Flattening the Curve: Understanding R_0

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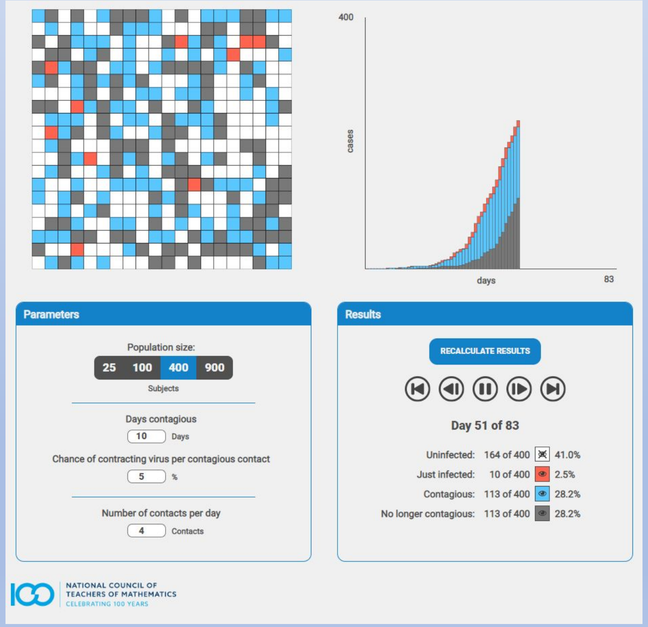
https://www.nctm.org/tools/pandemic2020/index.html



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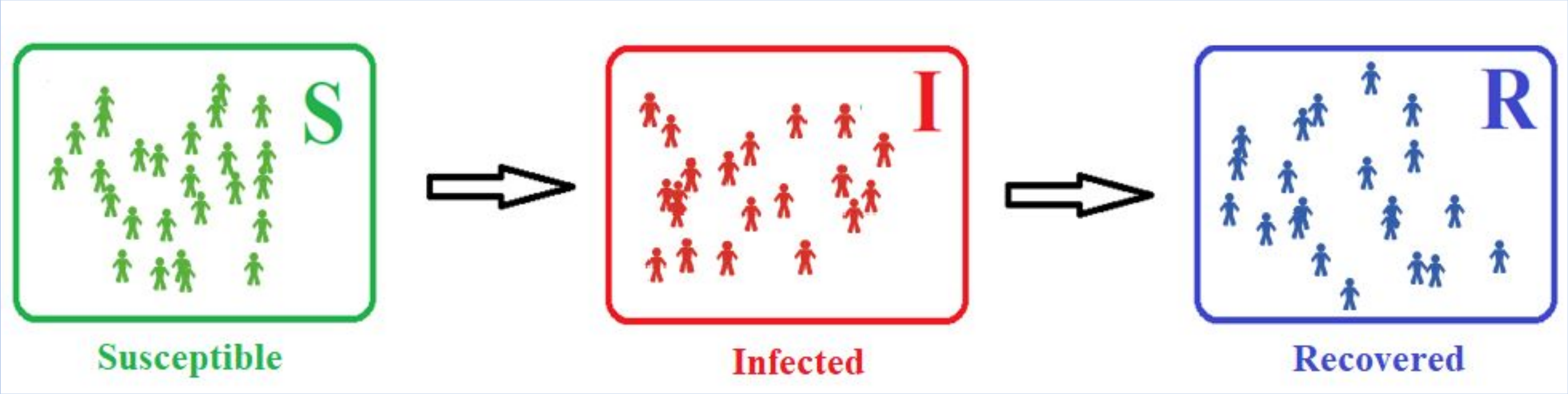
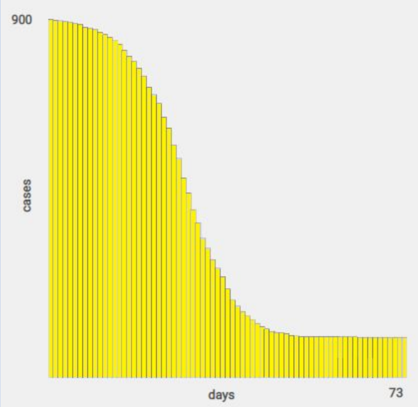
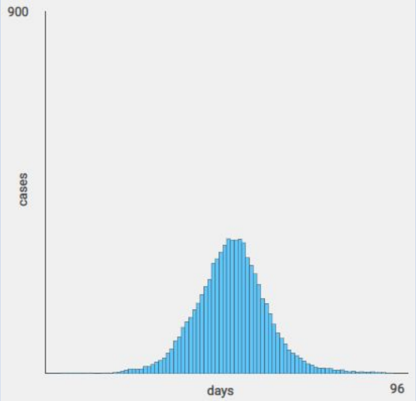
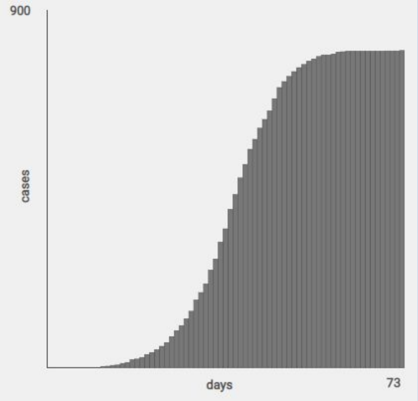
The Classic SIR Model









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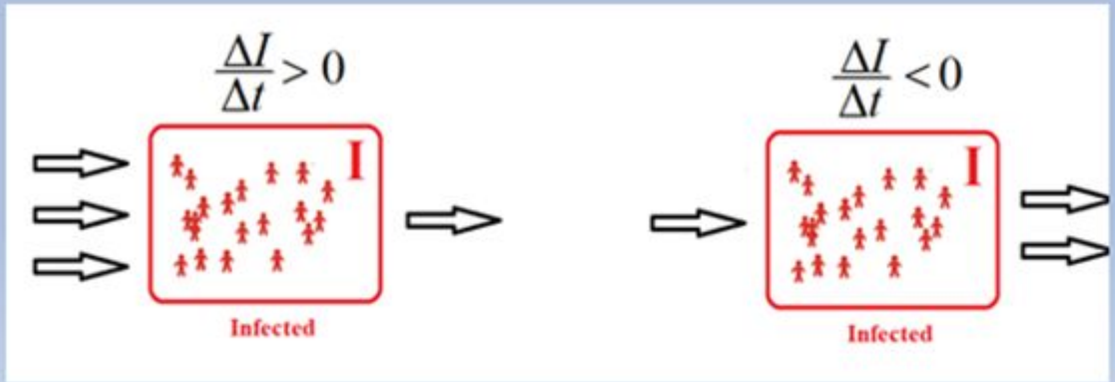
Differential Equations Model

$$\frac{dS}{dt} = -\beta\left(S \cdot I\right)$$

$$S + I + R = 1$$

$$\frac{dI}{dt} = \beta\left(S \cdot I\right) - \gamma I$$

This is the essential equation to understand



$$\frac{dR}{dt} = \gamma I$$

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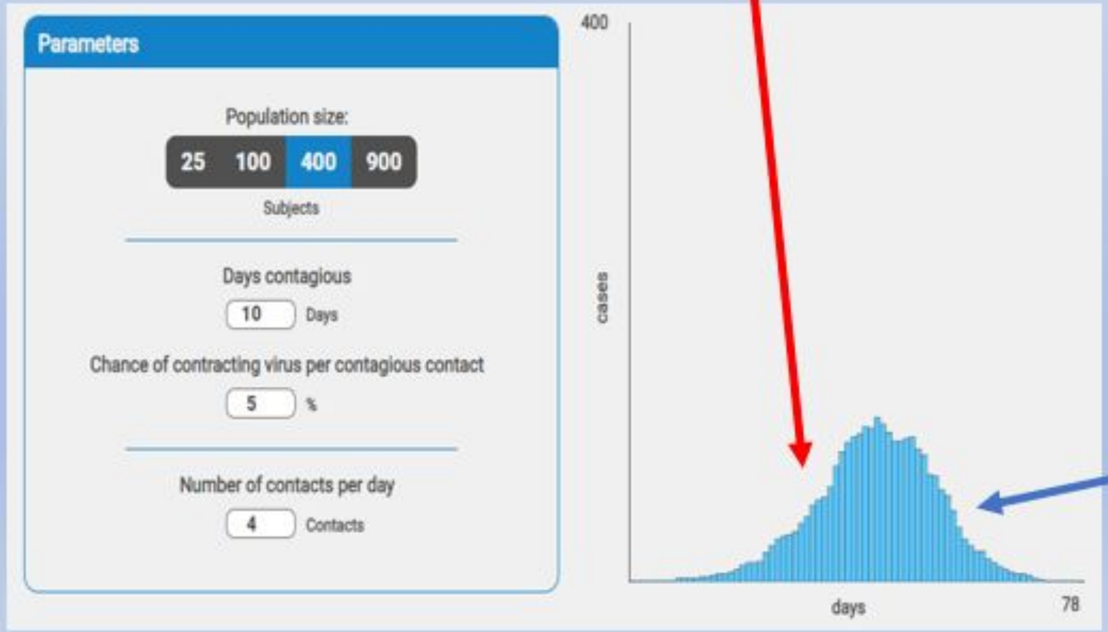
$$\frac{\Delta I}{\Delta t} \approx \beta(S \cdot I) - \gamma I \text{ is the important expression}$$

If $\beta(S \cdot I) - \gamma I > 0$, the number currently ill is increasing



What makes $\beta(S \cdot I) - \gamma I < 0$?

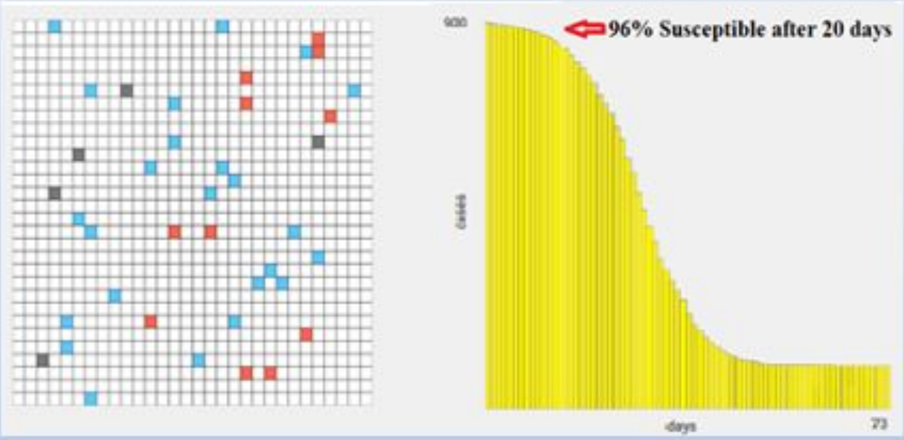
What makes $\beta(S \cdot I) - \gamma I < 0$?

$$\beta(S \cdot I) - \gamma I < 0$$

$$\beta(S \cdot I) < \gamma I$$

$$\beta S < \gamma$$

$$\frac{\beta \cdot S}{\gamma} < 1$$



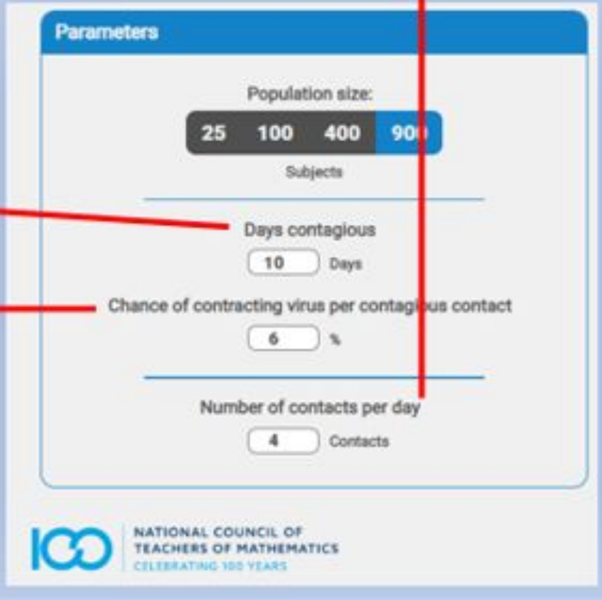
Early in the epidemic, the fraction of the population that is Susceptible is almost 1.

$\frac{\beta}{\gamma} = R_0$ The basic reproduction number.

The basic reproduction number, R_0, is interpreted as the average number of people infected by an Infective individual.

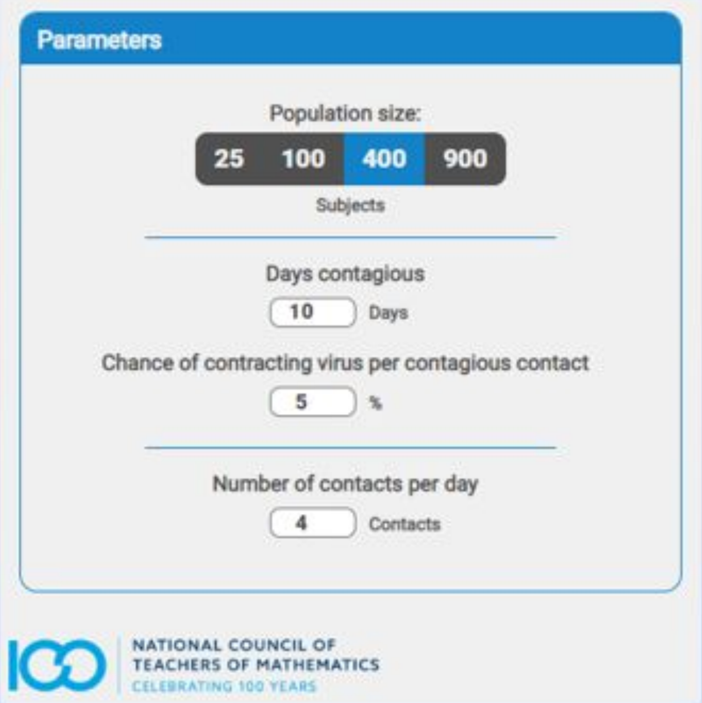
$$\beta = \underbrace{\text{Transmission Rate}}_{\text{Probability of Transfer per Contact}} \cdot \underbrace{\text{Average number of contacts}}_{\text{Contacts per Day}}$$

$$\gamma = \text{Recovery Rate} \approx \frac{1}{\text{Average length of illness}}$$



Interpreting $R_0 = \frac{\beta}{\gamma}$

$$R_0 = \frac{\beta}{\gamma} = \frac{\text{Transmission Rate} \cdot \text{Average number of contacts}}{\left(\dfrac{1}{\text{Average length of illness}}\right)}$$



$$R_0 = \underbrace{\text{Transmission Rate}}_{\substack{\text{probability per contact} \\ \downarrow \\ 0.05}} \cdot \underbrace{\text{Average number of contacts}}_{\substack{\text{contacts per day} \\ \downarrow \\ 4}} \cdot \underbrace{\text{Average length of illness}}_{\substack{\text{days} \\ \downarrow \\ 10}}$$

How do we make $R_0 < 1$?

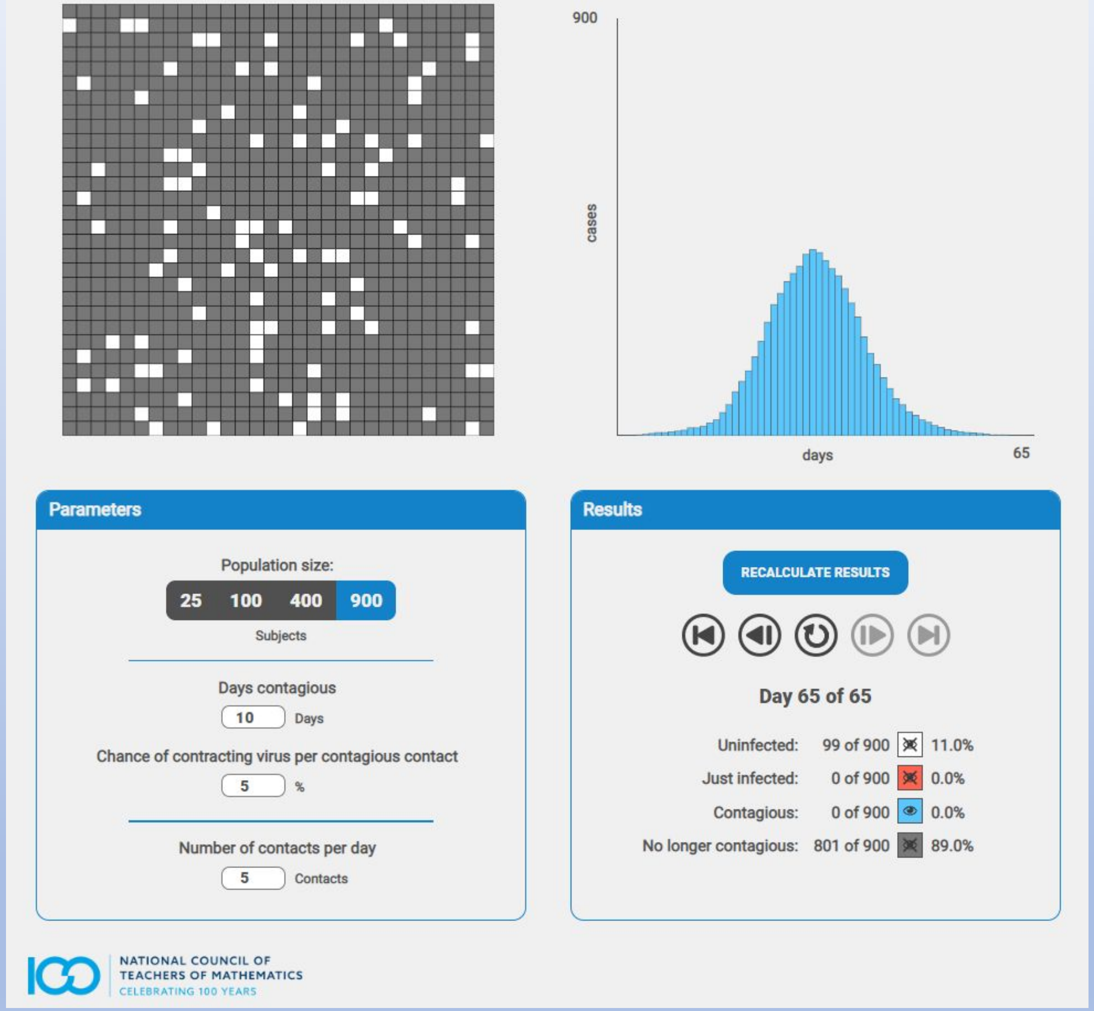
$$R_0 = \underbrace{\text{Transmission Rate}}_{\substack{\text{probability per contact} \\ \downarrow \\ 0.05}} \cdot \underbrace{\text{Avg number of contacts}}_{\substack{\text{contacts per day} \\ \downarrow \\ 4}} \cdot \underbrace{\text{Avg length of illness}}_{\substack{\text{days} \\ \downarrow \\ 10}}$$

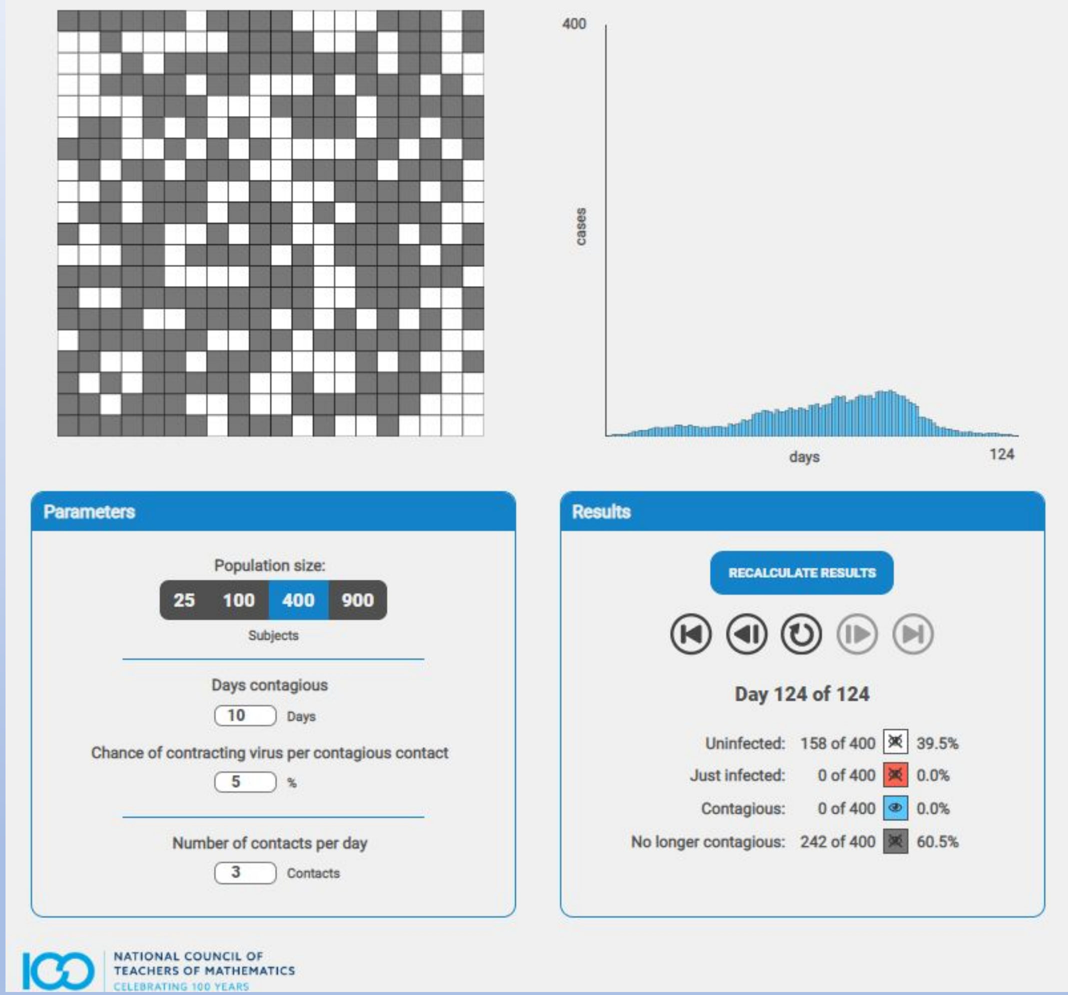
Transmission Rate can be reduced by washing hands, wearing masks, etc.

Average Length of Illness cannot be modified at this time (Tamiflu, isolation)

Reducing the average number of virus passing contacts is the only viable option:

Stay Home





- $R_0 = 2.5$ $\qquad\qquad$ $R_0 = 1.5$

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NCTM applet

This math simulation models the spread of a virus through social contact.

The Spread of Virus Simulation has four variable you can control.

Population Size:	Each small square represents a person

Virus variables include

Days Contagious	For the coronavirus this varies between 2 and 14 days before significant signs appear. A selection of 6-10 seems reasonable to start.
Chance of Contracting Virus per Contagious Contact	This is how likely an uninfected person is to catch the virus when contacted by a contagious person. This value is not known for coronavirus. Something between 10% and 4% may be reasonable.

Social variable is one we can control.

Number of Contacts per Day	This represents the number of contacts, other people, each person contacts per day. While in normal situations this could likely be in the 100s for students. In this setting between 10 and 2 seem reasonable.

NOTE: This is a simulation and it has been simplified to a small number of variables. These variables are applied uniformly across each person in the population. Real life is much more variable and complicated. Our goal is to help learners of all ages better understand how our actions impact the collective result and how mathematics can be used to better understand our world.

Each time you click CALCULATE RESULTS or RECALCULATE RESULTS the simulation recalculates. For some cases the spread ends quickly. This is by chance given the variables. Click RECALCULATE RESULTS again for a new simulation. You may need to run multiple times to get a sense of the variability and impact of the settings. This simulation/model is not predictive but is intended to show how our actions can impact our community.

Parameters

Population size:
25 | 100 | 400 | 900
Subjects

Days contagious
10 Days

Chance of contracting virus per contagious contact
6 %

Number of contacts per day
4 Contacts

Results

CALCULATE RESULTS

Day -- of --

Uninfected:	–		–%
Just infected:	–		–%
Contagious:	–		–%
No longer contagious:	–		–%

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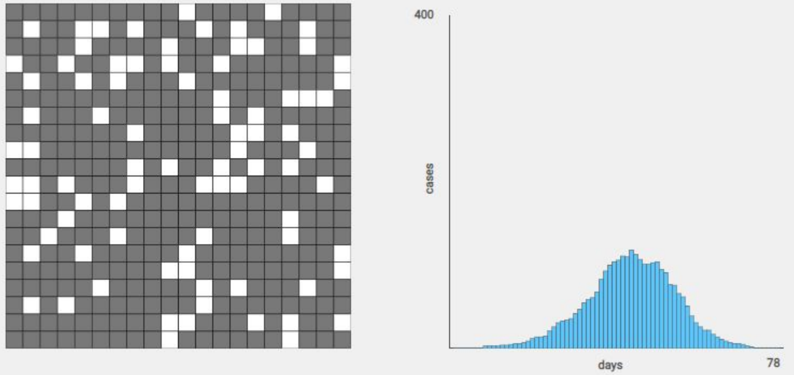

Parameters

Population size:
25 | 100 | 400 | 900
Subjects

Days contagious
10 Days

Chance of contracting virus per contagious contact
5 %

Number of contacts per day
4 Contacts

Results

RECALCULATE RESULTS

Day 78 of 78

Uninfected:	86 of 400	21.5%
Just infected:	0 of 400	0.0%
Contagious:	0 of 400	0.0%
No longer contagious:	314 of 400	78.5%

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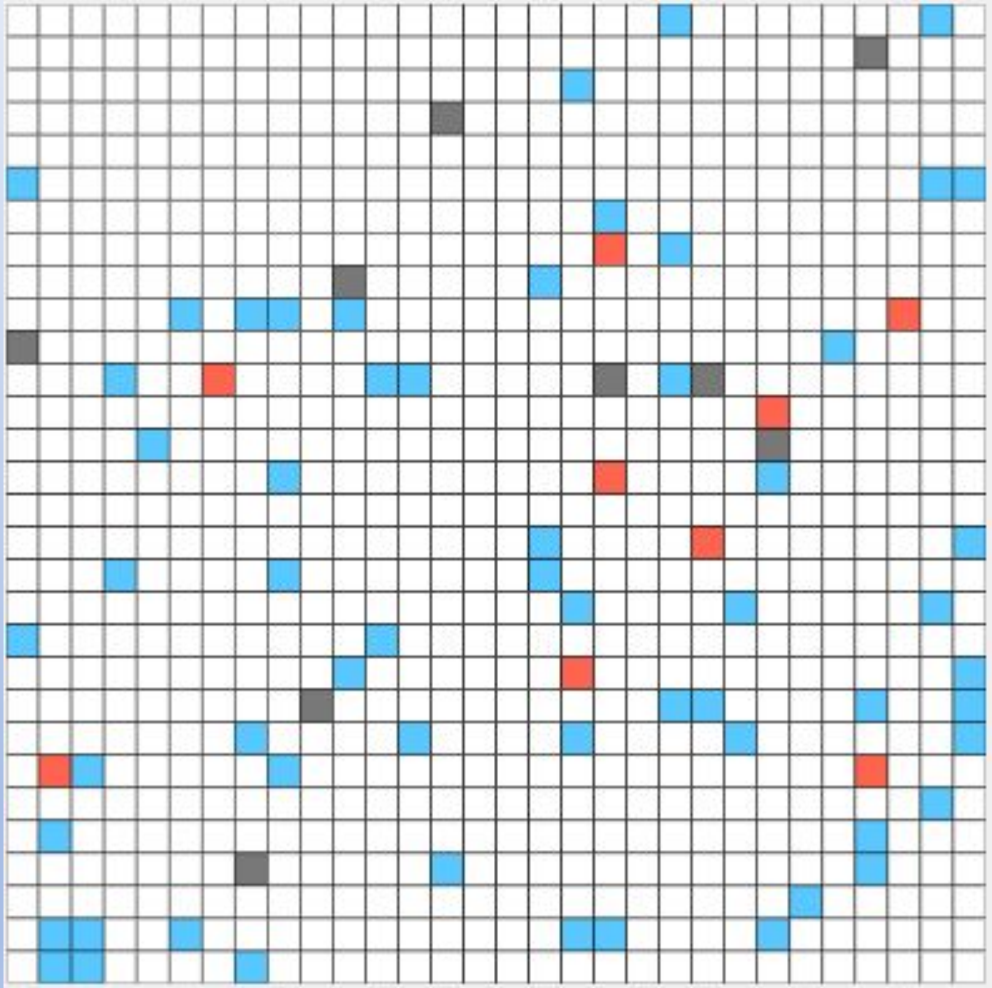
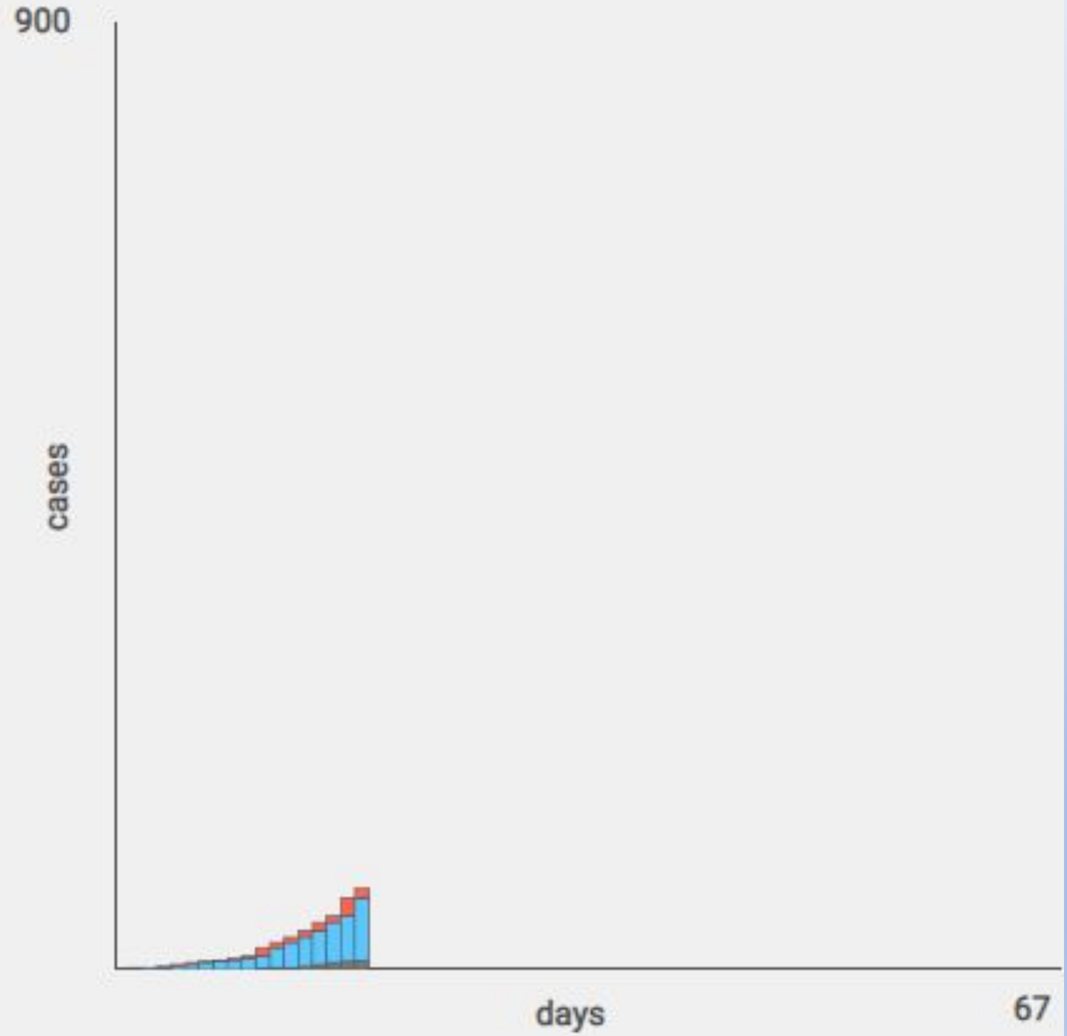
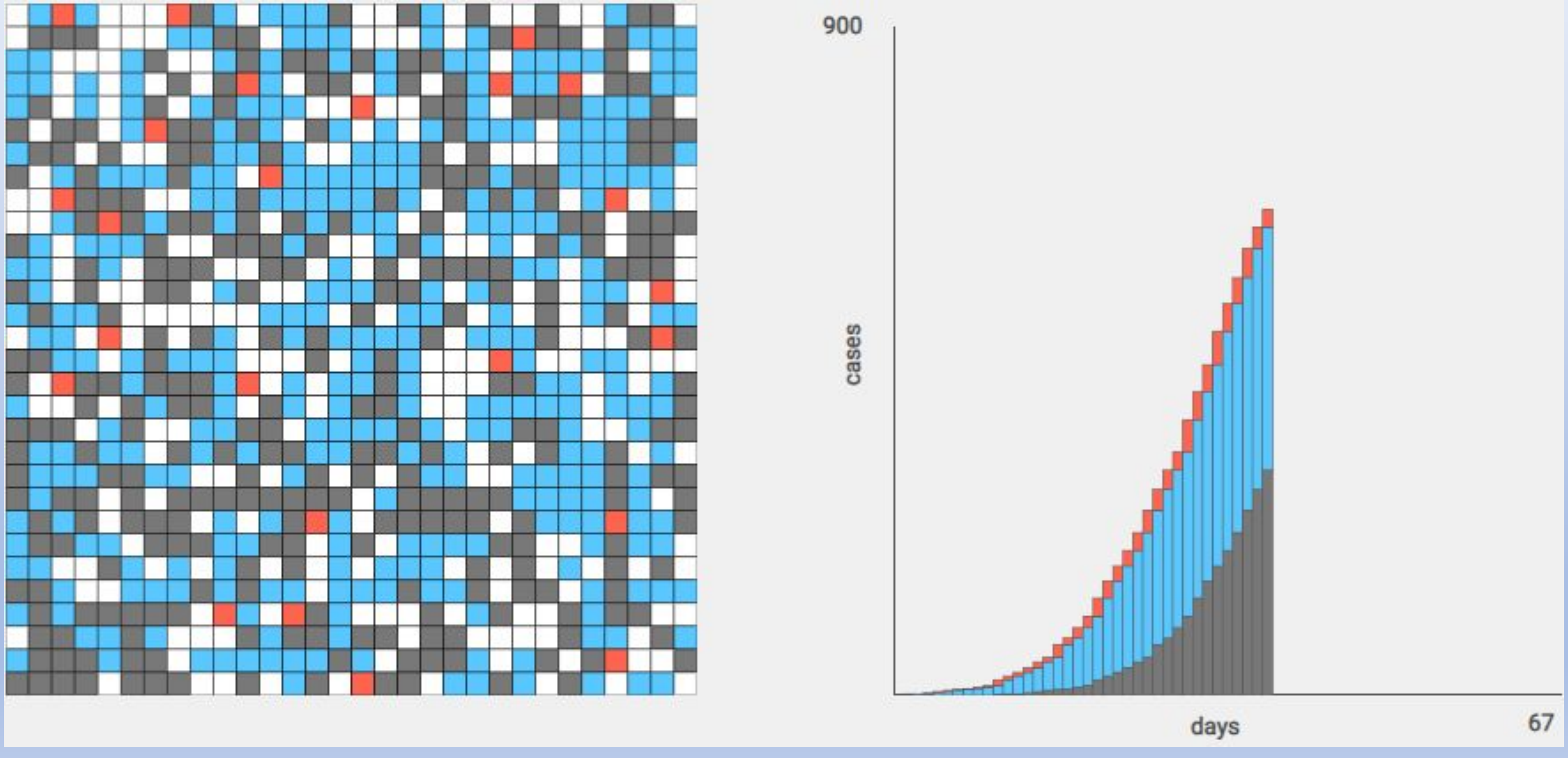
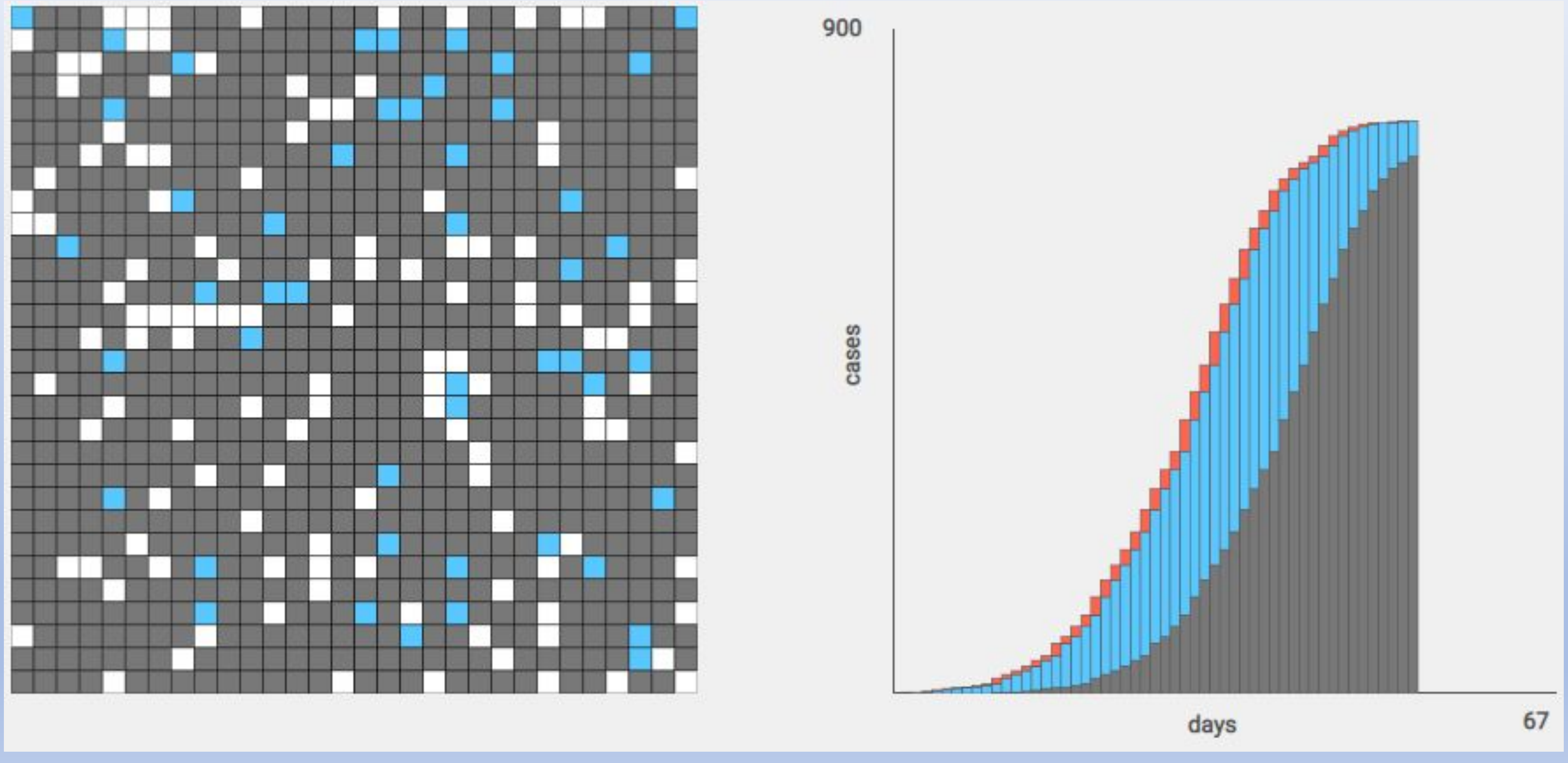
Illuminations Activity 3, 9-12

Pandemics: How are Viruses Spread




900
cases
days
67


900
cases
days
67


900
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