Why is that True? How does it Work? Finding Multiple Answers for Mathematical Classroom Situations

On a side note. - the file was too large so. I took out the photos and formatting. Also slides that did not change the content.

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Immediate Past-President NCSM
Emporia State University, Professor of Mathematics

Look at the expressions on these two. What would they be seeing in your classroom?

Do your students get excited to notice and wonder in mathematics?

On a side note. - the file was too large so. I took out the photos and formating.

Think about some of the WHY questions you have heard in your classroom?

Who asks WHY?

When do you hear WHY?

How often are we asking WHY questions?

What changes occur in your classroom when your students become comfortable answering WHY?

Some of my favorite WHY questions.

Why does $.\overline{9} = 1$?

Why isn't division by 0 allowed?

Why isn't 1 prime?

Why is the product of two negative numbers positive?

Why do we do proofs?

More Questions

Why does (x - 4)(x – 6) = 0 imply that (x - 4) = 0 or (x - 6) =0 but (x – 4)(x - 6) =4 does not imply that (x-4) = 4 or (x-6) = 4?

Why do we get extraneous roots?

Why is $\frac{\sqrt{2}}{2}$ simpler than $\frac{1}{\sqrt{2}}$?

Why are these statements Why are these statements wrong?

Parallel lines are lines that never intersect.

Linear equations graph to be a line. All lines are functions.

$a^2 + b^2 = c^2$ is the Pythagorean Theorem.

Goals for this presentation

- Explore
  - Explore the mathematical question why.
- Look
  - Look at examples of why and multiple ways to work with them.
- Share
  - Share the reference the book and guidebook outlining Situations taken from the resource, “Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations,”

Why is understanding important?

“We understand something if we see how it is related or connected to other things we know. “

Teaching Mathematics Through Problem Solving, Grades PreK-6. NCTM: Reston, VA, 2003.

When teachers are asked about understanding their answers often equate

Understanding

With

Skill Proficiency

Why do we need to know how to divide fractions?

$$\frac{4}{3} \div \frac{1}{2} = ?$$

Think of a real-world problem where you would use this equation to solve the problem.

What should our students be able to do to have a better understanding of fractions?

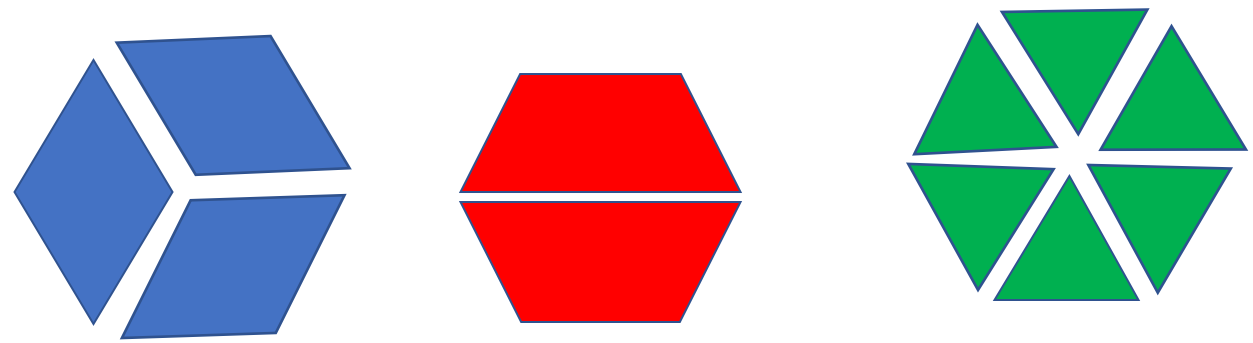
1. Estimate the quotient.
    a. Between what two integers is the exact answer?
    b. Which is it closer to and why?
2. Tell what the quotient means in this situation.

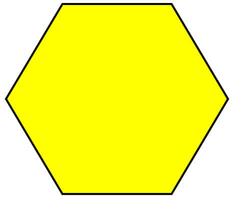
3. Make a model that shows how division works.

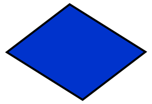
4. Explain why the “invert and multiply” rule makes sense.

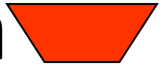
5. Give two ways of doing this calculation.

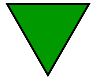
Explore with Pattern Blocks

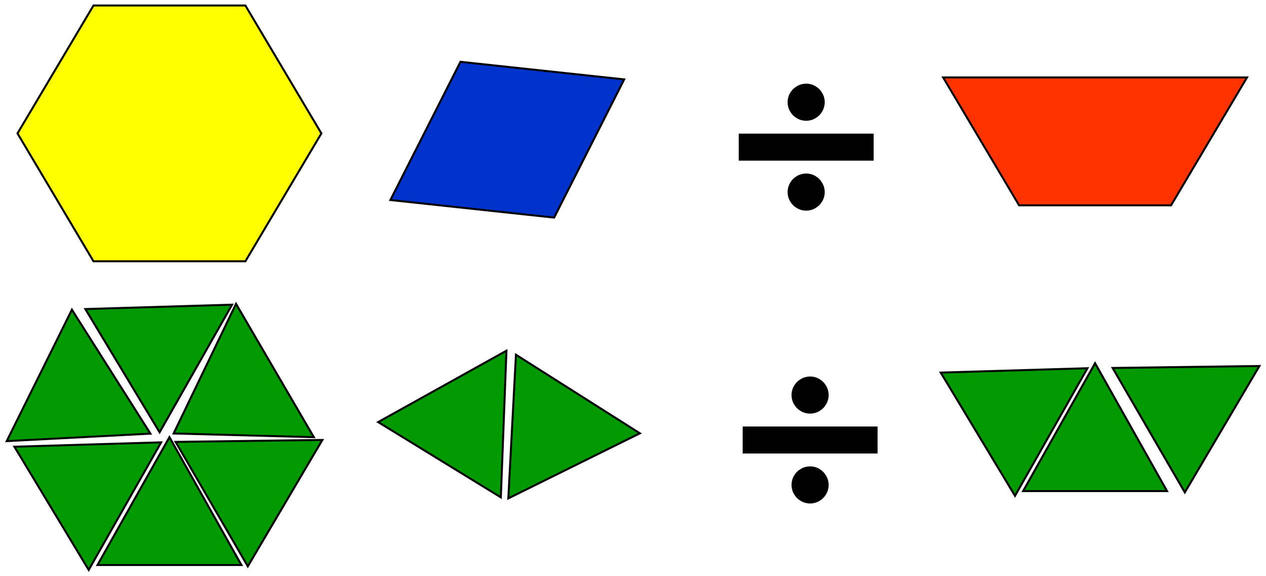


Let  represent 1.

What does a  represent?

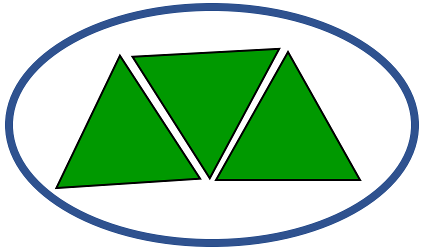
What does a  represent?

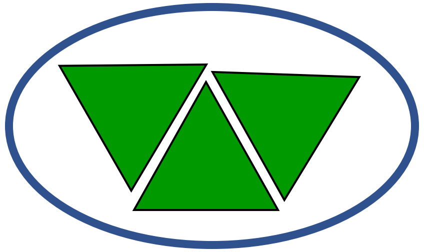
What does a  represent?

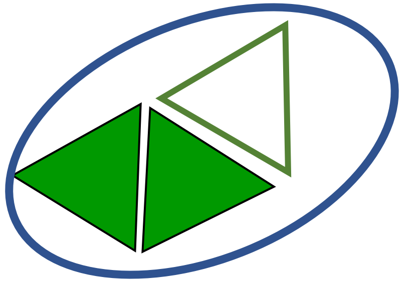


$$\frac{4}{3} \div \frac{1}{2} = \frac{8}{3}$$

$$= 2\frac{2}{3}$$

$1 =$ 

$1 =$ 

 $= \frac{2}{3}$

Why does invert and multiply work?

$$\frac{4}{3} \div \frac{1}{2} = \frac{\frac{4}{3}}{\frac{1}{2}} = \frac{\frac{4}{3}}{\frac{1}{2}} \cdot 1 = \frac{\frac{4}{3}}{\frac{1}{2}} \cdot \frac{\frac{2}{1}}{\frac{2}{1}} = \frac{4}{3} \cdot \frac{2}{1} = \frac{8}{3}$$

And WHY don't we just divide across?

$$\frac{4}{3} \div \frac{1}{2} = \frac{4}{3} \cdot \frac{2}{2} \div \frac{1}{2} \cdot \frac{3}{3} = \frac{8}{6} \div \frac{3}{6} = \frac{8 \div 3}{6 \div 6} = \frac{8}{3}$$

Understandings

1. You can use number sense and the meaning of division to estimate the quotient of two fractions.

2. The meaning of the quotient when dividing two fractions must interpreted relative to the divisor.

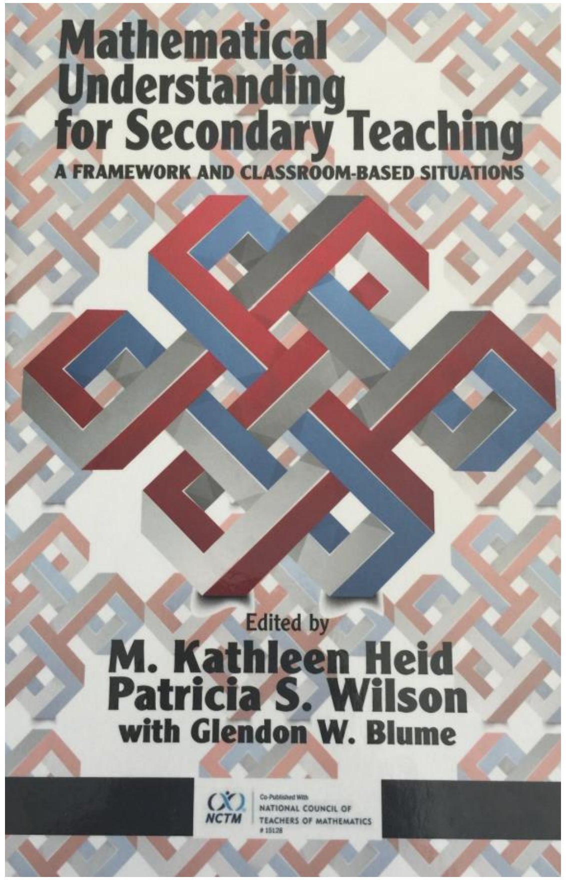
3. There are multiple ways to perform the operation.

4. Dividing a fraction by a fraction can be thought about as repeated subtraction.

5. You can use models or pictures to show the meaning of fraction division.

Situations: Project and Tool

- Began as a research project between Penn State and the University of Georgia: Glendon Blume, M. Kathleen Heid, Rose Mary Zbiek, Jeremy Kilpatrick, James W. Wilson, and Patricia Wilson.
  - ***What mathematics is useful for secondary mathematics teachers to know?***
  - Framework to complement frameworks for mathematics for elementary teachers.
  - Worked from teacher's practice to identify this mathematics.
  - "Situations" were the means by which the mathematics arose.
- Turned into a collaboration with NCSM leadership to leverage Situations for professional learning. NCSM members participating in the project: Diane Briars, M. Suzanne Mitchell, Connie Schrock, Steven S. Viktora
- Both the project book and the facilitator guide were endorsed by NCTM.



What's a Situation?

- Prompt (from practice)
- Commentary (overview)
- Foci (statement & explanation)
  - Focus 1
  - Focus 2
  - ….
  - Focus *n*
- Post-commentary (extended ideas)

Mathematical Understanding for Secondary Teaching (MUST)

1. Mathematical Proficiency

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Adaptive reasoning
- Productive disposition
- Historical and cultural knowledge

Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations
(Heid & Wilson, 2015)

Mathematical Understanding for Secondary Teaching (MUST)

2. Mathematical Activity

- Mathematical noticing
  - Structure of mathematical systems
  - Symbolic form
  - Form of an argument
  - Connect within and outside of mathematics
- Mathematical reasoning
  - Justify/proving
  - Reasoning when conjecturing and generalizing
  - Constraining and extending
- Mathematical creating
  - Representing
  - Defining
  - Modifying/transforming/manipulating
- Integrating strands of Mathematical Activity

Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations (Heid & Wilson, 2015)

Mathematical Understanding for Secondary Teaching (MUST)

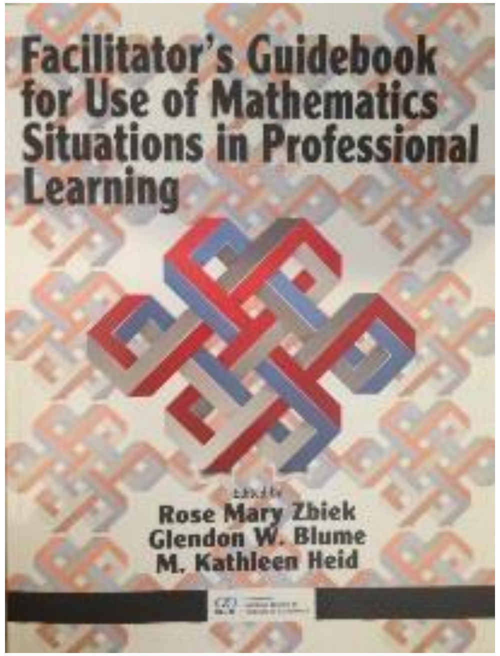
3. Mathematical Context of Teaching

- Probe mathematical ideas
- Access and understand the mathematical thinking of learners
- Know and use the curriculum
- Assess the mathematical knowledge of learners
- Reflect on the mathematics of practice

Mathematical Understanding for Secondary Teaching: A Framework and Classroom-Based Situations
(Heid & Wilson, 2015)

Facilitator's Guidebook

- Overview of Professional Learning
  - Situation prompt - Relevance
  - Goals - Key Mathematical Ideas
- Complete Copy of Situation
- Connections to Standards (Qs)
- Suggestions for Using This Situation
  - Tools - Time
  - Outline of Participant Activities
- Facilitator Notes
  - About the mathematics
  - Launch
    - *Time, Possible responses*
    - *Key points, Options*
  - Activities *1-n*
  - Reflect and assess learning (Qs)
- Resources
- References



The Six Situations in Professional Learning Guidebook

- Situation 1: Division Involving Zero
- Situation 2: Product of Two Negative Numbers
- Situation 21: Graphing Quadratic Functions
- Situation 34: Circumscribing Polygons
- Situation 35: Calculation of Sine
- Situation 38: Mean and Median

Why?

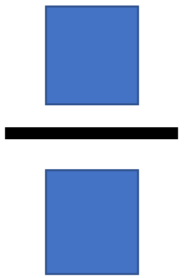
Why can't you divide by zero?

Why isn't $\frac{0}{3}$ less than $\frac{0}{5}$?

Why doesn't $\frac{0}{0} = 1$?

Up to 1

- Roll four number cubes with sides labeled 0 – 5. Write down your four numbers.
- Select any two numbers to make a fraction or decimal number less than 1.



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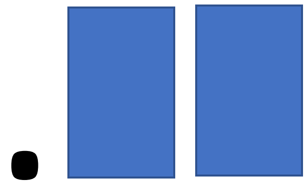


- On the next roll make a number greater than the first roll but less than or equal to one.
- Repeat until you cannot make a number less than or equal to 1.

Goal: To make as many rolls as possible.

What do you notice?

What do you wonder?

Up to 1 Record Sheet

Numbers Rolled	Number
3 4 0 1	0/1
5 0 3 3	0/3
1 5 0 2	0/5
1 4 2 4	.12
3 4 0 2	.23

Up to 1 Record Sheet

Numbers Rolled	Rational Number	Fractional Form	Decimal form
☐ ☐ ☐ ☐			
☐ ☐ ☐ ☐			
☐ ☐ ☐ ☐			
☐ ☐ ☐ ☐			
☐ ☐ ☐ ☐			
☐ ☐ ☐ ☐			

Another why question that came up during this activity.

Why is $\frac{1}{3}$ greater than .33?

Division Involving Zero

Prompt:

On the first day of class, pre-service middle school teachers were asked to evaluate $\frac{2}{0}, \frac{0}{0}, and\ \frac{0}{2}$ and to explain their answers. There was some disagreement among their explanations:

- Because any number over 0 is undefined;
- Because you cannot divide by 0;
- Because 0 cannot be in the denominator;
- Because 0 divided by anything is 0; and
- Because a number divided by itself is 1.

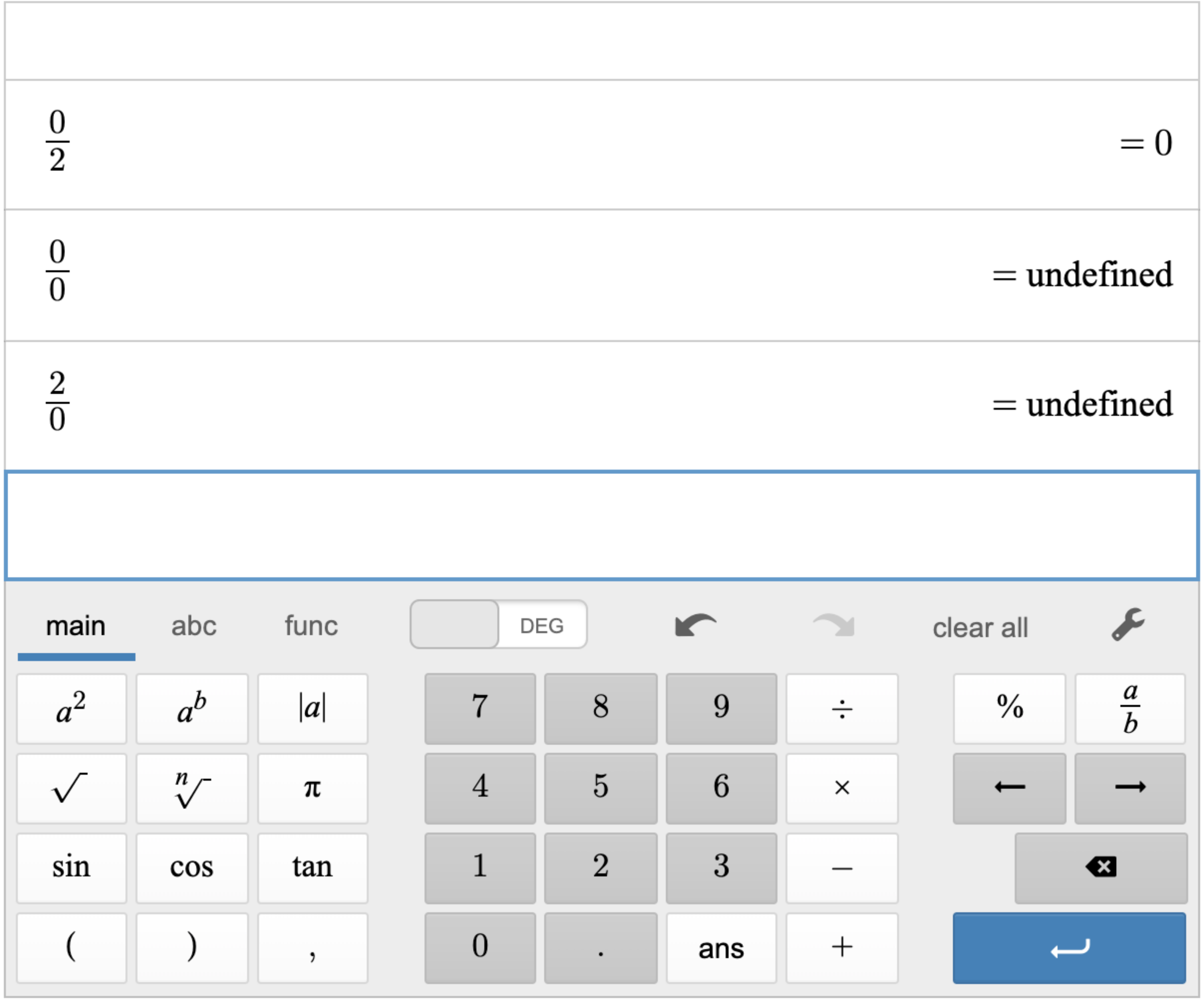
The facilitators guide provides examples of activities to use.

- Discuss
  - Ask students to discuss their solutions and responses
- Review
  - Review definitions; rational, irrational, division, slope of a vertical line, and exclusions involving 0.
- Explore
  - Explore technology and online answers (note many are wrong)
- Clarify
  - Clarify common misconceptions about division involving zero
- Distinguish
  - Distinguish between undefined and indeterminate.

Values for 0 ÷ 2, 0 ÷ 0, and 2 ÷ 0 displayed by various technological tools

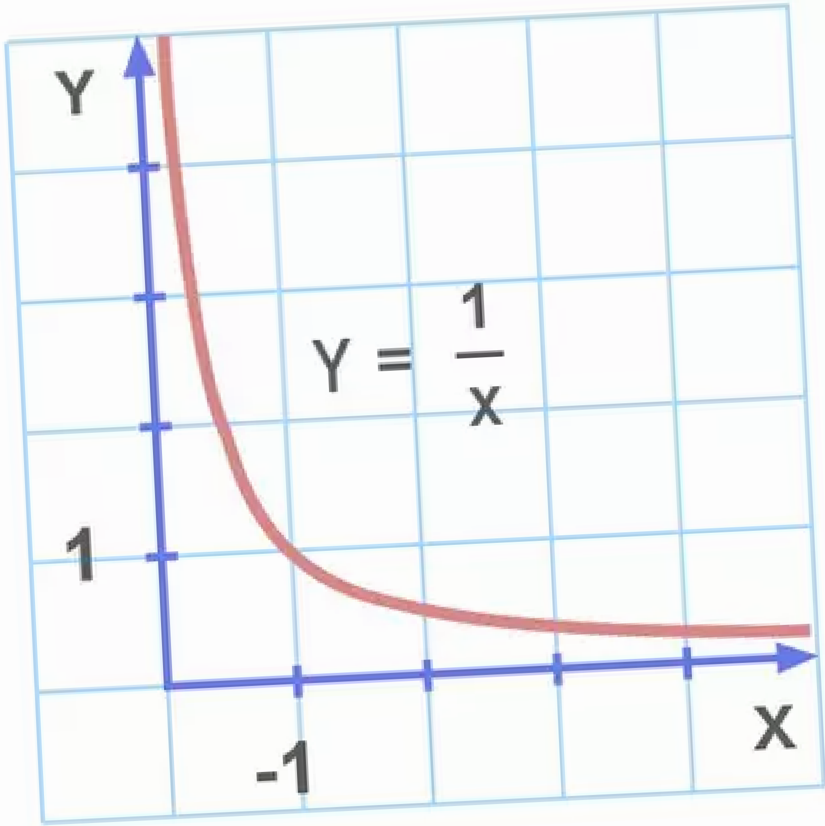
(Retrieved as of May 2017)

Tool	0 ÷ 2	0 ÷ 0	2 ÷ 0
Calculators			
Casio fx-CG10	0	Ma ERROR	Ma ERROR
TI-89	0	undefined	undefined
TI-Explorer Plus	0	Error Ari	Error Ari
Calculator Apps			
iPad app (HD calculator)	0	0	0
iPad app (pocketCAS)	0	undef	∞
Mac computer (calculator application)	0	Division by zero	Division by zero
Mac Calculator app in dashboard	0	error	error
Phones			
iPhone	0	error	error
Calculator on Android phone	0	Invalid operation	Invalid operation
Software (other than calculator apps)			
Excel spreadsheet	0	#DIV/0!	#DIV/0!



https://www.desmos.com/scientific

As the numbers get infinitesimally smaller, the trend can be traced toward infinity. However, *tending to* infinity and *being equal to* infinity are two very different notions. The result seems to be on its way to infinity, but it never arrives.



https://www.scienceabc.com/nature/why-cant-we-divide-by-zero.html

What is 0/0?

if anything over itself is 1
if anything over 0 is underfined
if 0 over anything is zero
what is zero over zero

Answer Save

17 Answers Relevance

a_liberal_economist
1 decade ago

Favorite Answer

it is an undefined quantity. Any number, even zero, divided by zero is undefined.
it is not infinity, it is not one, and it is not zero. it is undefined. period.

Source(s): Degree in mathematics.

0 3 1

Login to reply the answers Post

https://answers.yahoo.com/

WolframAlpha computational intelligence.

0/0

Extended Keyboard Upload Examples Random

Input:

$$\frac{0}{0}$$

Result:

(undefined)

Other indeterminate forms: More forms

$\frac{\infty}{\infty}$ | $0\,\infty$ | $\infty - \infty$ | 0^0 | 1^∞ | ∞^0

Indeterminate form »

Foci

Focus 1: An expression involving real number division can be viewed as real number multiplication, so an equation can be written that uses a variable to represent the number given by the quotient. The number of solutions for equations that are equivalent to that equation indicates whether the expression has one value, it is undefined or indeterminate.

Focus 2: One can find the value of whole number division expressions by finding either the number of objects in a group (a partitive view of division) or the number of groups (a quotative view of division).

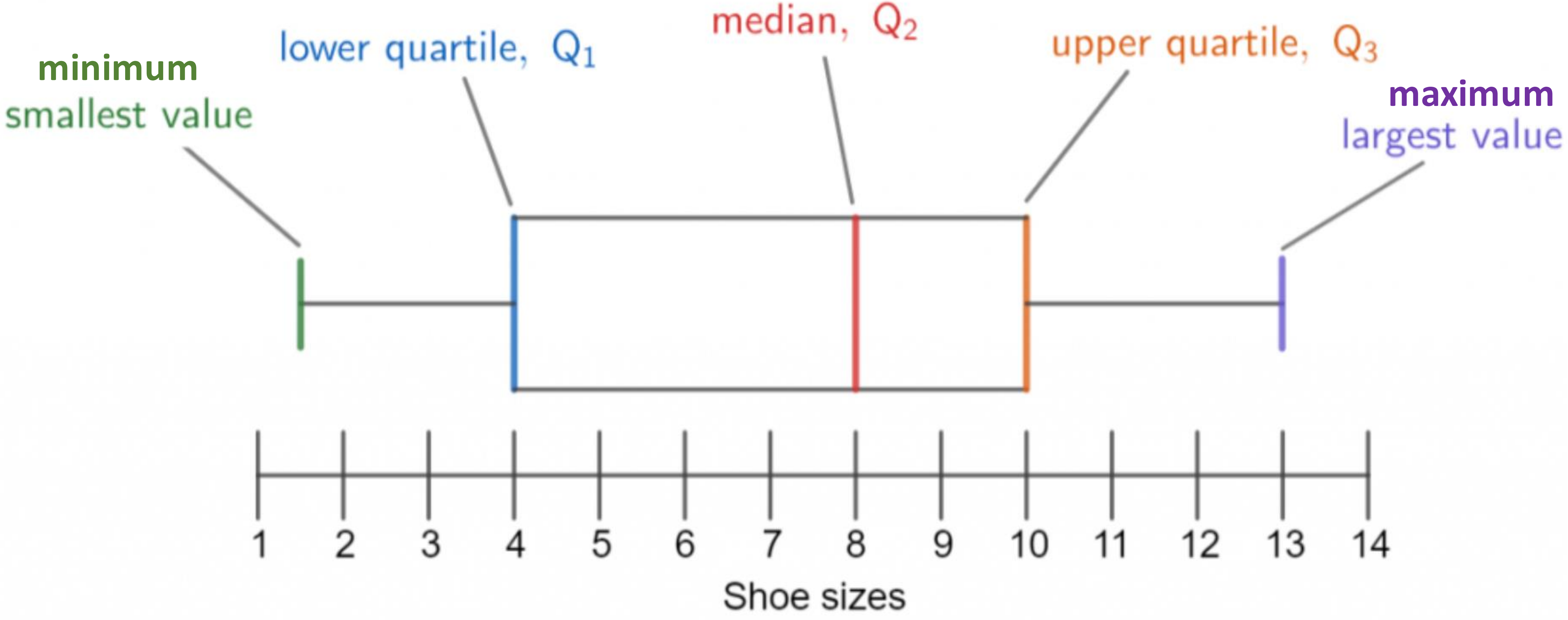
Focus 3: The mathematical meaning of $\frac{a}{b}$, arises in several different mathematical settings, including slope of a line, direct proportion, Cartesian product, factor pairs, and area of rectangles. The mean of $\frac{a}{b}$ for real numbers a and b should be consistent within any one mathematical setting.

Foci

Focus 4: Contextual applications of division or of rates or ratios involving 0 illustrate when division by 0 yields an undefined or indeterminate form and when division of 0 by a nonzero real number yields 0.

Focus 5: Slopes of lines in two-dimensional Cartesian space map to real projective one-space in such a way that confirms that the value of $\frac{a}{b}$ when b=0 is undefined if a $\neq$ 0 and indeterminate if a =0.

Why don’t we know the mean from looking at a box plot or using the five-number summary?
Can we figure it out?


minimum
smallest value
lower quartile, Q_1
median, Q_2
upper quartile, Q_3
maximum
largest value
1 2 3 4 5 6 7 8 9 10 11 12 13 14
Shoe sizes

How would your students fill out the following table? Think about what they know.

	Box plot	Five-number summary
What can we tell for certain looking at a display of this type?		
What can we approximate?		
What can not be determined?		

Mean and Median

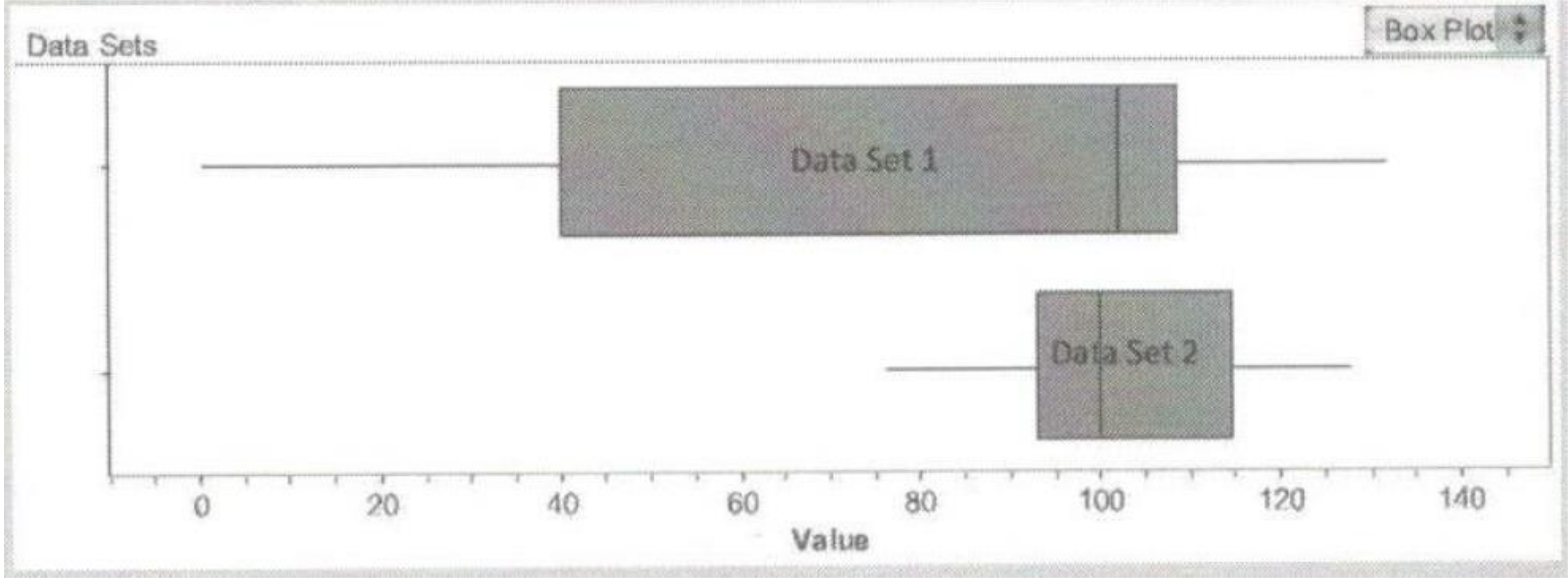
Consider the following box plots and five-number summaries for two finite distributions. Which of the distributions has the greater mean?

What are your first thoughts?

How could a middle school student tackle this problem?

What do they understand about box plots?

Consider the following box plots and five-number summaries for two finite distributions. Which of the distributions has the greater mean?



	min	Q1	median	Q3	max
one	0	40	102	109	132
two	76	93	100	115	128

One student’s approach to this problem was to construct what he thought were probability distributions for each data set and compare the corresponding expected values to determine which set had the greater mean.

Based on that, the mean of data set 2 is greater than that of data set 1.

From the situations book you find these foci.

- Focus 1
  - *The skewness of a data distribution affects the relationship between the mean and median of that set of data.*
- Focus 2
  - *A box plot display of data does not necessarily give the data values of information about the “distribution” of the data within each quarter.*
- Focus 3
  - *When exact values of two data sets are not known, comparisons between the two data sets can sometimes be made by comparing the ranges of their possible values.*
- Focus 4
  - *Stating a definitive conclusion about a comparison of the means using the five-number summaries and box plots is not always possible because the size of the data set may influence the relationship between the means for these distributions.*

Why is it important to have multiple ways to explain a mathematical concept?

Circumscribing Polygons

In a geometry class, after a discussion about circumscribing circles about triangles, a student asked, “Can you circumscribe a circle about any polygon?”

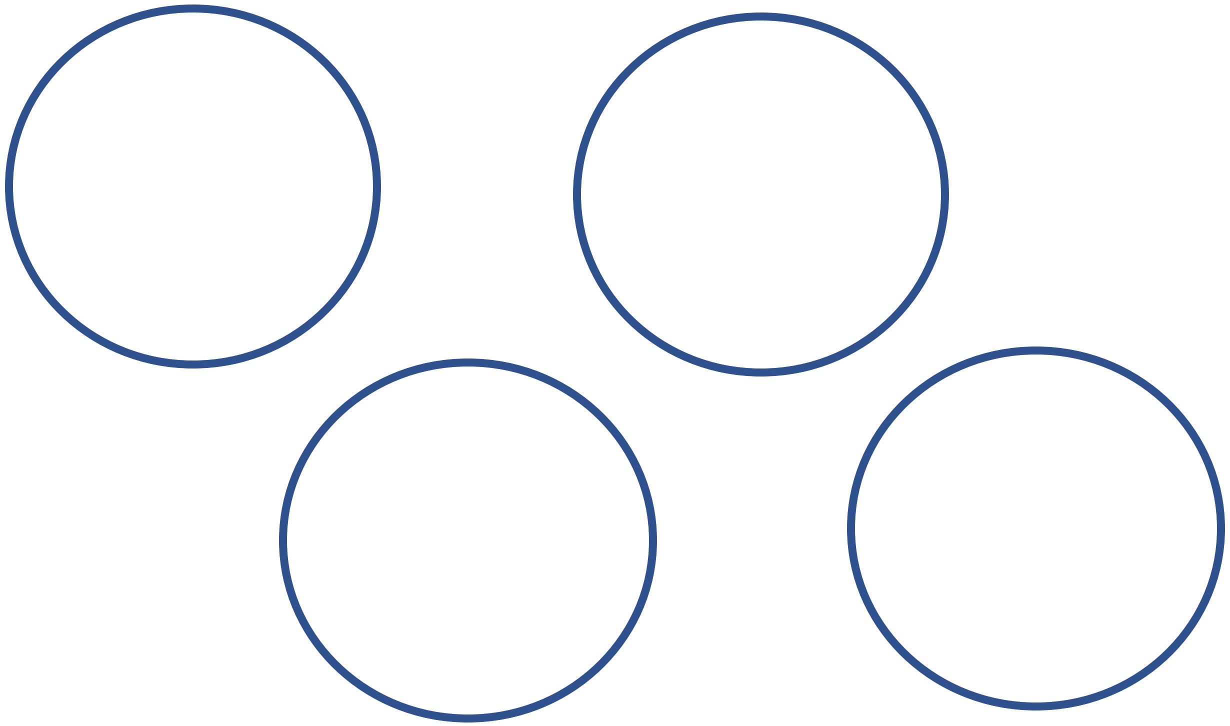
Think about it

- What do you do when questions like this come up?
- Is it better to give a brief answer or begin to explore the ideas?
- How do you decide which ideas are worth more of a time investment?

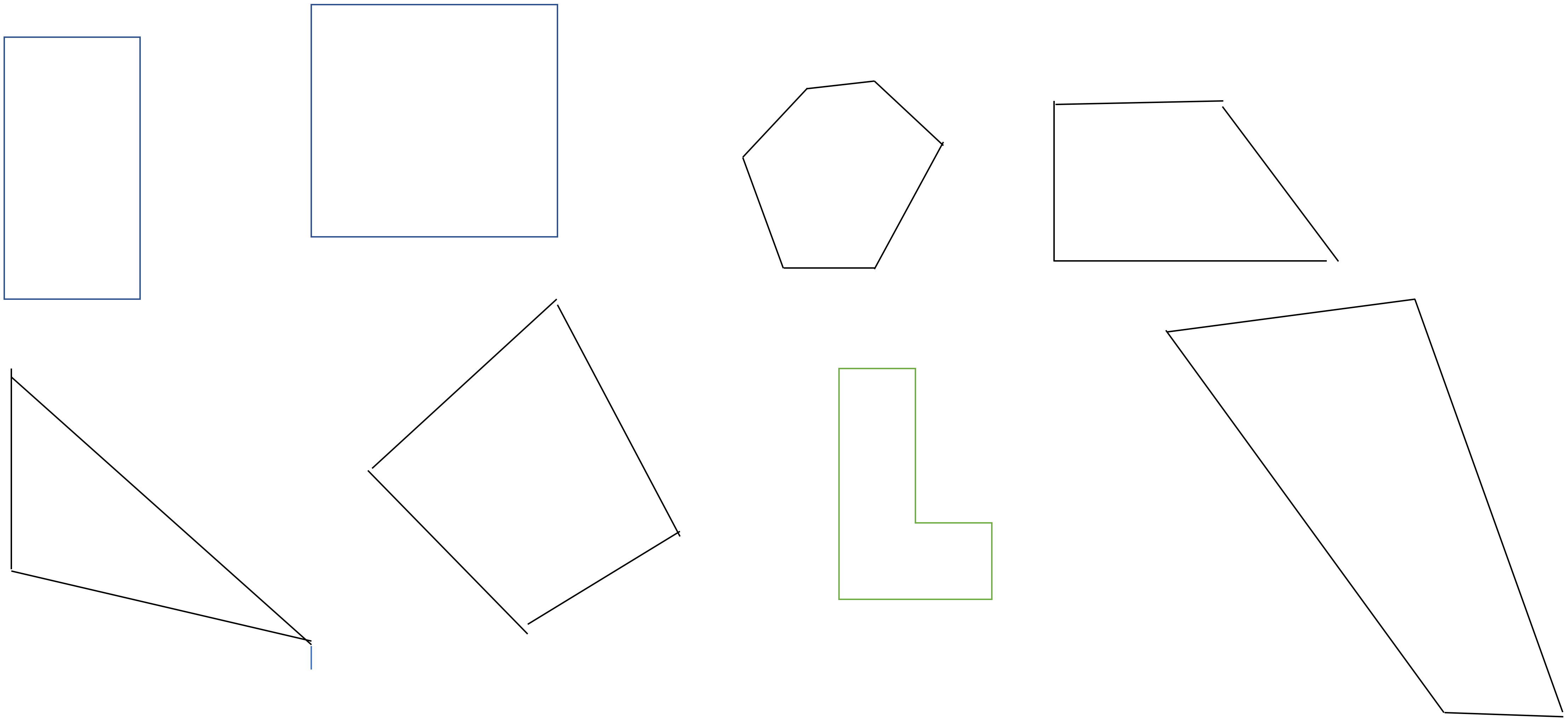
Launch Activity

Use the large congruent circles. Try to inscribe, using a straightedge each of the following types of polygons in a drawn circle:

- triangle,
- quadrilateral,
- rectangle,
- regular hexagon,
- parallelogram,
- trapezoid,
- star, and
- a polygon of their own design.



Try to Circumscribe the following shapes.



What did you learn?
Complete the following chart:

Figure	Can It Be Circumscribed?	Are There Special Conditions?
Triangle		
Scalene Triangle		
Rectangle		
Parallelogram		
Trapezoid		
Regular Polygon		
Concave Polygon		
Irregular Polygon		

So . . .

Can you circumscribe a circle about any polygon?

Product of Two Negative Numbers

Prompt:

A question commonly asked by students in middle school and secondary mathematics class is “Why is it that when you multiply two negative number together you get a positive answer?”

Exploration for Students and Teachers

- Knowing the rule is not enough, how can it be explained.
- Where is it used in mathematics from its introduction through high school?
- Have groups discuss why and report out which explanations are mathematical founded.
- Assign groups different foci to explore and then report out to the entire group.

Foci

Focus 1: Repeated addition suggests that the product of a negative integer and any negative number is a positive number.

Focus 2: Real-world instances that involve adding or subtracting positive or negative amounts can be used to suggest that the product of two negative numbers is a positive number.

Focus 3: Products of negative number can be represented as the composition of two reflections.

Focus 4: The distributive property of multiplication over addition can be used to illustrate and justify that the product of two negative number is a positive number.

Foci

Focus 5: Investigating patterns in real-valued functions yields insight into the product of two negative numbers.

Focus 6: A geometric model based on similar triangles can suggest that the product of two negative numbers is a positive number.

Focus 7: The product of two negative numbers can be shown to be a positive by using properties of the real number system, including the identity and inverse properties.

I cannot teach anybody anything, I can only make them think.

Socrates