAnnie, #NoticeWonder
annie.mathematicalthinking.org