President's Message

Hard Arithmetic Is Not Deep Mathematics

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Helping all students develop a high level of mathematical proficiency is more important than ever before. Nearly every state and province has raised high school graduation requirements for students, and almost everyone agrees that we must raise our standards and expect more of our students. Attempts to define what it means to raise standards or increase expectations have led to interesting, and sometimes contentious, discussions at the state or provincial and local levels.

The message of NCTM's *Principles and Standards for School Mathematics* is clear. Students need a balanced mathematics program that allows them to be actively engaged in mathematics lessons so that they can develop deep understanding, mathematical thinking, and the ability to apply what they learn to solve problems. Computational proficiency is an important part of such a balanced program. However, computational proficiency is not the primary goal of effective mathematics programs. Instead, it is a tool used in the service of deeper mathematics.

The kind of mathematics that students need today—that adult citizens need—goes far beyond what once was sufficient. In the past, it might have been enough for a literate citizen to know how to read, write, and do basic measurement and arithmetic in everyday life. In the past, it might also have been enough for students who were going to college to master a set of algebraic tools that enabled them to take higher-level mathematics or science courses. But in today's world, there is rapid change, pervasive technology, and jobs that didn't exist five years ago. These all call for a much broader set of mathematical skills, including the ability to reason and apply mathematics to an ever-changing range of problems. And the reality of life today is that many more of our students are likely to participate in some kind of postsecondary education than ever before.

In this environment, how do we raise the bar on the mathematical proficiency that we expect of all students? And how likely is it that all students can achieve the goals that we set?

In response to the first question, we can raise the bar on mathematical proficiency by choosing fewer topics to focus on at each grade level and by teaching those topics in great depth. "Depth" means, for example, that students know a lot about multiplication before they deal with an algorithm for performing multiplication. "Depth" means that when fractions are introduced, we teach in such a way that students really know what fractions represent, in what kinds of situations they might be useful, how they compare to one another, how they relate to what students know about whole numbers, what it means when the numerator or denominator increases or decreases, and so on. "Depth" means that before students confront the rules for operating with fractions—such as going straight across, turning upside down, cross multiplying, etc.—we ensure that they know a lot about fractions and a lot about operations. "Depth" means that students go beyond "solving proportions" to recognize and utilize proportional relationships in ways that powerfully connect the ideas of prekindergarten–grade 12 mathematics. And "depth" means that students earning credit for a high school algebra course know how to solve equations and how to use algebraic tools and representations to solve many kinds of problems both within and outside of mathematics.

"Depth" does not mean making all students master arithmetic procedures earlier or with more digits. A school system whose standards include the mastery of fraction operations earlier than the standards of another system does not necessarily have a more rigorous curriculum. "Depth" does not mean narrowing our curriculum down to numbers and operations alone at the expense of measurement, geometry, and data analysis, where those numbers and operations are actually used. "Depth" does not necessarily mean more exercises. Focusing on more arithmetic procedures or more digits at the expense of deeper explorations and problem solving is not the same as raising our expectations for all students. And "depth" does not have to be painful or boring.

In visiting schools, I have found many wonderful examples where students are learning mathematics in depth. In these classrooms, mathematics is taught in greater depth and students are actively engaged, which opens the door for all students to master challenging mathematics. "Depth" is not the same as difficult arithmetic. "Depth" comes when students "get it." This means that students need to see the contexts in which mathematical ideas arise, need to wrestle with those ideas in problems that take some time to solve, and need opportunities to represent and communicate what they learn. The next President's Message will address the nature of student engagement in these classrooms and how we can ensure that students learn what is taught.

If we define our mathematics curriculum—the standards developed in our states and provinces—in ways that focus on students knowing and using mathematics and not just doing hard arithmetic, we can achieve this depth. And if we make some accompanying shifts in how we structure our classrooms, we can ensure that all students have an opportunity to reach the ambitious goals that we set.

Is your state or province or school system shifting its curriculum and standards toward deep mathematics rather than hard arithmetic? Do you believe that all students can achieve high standards? What will it take to make this happen? What will keep it from happening? Share your thoughts during my next online Presidential Chat, scheduled for 4:00 p.m. ET, Tuesday, October 26. Visit www.nctm.org at that time to join the discussion.

Also, in November be sure to read the President's Message about student engagement and join a related discussion online at 3:00 p.m. ET on Tuesday, November 16.

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