Professional Development Guide for Creating, Critiquing and Revising Arguments (CCRA)

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An Introduction to Creating, Critiquing, and Revising Arguments (CCRA)

A key aspect of reasoning and sense making is creating, critiquing and revising mathematical arguments. These are central to NCTM’s Reasoning and Proof Standards (PSSM, 2000) and mathematical practice 3 of the Common Core State Standards for Mathematics, “Construct viable arguments and critique the reasoning of others” (CCSSI, 2010). These practices are all part of justifying a mathematical statement or process and provide students with opportunities to understand mathematics as a discipline that rests on reason and logic.

As students participate in creating, critiquing and revising mathematical arguments, they engage in a range of specific activities including:

1. investigating and generalizing relationships from patterns;
2. developing definitions of mathematical objects through examination of physical representations and comparisons of the similarities across representative cases or sets of similar examples;
3. stating generalized relationships as conjectures in precise terms;
4. using counterexamples to disprove and refine conjectures; and
5. explaining why a conjecture is true by using definitions and mathematical properties. (Connecting the NCTM Process Standards and the Common Core Standards for Mathematical Practice, p. 31)

Common Core Mathematics Practice 1 ends with a statement about the need for students to understand others’ solution strategies as well as understand the mathematical connections between different strategies. In fact in meeting the Standard, not only will students learn to understand and evaluate others’ strategies, but when they engage in mathematical arguments where they must justify their own solutions, they will gain a better mathematical understanding as they work to convince their peers about different points of view (Connecting the NCTM Process Standards and the Common Core Standards for Mathematical Practice, p. 15)

Note that making conjectures, which may involve inductive reasoning and pattern searching, is just the beginning of justifying a mathematical statement or process. Students engaged in creating, critiquing and revising arguments must also seek to make sense of why an observed pattern holds or does not hold true.

In the classroom, students’ reasoning will be presented at a variety of levels of sophistication. Over the course of a lesson, the level of sophistication and quality of argument may be refined and become more complete and mathematically sound. It is important that teachers are deliberate in promoting these practices within their classroom:
are true (Connecting the NCTM Process Standards and the Common Core Standards for Mathematical Practice, p. 32).

Pre-Activity: Understanding CCRA

Before diving into the activities that follow, ask participants to read the following articles and make note of key ideas that can be referenced in a discussion about the habit of creating, critiquing, and revising arguments.

- Article 1: The Power of Incorrect Answers
- Article 2: Promoting Equity through Reasoning

Using the description of CCRA above and the readings, ask participants to discuss the following prompts (either in small groups or as a whole group if there only a few present):

1. What characteristics are important to consider when choosing a task that will promote CCRA?
2. As students work on a task that engages them in CCRA, what are strategies the teacher can use to maintain the focus on students’ reasoning and sense-making?

CCRA Cluster 1: Ms. Hauser’s Hexagon Train Task

Ms. Hauser’s classroom offers a context for participants to consider how this community of students is actively creating, critiquing and revising their mathematical arguments. Several options for professional development are presented below. You may choose to do one or several of the activities depending upon your goals and time available. That said, whenever possible we strongly suggest that participants are presented with the Hexagon Task and work through the problem as though they were students (as in Activity 1).

Activity 1: Experiencing CCRA with a Perimeter Task

When discussing the pedagogy behind facilitating productive habits of mind in the classroom, it is important that participants experience the habit of mind from the perspective of a student. For this activity we offer two tasks and recommend that you use one of them with the participants. The first task is the Hexagon Task as presented in Ms. Hauser’s classroom. The second is an Extension of the Hexagon task, which you might opt if you want a more mathematically complex task. The idea of this activity is to have participants to reflect on their own thinking as they engage in CCRA in order to facilitate the development of this habit of mind in their classrooms.

Materials:
- Copies of the task
  a. Original Task: Hexagon Task from Ms. Hauser’s class
b. Alternative Task: Extension of the Hexagon Task that asks, “What would the perimeter be for the n\textsuperscript{th} figure of regular k-sided polygon (where each side is one unit in length)?”

- Video Clips from Ms. Hauser’s classroom

Suggested Sequence for Activity 1:
- Introduce the Creating, Critiquing and Revising Arguments (CCRA) Habit of Mind.
- Explain that this activity is meant to elicit this habit of mind, ask that they be mindful of how they are creating, critiquing and revising arguments as they work through the task with their colleagues.
- Introduce the task (either the original task or the extended task).
- In small groups have participants work on the given task. You may wish to give individuals “private think time” (individual work) before they collaborate with groups.
- While the participants are working you may want to discretely keep notes on things you notice that are representative of CCRA (e.g., statements such as “I see how that works in this case, but would it work with…?” or “Would it make more sense if we changed… to…?”). Some of the interactions that facilitate CCRA might be more apparent to you than to those who are actively solving a mathematical problem.
- Debrief the mathematics by having select participants post their solutions (you may want to reference Activity 3 to model effective ways to select and order work to share). Lead a discussion of the group’s work.
- Facilitate a discussion about CCRA. Some questions to help guide the discussion include:
  a. [Post the description of CCRA] Which of these habits of mind did your group practice while solving this problem? (If you took notes of things you observed that facilitated the habit of mind, now may be an appropriate time to share your observations.)
  b. Which of these aspects of this habit do you think are the easiest for students to engage in while doing mathematics? Which are the most difficult?
  c. With which of these aspects of this habit are do you think you are most proficient? Which ones do you find more challenging?
  d. What supported or impeded the habit in your group?
  e. What can we do, as educators, to make these aspects of the habit a more natural occurrence in the classroom?

- Depending on time you could watch the videos for instances of CCRA, or you might want to use Activity 1 as a launch into one of the following suggested professional development activities.
- This activity can also launch into a discussion into aspects of setting up the classroom environment of this habit of mind with selection of cognitively demanding tasks, anticipating student responses to mathematical tasks, monitoring student work, selecting and sequencing student work to discuss, and
facilitating discussions on mathematical connections. For more information the following publication may be referenced:


**Activity 2: A Faulty Argument Case**

Much of the work of creating arguments and critiquing reasoning lies in reflecting on reasoning and testing conjectures. In this brief exercise participants will first explore student work and discuss the ways that a teacher might respond to this student upon seeing her work. The teachers will then see a video clip of the pedagogical plays the teacher makes when talking with the student with a faulty argument. The activity ends with a discussion about the instructional decisions the teacher made and her options for proceeding.

Materials (from the “Eliciting Student Arguments” page of the Hauser Hexagon Train Task site):
- Copies of the Hexagon Task
- Copies of Amanda’s written work
- Video Clip 1

Suggested Sequence for Activity 2:
- If you have not already done so, have participants complete the Hexagon Task.
- You will be reviewing work samples from students seated near one another in Ms. Hauser’s class. These were arguments put forth early in the lesson. At this point, the students were working independently. As you review the samples think about how you would respond to the student(s).
- Launch the video (Clip 1) by asking participants to think about the pedagogical choices the teacher makes in the clip.
- Debrief the video with questions that focus both on how the situation played out as well as how Ms. Hauser might proceed.
- Facilitate a discussion on:
  - The mathematical arguments Amanda generated on paper.
  - The pedagogical choices the teacher can make that will foster CCRA in the face of the faulty argument.
- Watch the clip of Ms. Hauser reflecting on this excerpt from her classroom in [Part 1 - Eliciting Student Arguments of the cluster](#).
- Facilitate a discussion in relation to Ms. Hauser’s reflection. Consider including the following pedagogical questions:
  - Why did Ms. Hauser’s effort to get Amanda to shift her thinking not work at this point in time? Why did Ms. Hauser decide to move on without having convinced Amanda that her reasoning was flawed?
  - What other strategies might a teacher employ to get a student such as Amanda to reflect on her work in a meaningful way?
o Have participants read and discuss this reading on orchestrating mathematical discussions that can be used to prompt additional reflection:

Classroom Arguments

Activity 3: Using Specific Cases and/or Counterexamples to Test Conjectures
In Clips 1, 2A, and 2B, we have seen that there are many ways that students can reason about the perimeter of a chain of 25 hexagons. In Clip 3 the teacher chooses to bring several differing examples of student thinking to the attention of the whole class. Teachers choosing to elicit CCRA in their classrooms often have to make pedagogical choices as to what student work to bring to the whole class and how to order the presentation of student ideas. In this activity participants will review samples of student work and will discuss (1) which examples they would have students present to the class, and (2) the order in which they would choose to have students present their work.

Materials:
- Student Work for PD Activity 3 (with graphic organizer)
- Clip 3
- Ms. Hauser’s interview remarks on Clip 3
- Smith & Stein MTMS February 1998 article 344-50

Notes:
Prior to leading this activity, read the Smith and Stein article, as the Suggested Sequence for Activity 3 assumes that you are familiar with the terms on the graphic organizer found at the end of the student work file. You might choose to have the participants read the article prior to the professional development so that they have thought through the 5 Practices.

Suggested Sequence for Activity 3
- If they have not already done so, have teachers work through the Hexagon Task.
- In Smith & Stein (2011), they discuss five practices for orchestrating productive math discussions. The first two practices include anticipating student responses and monitoring student work. If you watched Clip 1 you have seen that Ms. Hauser has monitored the work of students. The next three practices include selecting student work to share, determining the order for sharing the work (sequencing), and connecting the student work.
- Remind participants that when asked about her goals for the lesson, Ms. Hauser stated, “In this lesson I was setting the stage for our unit on pattern-seeking and variables. I was also working on having student justify their answers and listen to each other.”
- Given this goal, ask teachers to consider the following:
  o Look at the student work and categorize students’ reasoning strategies. How are they similar? Different? What mathematics are they using to reason about this task? What kinds of arguments are they constructing?
Keeping Ms. Hauser’s goal for the lesson in mind, which three pieces of student would you select to share in order to reach (or at least make progress on) these goals?

- Given your chosen three pieces of work, what mathematical connections can be drawn among these pieces of work? That is, what do these three pieces of work allow you to discuss?

- Distribute student work samples and graphic organizer to participants and have them begin the task.
- Facilitate a discussion where participants share their responses to the three parts of the task.
- Watch Clip 3 to see the pieces of work selected to be made public by Ms. Hauser and presented by the students.
- Lead a final discussion on why Ms. Hauser may have chosen the three pieces of student work and why she ordered their presentation in this way.

Connections to Other NCTM Resources:

* 5 Practices for Orchestrating Productive Mathematics Discussions*

**Activity 4: Formative Assessment & Student Work Progressions**

Throughout the clips in this lesson we have seen evidence of productive and faulty (but revisable!) student reasoning. What we do not see in the videos is how the teacher extends the task in the days that follow this lesson. This activity engages teachers in examining student work progressions; each work progression is a student’s response to three prompts developed across the three days. The materials – found on the Students Critiquing and Revising page of the lesson – are organized into two sets. Depending upon your focus you may choose to use either one or both sets of the work progressions. Set 1 has work progressions from two students (including Amanda) illustrating how their understanding of the mathematical content evolves. The progressions in Set 2 illustrate how the arguments of three students become elaborated, revised, and/or more sophisticated over time.

**Materials:**

- Set 1: Student Work – Amanda and Abby
  - or
- Set 2: Student work – Bernardo, Brit and Bill

**Suggested Sequence for Activity 4:**

- Introduce the focus of the session, namely, considering how students may have revised their thinking and arguments over time. This process of revision is both a valued habit, and evidence of learning.
- Provide teachers with the background for the work sample sets they will see. Each set comprises three pieces of work from the same student.
The first piece of work is the student’s initial response to the question of the perimeter of the 25th figure, as shown in the video clips.

The second is the student’s response to a warm-up on the next day. Students were told that “Jacob” knew that Figure 3 of the hexagon pattern had a perimeter of 14, and so to get the perimeter of Figure 6, he multiplied 14 times 2. Students were asked to determine whether Jacob’s method was “complete” and explain their reasoning. The warm-up can be found on the Lesson Overview page, under the Lesson Graph.

The third is the student’s final write-up of his or her solution to the question: What’s the perimeter of the 25th figure? Students were asked to write up a CLEAR solution to this question, which is based on a rubric students use when writing responses to open-ended problems. CLEAR stands for: C – Calculations, L – Labels, E – Evidence, A – Answer, R – Reasons. The rubric can be found on Lesson Overview page, under the Jacob’s solution warm-up.

SET 1:
- Provide teachers with, or direct teachers to, Amanda’s and Abby’s work sets.
- Let them read through the arguments for 2-5 minutes.
- Pose the questions provided on the Students Critiquing and Revising Arguments page under the work sets.
- Let teachers work in teams for 5-10 minutes.
- Organize discussion. See notes below.

SET 2:
- Provide teachers with, or direct teachers to, Bernardo’s, Brit’s and Bill’s work sets.
- Let them read through the arguments for 2-5 minutes.
- Pose the questions provided on the Students Critiquing and Revising Arguments page under the work sets.
- Let teachers work in teams for 5-15 minutes. You might have different groups of teachers focus on different students, or have them consider all work sets.
- Organize discussion. See notes below.

Notes:

Set 1:
Both sets of student work show students who initially used a (faulty) proportional argument and then revised their thinking, showing some evidence, to different degrees, that they now understood the flaw with the argument. Both students are also able to produce a solid argument demonstrating clear thinking about an approach to finding the perimeter on the third work sample (that is applied to one case, but could be applied generally).

Amanda shows advancement in thinking about her initial proportional reasoning argument. On the Day 2 warm up, she seems to recognize there is an error with the
answer, but does not offer any analysis of the error. (She is showing Jacob is not correct by counting – direct evidence.) Her final work sample shows evidence that she understands the idea of the “connectors” and she has a (new) method that she seems to understand. Note that there perhaps remains a little bit of confusion in her thinking, as evidenced by her diagram, which shows 26 hexagons and notches the chain into six groups of four hexagons, plus one more, which was her original thinking.

**Abby** shows in the first work sample that she is unaware of the “overlapping sides” that should not be counted towards the perimeter. In the Day 2 warm-up, she demonstrates that she understands this issue now, and explains how Jacob counts “2 in the middle” which should not be counted. Her final solution offers a new approach, and she demonstrates that she can explain where each of the values “comes from” and connect those to a hexagon figure. Note that she retains across the work samples some sense of thinking about the full set of 25 and then breaking it down. Earlier work shows groups of 4 with one left over; the last work sample shows breaking into the groups of “insides” and “outsides” and appropriately counting the contribution of each type of hexagon towards the perimeter of the chain.

**Set 2:**
For this set, each student had the correct answer at the outset, but there is identifiable improvement in the argument the student creates to support his or her answer. The purpose of examining this set is to focus on how the *arguments* offered change over time.

**Bernardo** is a case of developing (or articulating) a more sophisticated argument. His reasoning on day 1 was expressed as “because that’s the pattern.” He even has a generalized equation expressed on Day 1. On his final solution, Bernardo shows connections with the diagram and clarity regarding how each value was relevant for finding the perimeter. Though not as well woven into a sequence, the second piece of work offers an argument that first shows Jacob is wrong by appealing to a “known rule” – you have to do $6 \times 4 + 2$. The last sentence demonstrates insight into understanding why Jacob’s solution is not correct – Jacob is finding the perimeter of two groups of 3 hexagons, and not one group of 6 hexagons.

**Brit’s** work shows revision of an argument that makes the argument more sophisticated. On day 1, we see more of a “brute force” method for finding the perimeter of the 25th figure. With the revision for the final solution, we see an argument that is more sophisticated, as it connects the contribution of each hexagon of the figure with calculations to find the result. This method also would lead itself to a generalization as it recognizes the general relationships that hold in the chain and distinguishes the inner hexagons from the ends. (She’s reasoning about a specific case, but in a more general way.)
Bill also represents a case of developing more sophisticated reasoning. Bill uses a proportional method with a compensation strategy correctly from the start. Initially, he does not offer reasoning to support his method, and it seems likely he noticed a pattern in what needed to be subtracted. This interpretation is supported by his Day 2 warm-up work and other evidence from the video (not included in these web pages). In the final solution, Bill shares multiple ideas. His top picture shows 2 hexagons coming together, which gives some indication that he understands more about why he must subtract, and not just that he must subtract because there’s a pattern. His new method for finding the perimeter, shown on figures 4 and 5, is readily generalizable. The table seems to be his evidence of “the right answer” and he is confirming that his approach on Figures 4 and 5 produces the right answers by comparing those results with the table of values.

Other possible discussion points:
- Bill’s final write up has a wide range of ideas. For the moment, focus on his written argument. Do you consider this a complete solution and a high quality CLEAR solution write up? What feedback might you give to Bill about strengths and areas to improve?
- What question(s) might you ask Amanda to gain further evidence that she understands the flaw with her original method?
- What feedback might you give to Amanda on her CLEAR solution write up? What are areas of strength? What are areas for improvement?
- Critique the CLEAR solution rubric. Could you use this directly in your classroom? What modifications, if any, would you make to a) suit your grade level, and b) better guide students in developing and expressing arguments?

Tools for Sharing Digital Representations
As part of your professional development practice you may want to use digital representations of practice (i.e., videos and pencasts) to reflect on your own practice. Using digital representations of practice to reflect on practice either individually or in a professional community is a powerful way to refine your own teaching.

- If someone is available to help record while you are teaching, have them help operate the camera. To the extent possible, try to point the camera in the direction of the main action in the room at any given moment, for example, the small group engaged in the liveliest math talk.
- If someone does help you record, ask him or her to keep the camera as steady as possible. Some might want to use a tripod but think about whether this may make it harder to capture close ups of student interaction.
• Capturing good quality audio in a classroom is a challenge with so many voices. It will help to use an external microphone particularly when recording conversation in small groups.
• Reduce the ambient noise in the room if possible. Turn fans, radios, and anything else that makes noise off. Also close windows and doors. This may seem simple, and it is, but the quality of the audio will improve dramatically.
• Move the camcorder as close as possible to the source of audio, while still capturing the video you need to see. For example, if you have a camcorder set up in the very back of the room with 5 rows of desks, but students only sit in the first two rows, move the camcorder closer to the front of the room, while still behind the students. This will result in louder and clearer audio.
• When possible, the camera operator should keep any windows behind him or her. Outdoor lighting is usually very bright; if the camera is aimed at the windows, the camera will adjust by darkening the subjects.

Resources for storing and sharing digital media:
• Free option for sharing video
  o YouTube private channel
• Fee-based options for sharing video
  o Teacher Studio (www.teacherstudio.com)
  o Vimeo (video sharing site)
  o Weebly Pro account (personal website with storage for media files)

Protocols for Using Digital Representations of Practice

The High Tech High Graduate School of Education has some protocols for structuring professional conversations about teacher practice. Particularly useful for sharing video clips as a means for professional development are the Video Clip Protocol and the Focus Point Protocol (which you can adapt from a live classroom observation to the viewing of a recording).