Place Value – Selected Solutions

1. (a) Any power of 6 is used at most 5 times. This can be thought of in general as $5(6^P)$. If the power of 6 was used 6 times, it would be written as $6(6^P)$. But this could be rewritten as 6^{P+1}, which is the next highest power of P.

(b) In the repeated division method, you divide by base b. Here we divide by 6. When dividing by 6 the maximum value of the remainder is 5. If the remainder is more than 5, one or more groups of 6 remain in the quotient.

3.

n	binary (base-2)	base-4	base-8
104	1101000	1220	150
105	1101001	1221	151
106	1101010	1222	152
107	1101011	1223	153
108	1101100	1230	154
109	1101101	1231	155
110	1101110	1232	156
111	1101111	1233	157
112	1110000	1300	160

4. $2006 = 2668_9$

5. If N is odd, then d_1 is 1. This is because all previous factorials have a factor of 2 in the product which would produce even addends. To make the sum odd, a 1 must be added.

$2005 = 243201_C$. 2005 divided by 6! is **2** with a remainder of 565. 565 divided by 5! is **4** with a remainder of 85. 85 divided by 4! is **3** with a remainder of 13. 13 divided by 3! is **2** with a remainder of 1. 1 divided by 2! is **0** with a remainder of 1. 1 divided by 1! is **1** with no remainder. Here it is the quotients that are needed to form the factorial representation. $3005 = 410021_C$, $4005 = 531311_C$, $5005 = 653201_C$

6.

A	*B*	*C*	*D*	*E*	*F*
13	*100000*	*100000*	*201*	*111*	*11101*
14	*21*	*100001*	*210*	*112*	*111110*
15	*102*	*100010*	*211*	*120*	*111111*

7. Column C in the previous question shows the first 15 Fibonacci numbers.

8. 11000001_f

Challenge 1: $2006 = 23222_{-6}$ This one will challenge most students. When thinking about the repeated subtraction method, students need to realize that even powers will result in positive addends and odd powers will result in negative addends. Therefore, start with the first positive power of n that is less than 2006. This is $(-6)^4$. Using $2(-6)^4 = 2592$ is too large, but that will be remedied when using $(-6)^{-3}$. 2592 is 586 more than 2006. We need to add a value that is -586 or smaller. That would mean adding $3(-6)^{-3}$. So far we have 2592 + -648 or 1944. We need to add 62 to get exactly 2006. This process of going above and below the targeted base 10 number will eventually result in the base -6 representation.

Challenge 2:(a) To change from base 2 to base 4, group each pair of binary digits. Determine the base 10 representation for each pair of binary digits. For example, 110011_2 becomes 303_4 when grouped as 11, 00 and 11. This is because 11 in binary is the same as 3 in base 10. A similar process can be done to change from base 2 to base 8, but instead of pairing digits, they are grouped by threes. For example 110011_2 becomes 63_8. This is because 110 in binary is 6 in base 10 and 011 in binary is 2 in base 10.

(b) Take each base 4 digit and write it as its two binary equivalent. For example, 12_4 becomes 0110_2. This is because 1 is binary for 1 in base 4 and 10 is binary for 2 in base 4.

(c) Take each base 8 digit and write it as its three digit binary equivalent. For example, 421_8 becomes 100010001_2. This is because 4 is 100 in binary 2 is 010 in binary and 1 is 001 in binary.

Challenge 3: $a = 2$, $b = 0$, $c = 0$, $d = 5$