

Problems to Ponder: Worthy of your Effort

NCTM On-Line Session for 100th Anniversary

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Handout 1. Some initial strategies, and extensions for our Problems to Ponder

1. Covid Café

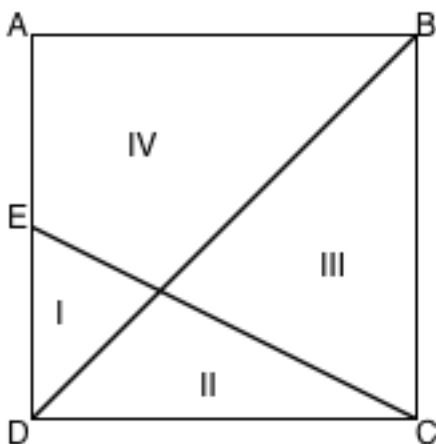
In a neighborhood café there are 10 seats in a row at the counter. The owners have planned for what to do when they are allowed to re-open after the Covid 19 virus has subsided. When customers enter the café for their morning coffee, they will have to sit so that they are not next to one another at the counter. Two people enter the café when it opens up.

How many different ways can those two customers sit at the counter with 10 seats so that they are *not* next to each other?

2. Compare Regions in a Square

In Figure 1. below, quadrilateral ABCD is a square, and E is the midpoint of the side AD. How do the areas of regions I, II, III, and IV compare? That is, what are the respective ratios of the areas of the four regions I:II:III:IV ?

Figure 1.



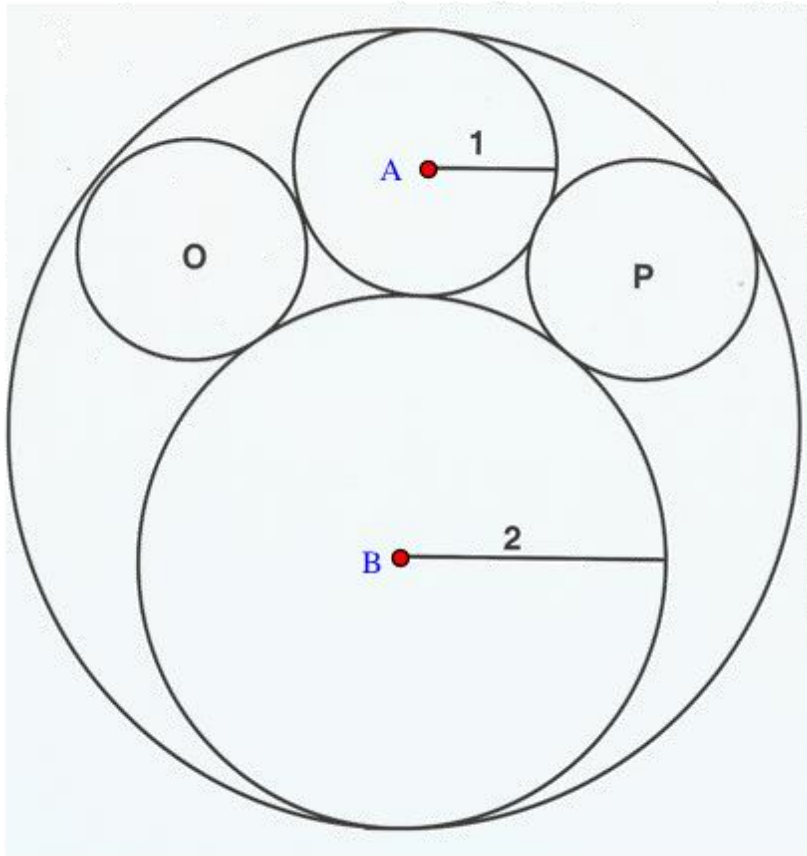
3. Consecutive Sums—Staircases

Which whole numbers can be written as a sum of consecutive whole numbers?

For example, $6 = 1 + 2 + 3$ so it can be written as a consecutive sum. Also, $15 = 4 + 5 + 6$ is the sum of consecutive whole numbers, and $15 = 7 + 8$, so 15 can be written as a consecutive sum in more than one way. So, a natural extension of this problem is: How many *different* ways can a given whole number be written as a consecutive sum?

4. Circling Around

The circles in the figure below are mutually tangent to one another. The radius of the circle centered at A is one unit, and the radius of the circle centered at B is two units. What are the radii of circles O and P?



5. Locks & Keys—protecting the treasure

In medieval times the inhabitants of a village decided to lock the village valuables in a giant chest to protect them. For insurance the villagers had a number of locks put on the chest, and distributed keys to the villagers so that no two people could open the chest, but any three people always could open it.

How many locks, and how many keys will they need?

6. Water Bucket Conundrums

At a rural cabin water must be drawn from a well using buckets. The cabin has only a 4 - gallon bucket and a 9 - gallon bucket. In one trip to the well, what whole number amounts of water in gallons could you bring back to the cabin in the buckets? There are no other markings on the pails, no estimating allowed, exact whole number amounts only!

7. Cevian Triangles

In the triangle below, segments AG , BF , and CE are drawn from each vertex to the opposite side. The pairwise intersections of these segments then creates another triangle, IJK , in the interior of triangle ABC .

What is the relationship between the areas of triangles ABC and IJK ?

