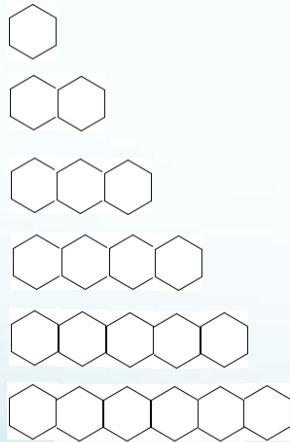


Generalizations and Extensions to the Hexagon Train Task

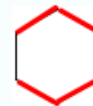
Standard Approaches

Many students create a table of perimeters of the hexagons by simply counting the number of sides or by visualizing the perimeter as the tops and the bottoms plus the two ends (left and right).

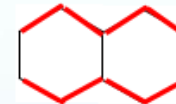


No. of Hexagons	Perimeter
1	6
2	10
3	14
4	18
5	22
6	26

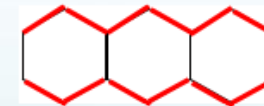
OR



$$4 \cdot 1 + 2$$



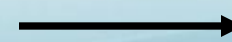
$$4 \cdot 2 + 2$$



$$4 \cdot 3 + 2$$

⋮

The sequence of the number of hexagons 1, 2, 3... can be represented by k .



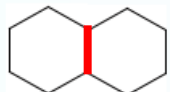
$$4k + 2$$

Adding Five Sides

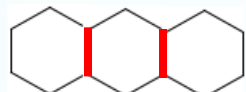
Add five sides to the perimeter each time a hexagon is added to the train and subtract a side or sides (in the middle).



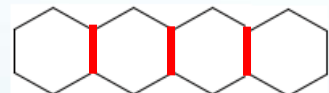
$$5 \cdot 1 + 1 = 5 \cdot 1 - (-1)$$



$$5 \cdot 2 + 0 = 5 \cdot 2 - (0)$$



$$5 \cdot 3 + (-1) = 5 \cdot 3 - (1)$$



$$5 \cdot 4 + (-2) = 5 \cdot 4 - (2)$$

⋮

$$5 \cdot k + (2 - k) = 5 \cdot k - (k - 2)$$

The sequence
-1,0,1,2 can be
represented by $(k-2)$.



A Proportional Reasoning Approach

Although a proportional reasoning approach yields an over count of the sides involved in the perimeter, it can be revised by considering the pairs of middle sides where the groups are joined.

Using Proportional Reasoning with Correction

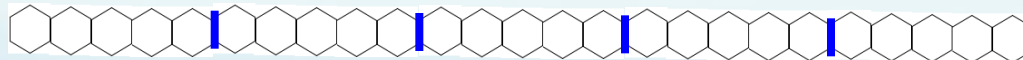
$$5 \times \text{[Diagram of 5 hexagons in a row]} = \text{A train of 25 hexagons}$$

From the table, we take the perimeter for the 5 hexagon train and multiply it by 5 to get (close to) the perimeter of the 25 hexagon train.

No. of Hexagons	Perimeter
4	18
5	22
6	26

$$5 \times 22 = 110 \quad (\neq 102)$$

To revise this approach, a number of pairs of middle or shared sides need to be subtracted.



$$5 \times 22 = 110 - 4 \times 2 = 102$$

Generalizing the Proportional Reasoning Approach

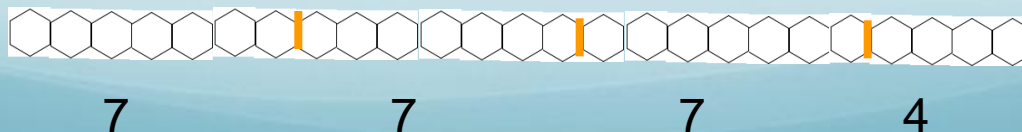
A similar approach works based on the division algorithm $(a = b \cdot q + r)$ where a correction is made for the remainder, r .

Again for a 25 hexagon train, Using 3 groups of 7 hexagons, we have $25 = 7 \cdot 3 + 4$. We start with building a $7 \cdot 3 = 21$

hexagon train that has a perimeter of 30 for each group of 7, then append the perimeter of the 4 additional hexagons which is $(4 \cdot 4 + 2 = 18)$. We now have $3 \cdot 30 + 18 = 108$.

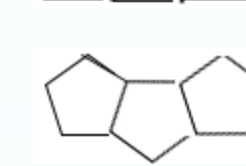
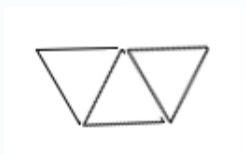
Finally, subtract 3 pairs of middle sides $(2 \cdot 3)$:

$$3 \cdot 30 + 18 - (2 \cdot 3) = 102$$



Extension: k -sided polygons (k -gons)

Can our work with hexagons be extended to other regular polygons?



Here are examples of trains of length 3 for triangles, squares, and pentagons.

Which of the solution methods used with the hexagon problem will work with k -gons?

Is there a generalized formula for an k -gon train?

Is the conjecture $k \cdot n + 2$ correct where n is the length of the train? Why or why not?

Additional references related to pattern tasks

- Huntzinger-Billings, E. (2008). Exploring generalization through pictorial growth patterns (279 – 293). In Greenes, C., & Rubenstein, R., *Algebra and Algebraic Thinking in School Mathematics, 70th Yearbook*. Reston, VA, NCTM
- Friel, S., & Markworth, K. (2009). A framework for analyzing geometric pattern tasks. *Math teaching in the middle school*, 24 – 33.