President's Message

A Journey in Algebraic Thinking

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The United States is one of the only countries in the world that is teaching courses with names like Algebra I or Algebra II. The rest of the world, including our colleagues in Canada, teach mathematics, not as separate courses, but as a continuous program from elementary through secondary school. In the United States, some schools offer an alternative, such as an integrated program that incorporates algebra as a strand blended with geometry and other advanced topics. Others continue to offer a course sequence that includes Algebra I, Geometry, and Algebra II. In an increasing number of states, the study of algebra in some form is required of all students for high school graduation. Regardless of whether a school's secondary curriculum includes a separate course in Algebra I or is more integrated, we can take concrete steps to ensure that students will flourish and succeed when they arrive at the formal study of algebra. A key to this success is the development of algebraic thinking as a cohesive thread in the mathematics curriculum from prekindergarten through high school.

Algebraic thinking includes recognizing and analyzing patterns, studying and representing relationships, making generalizations, and analyzing how things change. Of course, facility in using algebraic symbols is an integral part of becoming proficient in applying algebra to solve problems. But trying to understand abstract symbolism without a foundation in thinking algebraically is likely to lead to frustration and failure. Algebraic thinking can begin when students begin their study of mathematics.

At the earliest grades, young children work with patterns. At an early age, children have a natural love of mathematics, and their curiosity is a strong motivator as they try to describe and extend patterns of shapes, colors, sounds, and eventually letters and numbers. And at a young age, children can begin to make generalizations about patterns that seem to be the same or different. This kind of categorizing and generalizing is an important developmental step on the journey toward algebraic thinking.

Throughout the elementary grades, patterns are not only an object of study but a tool as well. As students develop their understanding of numbers, they can use patterns in arrays of dots or objects to help them recognize what 6 is or whether 2 is larger than 3. As they explore and understand addition, subtraction, multiplication, and division, they can look for patterns that help them learn procedures and facts. Patterns in rows and columns of objects help students get a sense of multiplication and see that facts make sense. Patterns within the multiplication table itself are interesting to children and help them both learn their facts and understand relationships among facts. The process of noticing and exploring patterns sets the stage for looking at more complex relationships, including proportionality, in later grades.

As students move into the middle grades, their mathematics experience can focus on connecting their work with numbers and operations to more symbolic work with equations and expressions. At this level, the focus of the mathematics program should be on proportionality, perhaps the most important connecting idea in the entire pre-K–12 mathematics curriculum. This concept should take students well beyond the study of ratios, proportions, and percent. A real understanding of proportionality allows students to connect their experience with numbers and operations to ideas that they have studied in geometry, measurement, and data analysis. They begin to get a sense of how two quantities can be related proportionally, as seen on maps, scale drawings, and similar figures, or in calculating sales tax or commissions.

A solid understanding of proportionality sets the stage for students to succeed in the more formal study of algebra. From this base, notions of linearity and linear functions emerge naturally. As students explore how to use linear functions to solve problems, the bigger world of functions that may not be linear begins to open for them. Looking at what is the same and what is different among functions lies at the heart of understanding algebraic skills and processes.

The journey doesn't end with a student's first formal study of algebra at high school. Continuous development of increasingly sophisticated algebraic reasoning can provide an avenue into the study of geometry and advanced mathematics. In the world outside school, these topics are not separated. When higher-level courses regularly incorporate opportunities to build on students' algebraic understanding, students are far more likely to succeed than if the courses present just one mathematical perspective.

The development of algebraic thinking is a process, not an event. It is something that can be part of a positive, motivating, enriching school mathematics experience. "Developing Algebraic Thinking: A Journey from Preschool to High School" is the Professional Development Focus of the Year for NCTM during the 2004–05 school year. Watch for opportunities to develop your own understanding of this important topic throughout the year, including as part of conferences, journals, publications, and the NCTM Web site.

For this month's questions, consider the following: How can we build algebraic thinking into the pre-K–12 curriculum at all levels? How should secondary school mathematics be organized to capitalize on the inclusion of algebraic thinking throughout the elementary and middle grades? What can NCTM do to support teachers in fostering the development of algebraic thinking? Ω

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