The Five Practices in Practice: Orchestrating Productive Mathematics Discussions in High School

NCTM 2020 100 Days of Professional Learning
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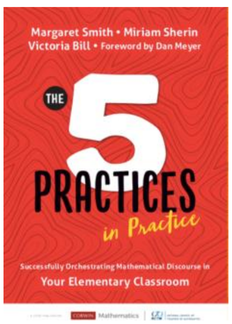

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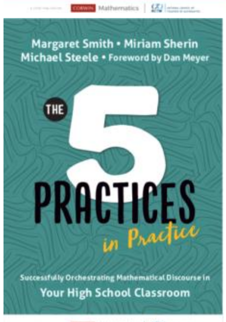

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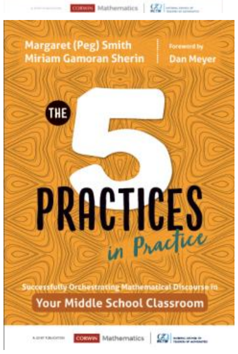

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Challenges Associated with Implementing the Five Practices

Setting Goals and Selecting Tasks	1.	Identifying learning goals
	2.	Identifying a doing-mathematics task
	3.	Ensuring alignment between task and goals
	4.	Launching a task to ensure student access

Anticipating Student Responses	5.	Moving beyond the way *you* solve a problem
	6.	Being prepared to help students who cannot get started on a task
	7.	Creating questions that move students toward the mathematical goals

Monitoring Student Work	8.	Trying to understand what students are thinking
	9.	Keeping track of group progress—which groups you visited and what you left them to work on
	10.	Involving all members of a group

Selecting and Sequencing Student Solutions	11.	Selecting only solutions that are most relevant to learning goals
	12.	Expanding beyond the usual student presenters
	13.	Deciding what work to share when the majority of students were not able to solve the task and your initial goal no longer seems obtainable
	14.	Moving forward when a key strategy is not produced by students
	15.	Determining how to sequence incorrect and/or incomplete solutions

Connecting Student Solutions	16.	Keeping the entire class engaged and accountable during individual presentations
	17.	Ensuring key mathematical ideas are made public and remain the focus
	18.	Making sure that you do not take over the discussion and do the explaining
	19.	Running out of time

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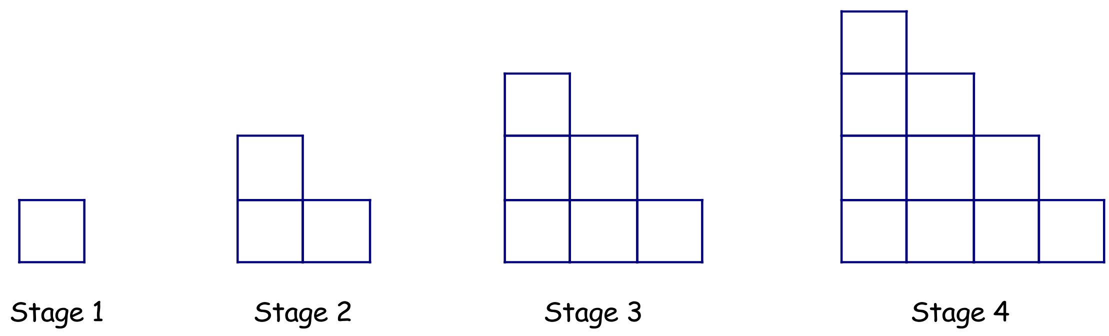
Staircase Problem

The first four stages of a pattern are shown below.



1. Determine the number of small squares in each of the first four stages.

2. Draw the fifth stage and determine the number of small squares in that stage.

3. Determine the number of small squares in the tenth stage without drawing or building it.

4. What function describes the relationship between the total number of small squares and the stage number? Explain how you know.

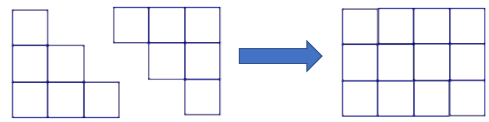
Ms. Moran's Anticipated Solutions to the Staircase Problem

A. Identify the Recursive Pattern

Stage number	Number of squares	Number of squares added to get next stage
1	1	
2	3	1 + 2 = 3
3	6	3 + 3 = 6
4	10	6 + 4 = 10

Student uses a table or visual inspection of staircases to determine that the number of squares in a stage is the number of squares in the previous stage plus the stage number.

B. Double the Staircase to Create a Rectangle



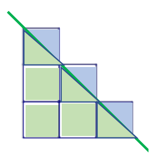
Stage 3 Stage 3 (rotated 180°)

Student creates the $n \times (n+1)$ rectangle by putting two copies of the same staircase together.

Student finds the area of the original staircase by taking $\frac{1}{2}$ of the area of the resulting rectangle.

Area of staircase in Stage $n = \frac{n(n+1)}{2}$

C. Divide the Staircase Into Triangles



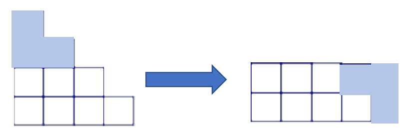
Student draws a diagonal through the staircase to create one large triangle and several smaller triangles.

Student uses $A = \frac{1}{2}(bh)$ to determine the area of a large triangle. Since base and height are the same as the stage number, $A = \frac{1}{2}n^2$.

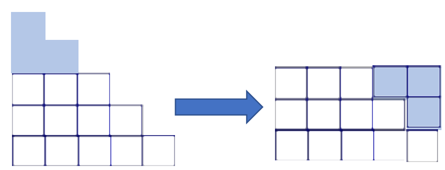
Student determines that the area of each small triangle is $\frac{1}{2}$ and that the number of small triangles is the same as the stage number. This means that the combined area of the small triangles is $\frac{1}{2}n$.

Total area = $\frac{1}{2}n^2 + \frac{1}{2}n$

D. Rearrange the Staircase Into a Rectangle



Stage 4 L = ½n, w = n+1



Stage 5 L = ½(n+1), w = n

Student rearranges the squares in a staircase to create a rectangle. Student determines the area of the original staircase by calculating the area of the resulting rectangle. Student uses a slightly different approach for even-number stages and for odd-number stages.

E. Suggest Growth Is Exponential

Student recognizes that the difference in the number of squares in each stage is not constant and that the growth is not linear.

Because the amount of each staircase keeps getting bigger, the student decides that the growth must be exponential.