



Catalyzing Change: Initiating Critical Conversations in Mathematics Teaching and Learning

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Catalyzing Change Series





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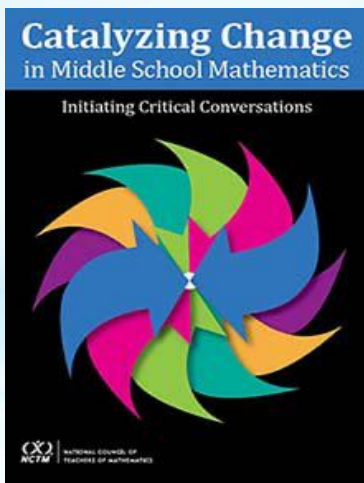
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Catalyzing Change 2020





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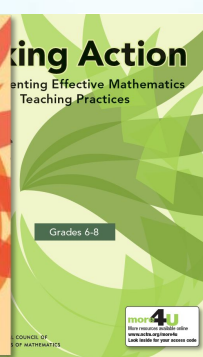
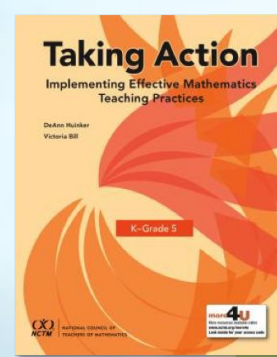
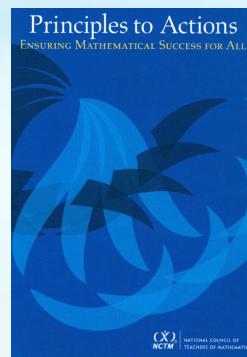
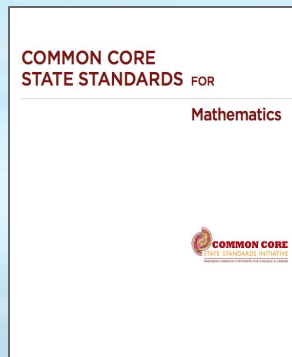
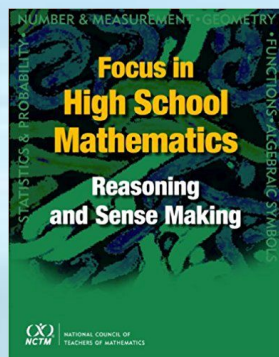
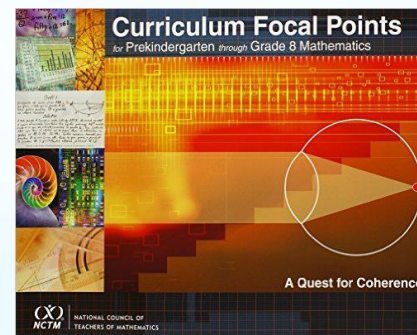
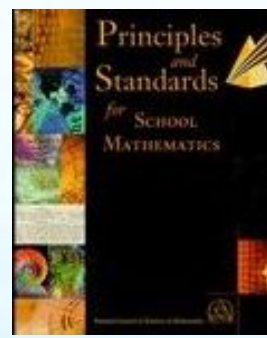
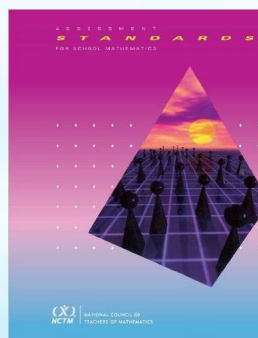
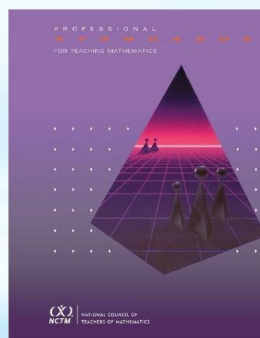
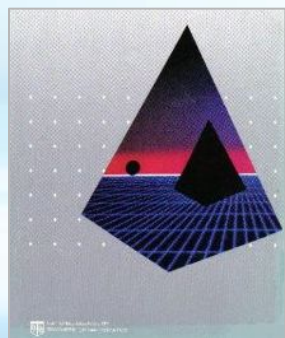
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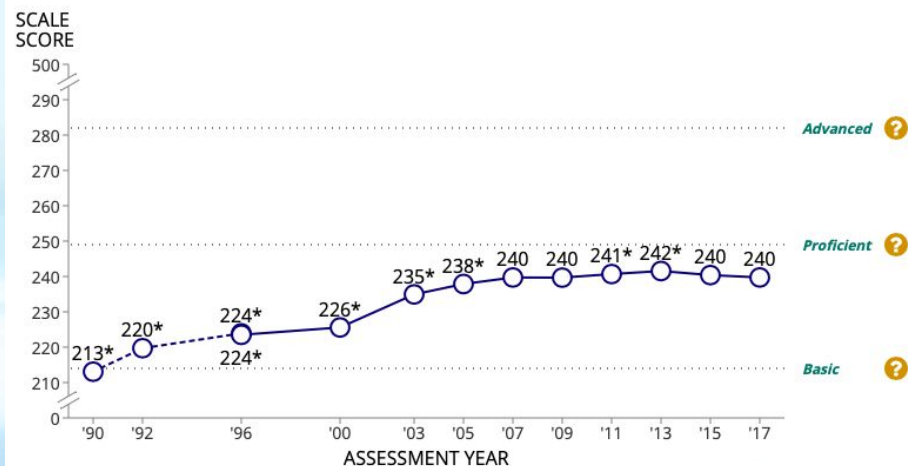


The Last Three Decades Have Seen Significant Progress in the Teaching and Learning of Mathematics

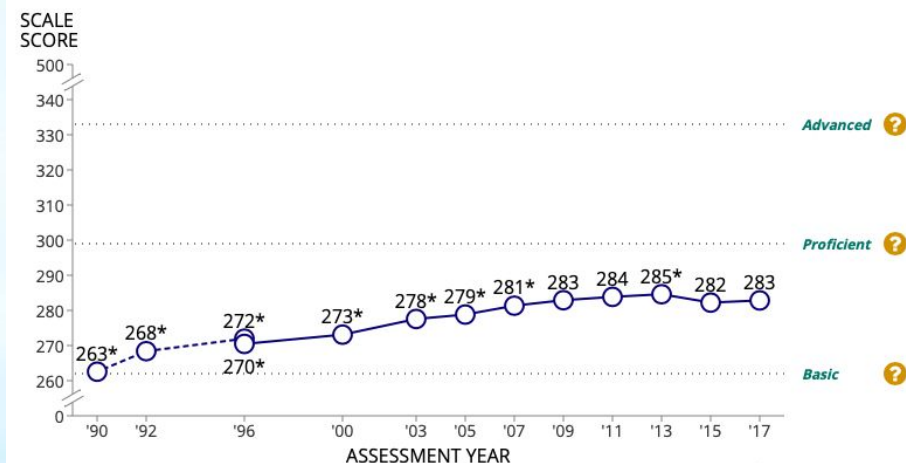


Why Catalyzing Change now? (4th & 8th Grade Data)

Trend in fourth-grade NAEP mathematics average scores



Trend in eighth-grade NAEP mathematics average scores



SOURCE: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), various years, 1990–2017 Mathematics Assessments.



Why we need *Catalyzing Change* in Early Childhood and Elementary Mathematics?

- Children's growth in mathematical knowledge in kindergarten and first grade is a **strong predictor** of later mathematics success.
- Mathematics instruction in early childhood and elementary school places **too much emphasis** on memorizing basic number facts and following procedures at the **expense of** developing deep conceptual understanding.
- Mathematically powerful instruction in early childhood and elementary school is **reaching too few** children, particularly those **most marginalized** in our society, leading to differential and unjust mathematics learning environments and outcomes.

Why we need *Catalyzing Change in Middle School Mathematics*?

- Structures and traditions in mathematics education are **deeply rooted**. We must reconsider legacy practices and structures **impacting students'** mathematical **identities** and sense of mathematical **agency**.
- Instructional practices must be examined in order to systemically support, enhance, and adopt practices that are **equitable and provide high-quality learning opportunities** to motivate and engage students in learning mathematics.
- Mathematical **learning experiences** that engage students in rich investigations reinstate mathematics to its rightful position as a magnet to **STEM**.

Four Key Recommendations

1. Broaden the Purposes of Learning Mathematics
2. Create Equitable Structures in Mathematics
3. Implement Equitable Mathematics Instruction
4. Develop Deep Mathematical Understanding





NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

Catalyzing Change in School Mathematics Key Recommendations



	Early Childhood and Elementary	Middle School	High School
Broaden the Purposes of Learning Mathematics	Each and every child should develop deep mathematical understanding as confident and capable learners; understand and critique the world through mathematics; and experience the wonder, joy, and beauty of mathematics.	Each and every student should develop deep mathematical understanding, understand and critique the world through mathematics, and experience the wonder, joy, and beauty of mathematics, which all contribute to a positive mathematical identity.	Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the wonder, joy, and beauty of mathematics.
Create Equitable Structures in Mathematics	Early childhood and elementary mathematics should dismantle inequitable structures, including ability grouping and tracking, and challenge spaces of marginality and privilege.	Middle school mathematics should dismantle inequitable structures, including tracking teachers as well as the practice of ability grouping and tracking students into qualitatively different courses.	High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.
Implement Equitable Mathematics Instruction	Mathematics instruction should be consistent with research-informed and equitable teaching practices that nurture children's positive mathematical identities and strong sense of agency.	Mathematics instruction should be consistent with research-informed and equitable teaching practices that foster students' positive mathematical identities and strong sense of agency.	Classroom instruction should be consistent with research-informed and equitable teaching practices.
Develop Deep Mathematical Understanding	Early childhood settings and elementary schools should build a strong foundation of deep mathematical understanding, emphasize reasoning and sense-making, and ensure the highest-quality mathematics education for each and every child.	Middle schools should offer a common shared pathway grounded in the use of mathematical practices and processes to coherently develop deep mathematical understanding, ensuring the highest-quality mathematics education for each and every student.	High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts, to ensure the highest-quality mathematics education for all students.



Broadening the Purposes of Learning Mathematics

	Early Childhood and Elementary	Middle School	High School
Broaden the Purposes of Learning Mathematics	Each and every child should develop deep mathematical understanding as confident and capable learners; understand and critique the world through mathematics; and experience the wonder, joy, and beauty of mathematics.	Each and every student should develop deep mathematical understanding, understand and critique the world through mathematics, and experience the wonder, joy, and beauty of mathematics, which all contribute to a positive mathematical identity.	Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the wonder, joy, and beauty of mathematics.
In what ways can individuals in our school and district communicate with educators, families, and children about broadening the multiple purposes of school mathematics and related shifts in learning?			



Creating Equitable Structures in Mathematics

	Early Childhood and Elementary	Middle School	High School
Creating Equitable Structures in Mathematics	Early childhood and elementary mathematics should dismantle inequitable structures, including ability grouping and tracking, and challenge spaces of marginality and privilege.	Middle school mathematics should dismantle inequitable structures, including tracking teachers as well as the practice of ability grouping and tracking students into qualitatively different courses.	High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.
What are the support structures needed to dismantle and disrupt policies, practices, and procedure that limit students' access to high-quality mathematics teaching and curriculum?			



Implementing Equitable Mathematics Instruction

	Early Childhood and Elementary	Middle School	High School
Implementing Equitable Mathematics Instruction	Mathematics instruction should be consistent with research-informed and equitable teaching practices that nurture children's positive mathematical identities and strong sense of agency.	Mathematics instruction should be consistent with research-informed and equitable teaching practices that foster students' positive mathematical identities and strong sense of agency.	Classroom instruction should be consistent with research-informed and equitable teaching practices.
What partnerships should be fostered for educators' professional learning to strengthen understanding of mathematics and equitable mathematics teaching practices?			



Develop Deep Mathematical Understanding

	Early Childhood and Elementary	Middle School	High School
Develop Deep Mathematical Understanding	Early childhood settings and elementary schools should build a strong foundation of deep mathematical understanding, emphasize reasoning and sense-making, and ensure the highest-quality mathematics education for each and every child.	Middle schools should offer a common shared pathway grounded in the use of mathematical practices and processes to coherently develop deep mathematical understanding, ensuring the highest-quality mathematics education for each and every student.	High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts, to ensure the highest-quality mathematics education for all students.

What supports are needed to ensure students' development of the mathematical practices and processes within their daily mathematics instruction?

Broaden the Purposes of Learning Mathematics: Middle School

Develop deep mathematical understanding

“Developing a deep understanding of mathematics and a positive mathematical identity are connected and not mutually exclusive.” (p. 9)

Understand and critique the world through mathematics

“Tasks that honor the uniqueness of middle school students as individuals while also allowing them to experience mathematics in personal and socially meaningful ways are tasks that empower students to maximize the utility of mathematics in a variety of ways to understand and critique the world.” (p. 14)

Experience the wonder, joy, and beauty of mathematics

“Experiencing the joy of mathematics is about a student seeing the mathematics they encounter inside and outside of the classroom, being in the moment with mathematics, and being proud of their mathematical accomplishments, as well as the intrinsic reward a student has from solving mathematical problems.” (p. 16)



NCTM. (2020). *Catalyzing change in middle school mathematics: Initiating critical conversations*. Reston, VA: NCTM.



Broaden the Purposes of Learning Mathematics: Early Childhood & Elementary

Develop deep mathematical understanding as confident and capable learners.

“Children should be positioned with the authority to draw upon their resources (e.g., strategies, tools, and prior experiences) to explore and discuss tasks and delve deeper into the mathematics” (p. 12).

Learn to understand and critique the world through mathematics.

“Understanding and critiquing the world with mathematics should not only raise children’s awareness of social issues but also develop their power with mathematics and their sense of self as mathematical thinkers and doers” (p. 17).

Experience the wonder, joy, and beauty of mathematics.

“Each and every child must be afforded opportunities to not only feel confident as doers of mathematics but also to experience joy and see the beauty in their mathematical discoveries” (p. 17).



NCTM. (2020). *Catalyzing change in early childhood and elementary mathematics: Initiating critical conversations*. Reston, VA: NCTM.

Catalyzing Change: Notice & Wonder

Catalyzing Change in Early Childhood & Elementary

- Naturally curious
- Enter school with rich informal mathematical knowledge
- Many children leave elementary school without the same curiosity and joy
- A just, equitable, and inclusive system for each and every child is needed **and** attainable.

Catalyzing Change in Middle School Mathematics

- Create the highest-quality experiences for students, their teachers, and their families
- Mathematical literacy they require and deserve for both their current and future personal and professional lives.
- Must be equitable, just, and inclusive
- Empowered and inspired students



Create Equitable Structures in Mathematics: Middle School

Barriers to Equitable Structures	Strategies to Build Equitable Structures
Deficit views about students that are deeply rooted in the broader culture of mathematics education and grounded in long-standing structures and practices.	<ul style="list-style-type: none">Engage in this work with colleagues to initiate the confrontation, identification, and examination of deficit-based beliefs about students (or their families and communities).
The inequitable practices of student ability grouping and the tracking of both students and teachers have been prominent since the beginning of the 20th century.	<ul style="list-style-type: none">Immediately stop the use of “deterministic” labels for students, such as “gifted,” “honors,” “college prep,” “regular,” “intensive,” or “remedial.” Similarly, while you may have students who are currently struggling with a concept, avoid labeling students as “struggling.”Recognize that de-tracking students takes time. Make it a long-term goal. The plan should start by de-tracking at the grade level tracking begins in the school or district. Teachers should receive ongoing professional development and support during this process.



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Create Equitable Structures in Mathematics: Early Childhood & Elementary

Barriers to Deep Mathematics Learning	Structures to Access Deep Mathematics Learning
Ability grouping and tracking of children lead to differential learning opportunities that not only widen achievement gaps but also impact how children see themselves in relationship to mathematics learning.	<ul style="list-style-type: none">• Use grouping that avoids “tracking” and provides shared mathematics learning experiences where children interact and support each other and learn from their varied approaches.
High-stakes assessments and readiness measures lead to the labeling and sorting of children, resulting in segregation, marginalization, or privileging that is strongly correlated with race, language, class, and ability status.	<ul style="list-style-type: none">• Use multiple forms of evidence (i.e., observation, conversations, written work) to accurately determine children’s understanding and learning needs.
Curriculum implementation that is not flexible and responsive to local contexts denies children access to rigorous and relevant mathematics learning opportunities.	<ul style="list-style-type: none">• Analyze and enhance alignment between curriculum materials and children’s needs, interests, and lived experiences.



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Let's reflect!

What come to mind for you related to broadening the purposes of mathematics and/or equitable structures?

Enter up to three words.

Use hyphen to count as one word.

Can enter more than once.



Implement Equitable Mathematics Instruction: Middle School

- How will we know that each and every student in our setting is receiving the highest quality mathematics instruction?
- In what ways does this lesson empower students and develop their mathematical agency?
- In what ways are students asked to think about and connect mathematical ideas within and across multiple representations?
- In what ways are students provided time and opportunities for meaningful discourse as they develop their mathematical ideas?
- What actions are we taking to ensure there is shared mathematical authority in our classrooms?
- How is our mathematics instruction building on students' strengths, developing who students are as young adolescents, and who they will become as adults?

Catalyzing Change
in Middle School Mathematics
Initiating Critical Conversations



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Implement Equitable Mathematics Instruction: Early Childhood & Elementary

Mathematical identity is . . .

- A view of oneself as a doer, knower, and sense maker of mathematics.
- A deeply held belief about one's own ability to engage successfully with mathematics.
- Shaped by children's mathematics learning experiences and interactions with peers and adults.
- Affected by beliefs about the nature of mathematics and the learning of mathematics.

Mathematical agency is . . .

- One's mathematical identity in action both inside and outside the classroom.
- Revealed in one's confidence, capacity, and willingness to engage mathematically.
- Shaped by children's opportunities to choose, use, and discuss their own strategies.
- Affected by the positioning of children as capable of working through mis-steps and confusions.

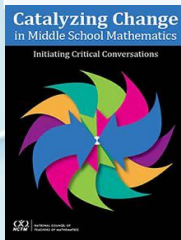


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Develop Deep Mathematical Understanding: Middle School

Strands of Mathematical Proficiency (NRC 2001)	Standards for Mathematical Practice (NGA Center and CCSSO 2010)	Process Standards (NCTM 2000)
<ul style="list-style-type: none"> • Conceptual Understanding • Procedural Fluency • Strategic Competence • Adaptive Reasoning • Productive Disposition 	<ol style="list-style-type: none"> 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning 	<ul style="list-style-type: none"> • Problem Solving • Reasoning and Proof • Communication • Connections • Representation

“Engaging students in the mathematics of relevant, often sensitive or controversial topics, requires careful attention and thoughtful implementation, but should and needs to be a part of students’ middle school mathematical learning experience.” (p. 68)



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Develop Deep Mathematical Understanding: Middle School

Emphasize More of This

Number: Highlight and explore the connections among the structures, properties, relationships, operations, and representations of number systems.

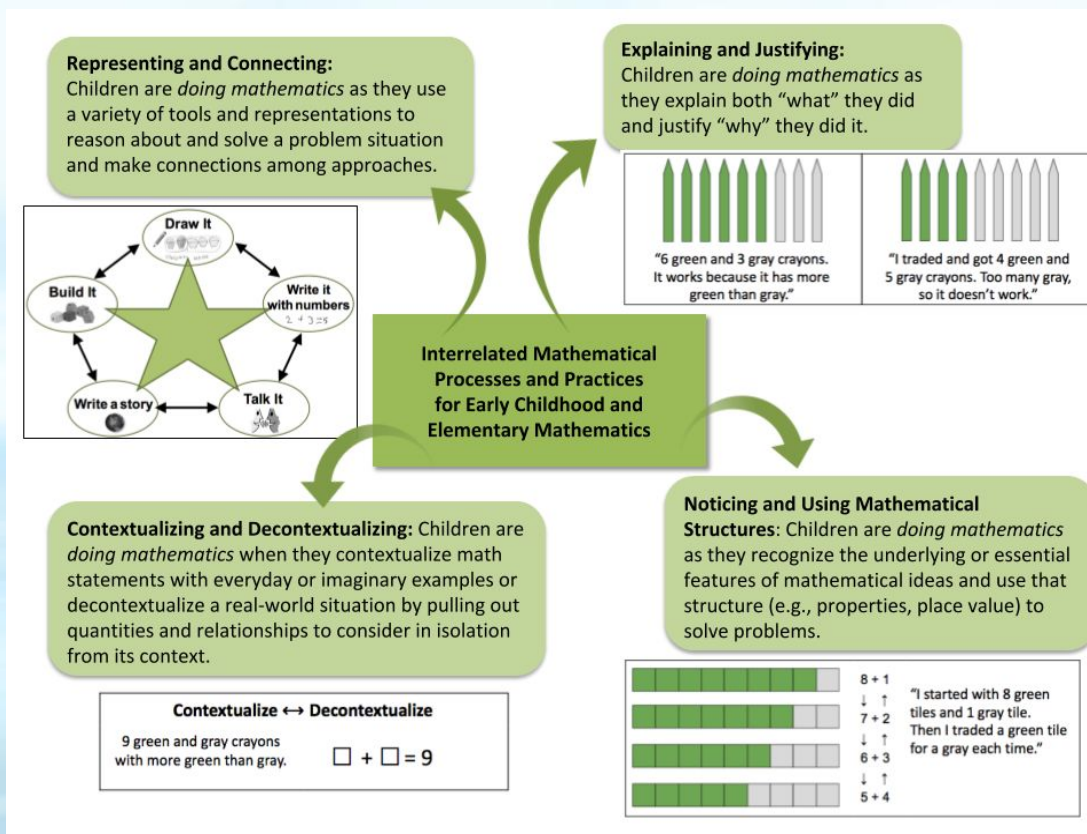
Ratio & Proportion: As students become more comfortable reasoning proportionally and understanding the relationships as multiplicative, they can build comprehension of finding a unit rate, connect that to the proportional relationship, and understand the idea of constant of proportionality.

Algebra & Functions: Students should engage in key mathematical ideas, including writing, interpreting, using, and evaluating algebraic expressions and equations; developing an understanding of linear equations that includes systems of equations and work with relationships in bivariate data; and understanding the concept of a function that includes the ability to identify those that are linear and those that are nonlinear.

Statistic & Probability: Students should develop an understanding of statistical variability, an ability to summarize and describe distributions for both categorical and quantitative variables, the skill to compare two or more groups with respect to the distribution for a categorical variable or for a quantitative variable, and the capability to investigate patterns of association in bivariate categorical or bivariate quantitative data.

Geometry & Measurement: Students should experience geometry and measurement in a manner that is integrated and active. Such experiences could investigate building and design; art and aesthetics; visualization; everyday applications of distance, angles, area, surface area, and volume; and transformations.

Develop Deep Mathematical Understanding: Early Childhood & Elementary Children as *Doers* of Mathematics



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Develop Deep Mathematical Understanding: Early Childhood & Elementary

Children as *Knowers and Sense Makers* of Mathematics

More of This

Whole Number Concepts and Operations

- Weave number and operation sense into the culture of the mathematics classroom.
- Foster flexibility in reasoning with number and operation relationships.
- Use subitizing activities across the grades to develop quantitative relationships.
- Provide opportunities to learn basic number combinations through sense making, not memorization.
- Support transitions from additive to multiplicative thinking.

Fraction Concepts and Operations

- Use unit fractions as the building blocks for developing fraction knowledge.
- Emphasize fractions as numbers whose magnitude can be represented on a number line.
- Focus on real-world contexts for understanding fraction operations conceptually.

Early Algebraic Concepts and Reasoning

- Develop meaning for the equals sign as stating two expressions have the same value.
- Discuss observations and intuitions about the properties and behaviors of operations.
- Find opportunities for algebraic thinking across the mathematics curriculum.

Data Concepts and Statistical Thinking

- Emphasize data analysis as describing the variability within our world.
- Allow for creation of data displays to organize, analyze, and communicate information.
- Use data distributions to answer questions and pose further questions.

Geometry and Measurement Concepts and Spatial Reasoning

- Develop spatial reasoning as an essential core of children's mathematical development.
- Build from children's thinking to co-construct meaning for attributes of two- and three-dimensional geometric shapes.
- Provide opportunities to examine measurable attributes of shapes and quantify "how much" objects possess.



Let's reflect!

What come to mind for you related to equitable instruction and/or developing deep mathematical understanding ?

Enter up to three words.

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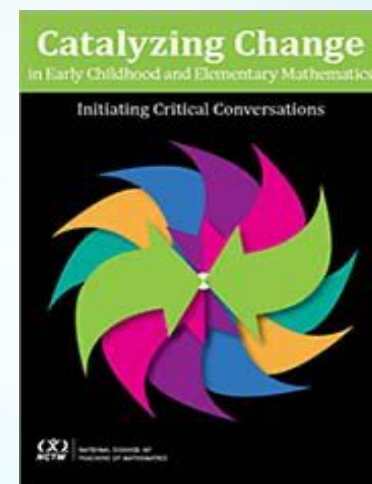
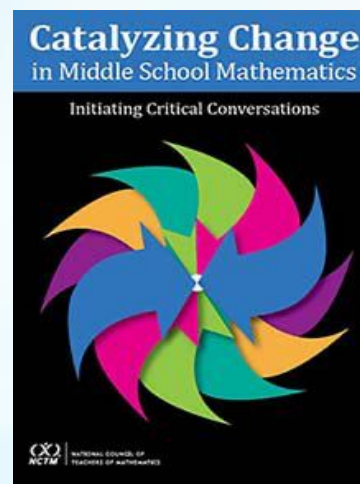
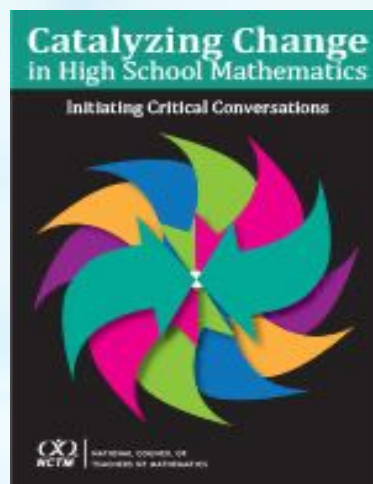
The *Catalyzing Change* series is intended to initiate critical conversations centered around essential areas to ensure the highest-quality mathematics education for each and every student.

What will be your next steps? What will be our next steps?

We must work together on this journey.

Catalyzing Change Series

- Ordering information
- Books
- Resource Guides
- Book study guides
- Case studies
- And more



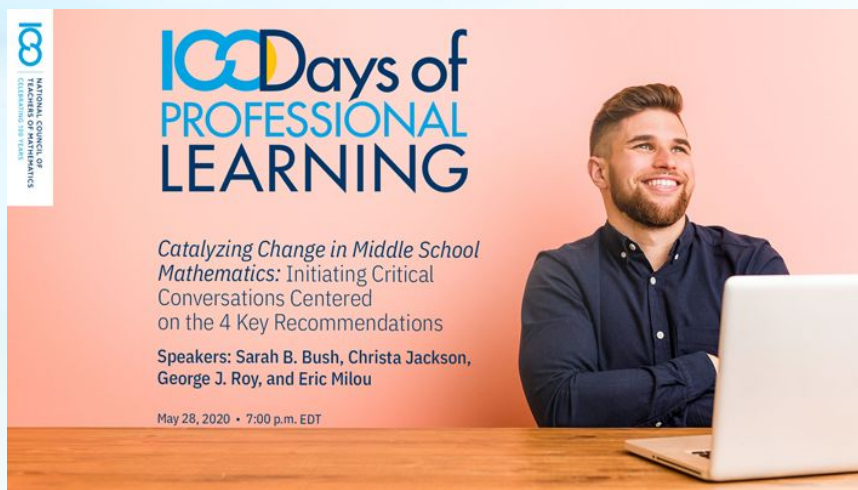
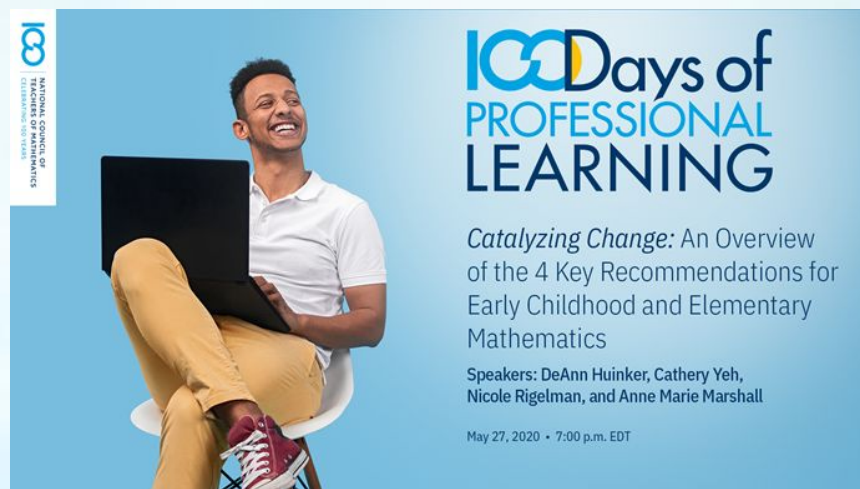
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Catalyzing Change Webinars

- The *Catalyzing Change* series empowers you to lead critical conversations to improve math education and student readiness.
- We all have a stake in the future of mathematics





Thank You!

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