



# Book Study Guide for

## *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations*

The purpose of this book study guide is to provide stakeholder groups with a tool to initiate needed critical conversations and actions in early childhood and elementary mathematics.

### Key Recommendations

*Catalyzing Change in Early Childhood and Elementary Mathematics* identifies four key recommendations for creating high-quality and equitable mathematics programs for each and every child.

1. **Broaden the Purposes of Learning Mathematics.** Each and every student should develop deep mathematical understanding, understand and critique the world through mathematics, and experience the wonder, joy, and beauty of mathematics, which all contribute to a positive mathematical identity.
2. **Create Equitable Structures in Mathematics.** Middle school mathematics should dismantle inequitable structures, including tracking teachers as well as the practice of ability grouping and tracking students into qualitatively different courses.
3. **Implement Equitable Mathematics Instruction.** Mathematics instruction should be consistent with research-informed and equitable teaching practices that foster students' positive mathematical identities and strong sense of agency.
4. **Develop Deep Mathematical Understanding.** Middle schools should offer a common shared pathway grounded in the use of mathematical practices and processes to coherently develop deep mathematical understanding, ensuring the highest-quality mathematics education for each and every student.

## AUDIENCE

This book study brings to the surface issues relevant for teachers, coaches, and administrators in early childhood settings and elementary schools as well as for other stakeholders, including leaders, policymakers, teacher educators, and community members.

## FORMAT

The book study guide is designed to support multiple formats, including face-to-face, online, and hybrid settings. Each book study session consists of three sections: (1) Before You Read, (2) After You Read, and (3) Action Steps. The suggestions for each session can be adapted to best meet the needs of the study group participants. In addition, facilitators are encouraged to incorporate any of the many conversation starters found throughout the book into the sessions.

## BOOK CITATION

National Council of Teachers of Mathematics (NCTM). 2020. *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations*. Reston, VA: NCTM.

## NOTE

The book study guide was developed by the *Catalyzing Change in Early Childhood and Elementary Mathematics* writing team: DeAnn Huinker, Anne Marie Marshall, Nicole Rigelman, and Cathery Yeh.

## LEARN MORE

Find more resources at [www.nctm.org/change](http://www.nctm.org/change) for the Catalyzing Change series (Early Childhood and Elementary, Middle School, and High School), including information on ordering copies of the books.

## ABOUT NCTM

The National Council of Teachers of Mathematics is the world's largest mathematics education organization. NCTM advocates for high-quality mathematics teaching and learning for each and every student. *Catalyzing Change in Early Childhood and Elementary Mathematics* is an official position of the National Council of Teachers of Mathematics (NCTM), as approved by the NCTM Board of Directors, October 2019.

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# Chapter 1

## Why We Need to Catalyze Change

*Catalyzing Change in Early Childhood and Elementary Mathematics* raises issues for critical examination by all stakeholders and calls for systemic approaches that ensure access, equity, and excellence for each and every child. (pp. 1–2)

Before You Read

After You Read

Action Steps

Children’s experiences in learning mathematics shape both their views toward the discipline and their own personal identities as mathematics learners.

- Reflecting on your own experiences as students, what are three ways mathematics teaching and learning has changed at the early childhood and elementary school levels?
- How might these changes be beneficial for children, both currently and for their mathematical futures?

Before You Read

After You Read

Action Steps

After reading chapter 1, engage in conversations centered on the following tasks and questions about the need to catalyze change in mathematics.

1. NCTM acknowledges that “many educators are purposefully working to transform early childhood and elementary classrooms into mathematically powerful spaces” (p. 1). What characteristics would you include on your list to describe mathematically powerful learning spaces for children?
2. How might the children in your setting respond if you asked them, “What does it mean to be smart in math class?” Do you think your students would have similar responses to those of Ms. Robinson’s second-grade students as shown in figure 1.1 (p. 2)? Why or why not?
3. Review the four encouraging findings from the National Center for Education Statistics on page 4 related to children’s experiences in learning mathematics.
  - Which finding is most surprising or affirming to you and why?
  - Unfortunately, many children are not afforded such rich learning opportunities. Why might this be the case?

4. Revisit the Grade 4 NAEP results in figure 1.2 (p. 6). The average mathematics score has been relatively the same since 2007, hovering just below the proficiency level. What needs to occur to propel children into the proficiency category?
5. Children’s early mathematical knowledge and learning gains predict later success in mathematics, as well as later success in reading.
  - What are some important messages for your school related to these findings?
  - What are some ways for your school to place greater priority on strengthening mathematics teaching, programs, and coaching support in prekindergarten, kindergarten, and first grade?
6. The four key recommendations listed on page 9 serve as a catalyst to initiate necessary and critical conversations about needed shifts in early childhood and elementary mathematics.
  - What will be a challenging but important and needed conversation related to each of the four key recommendations?
  - How will you invite other stakeholders into the conversations and actions for catalyzing change in mathematics in your school or district?

Before You Read

After You Read

Action Steps

1. What are your key takeaways from our discussion of chapter 1 on the need to catalyze change?
2. What is an initial action you can take related to the ideas from this chapter?

## Chapter 2

### Broadening the Purposes of School Mathematics

**Key Recommendation:** Each and every child should develop deep mathematical understanding as confident and capable learners; understand and critique the world through mathematics; and experience the wonder, joy, and beauty of mathematics.

Before You Read

After You Read

Action Steps

Reflect on your own experiences as a student of mathematics. What opportunities did you have as a learner to—

- develop deep mathematical understanding while feeling confident and capable;
- understand and critique the world through mathematics; and
- experience the wonder, joy, and beauty of mathematics?

Can you identify missed opportunities in your own education for experiencing any of these broader purposes of learning mathematics? If so, consider the conditions that were missing for these opportunities to have been created.

Before You Read

After You Read

Action Steps

After reading chapter 2, engage in conversations centered on the following tasks and questions about broadening the purposes of learning mathematics.

1. Which of the three purposes—develop deep mathematical understanding; understand and critique the world; or experience wonder, joy, and beauty—needs more emphasis in your school or district? What might be a first action step?
2. Consider the Flag Day task in figure 2.1 (p. 13). What are the characteristics of this task that provide opportunities for teachers to deepen students' mathematical understanding?
3. Revisit the following quote on page 14 about going deep with mathematics as a major goal of equity-based mathematics programs:

All students in light of their humanity—their personal experiences, backgrounds, histories, languages, and physical and emotional well-being—must have the opportunity and support to learn rich mathematics that fosters meaning making, empowers decision making, and critiques, challenges, and transforms inequities and injustices. (Aguirre, Mayfield-Ingram, and Martin 2013, p. 9)

In what specific ways is your school or district providing opportunities for students to reach this goal? What further actions need to be considered?

4. Learning experiences should foster mathematical curiosity in children as they experience, and learn to appreciate the wonder, joy, and beauty of mathematics across all grades.
  - What conversations are occurring in your school or district about providing children with opportunities to experience mathematics from a playful orientation or to express their mathematical curiosities?
  - What messages need to be shared with all stakeholders, including with parents and caregivers, that playing with mathematical ideas is part of our human nature and has a place in the mathematics classroom?
  - What challenges have you encountered or do you anticipate in placing more emphasis on wonder, joy, and beauty in mathematics learning?
5. Mathematics is all around us and can provide many opportunities for children to better understand their world, to question their world, and to work toward improving their world.
  - What current social issues are affecting your school community?
  - In what ways are children being provided opportunities to understand and critique these issues through meaningful mathematical inquiry?
  - What additional opportunities might you consider adding to your mathematics programs?
6. Why is it critical that stakeholders work together to broaden the purposes of school mathematics? Which stakeholders would you invite into your conversations?

Before You Read

After You Read

Action Steps

1. Challenge yourself to have a conversation with at least one colleague outside of this study group about the ideas in this chapter. Perhaps ask them to reflect on their own opportunities and experiences as learners to—
  - develop deep mathematical understanding while feeling confident and capable;
  - understand and critique the world through mathematics; and
  - experience the wonder, joy, and beauty of mathematics.
2. Find a headline using relevant data that highlights an important current issue. Talk with a colleague about how you might use the data at various grade levels to support students in understanding and critiquing the world with mathematics.

## Chapter 3

### Creating Equitable Structures in Mathematics

**Key Recommendation:** Early childhood and elementary mathematics should dismantle inequitable structures, including ability grouping and tracking, and challenge spaces of marginality and privilege.

Before You Read

After You Read

Action Steps

Reflect on your experiences as a learner of mathematics and consider how your experiences may have affected your current beliefs about ability grouping and tracking.

- Were you ever placed in a group based on a teacher’s perception of your ability in mathematics?
- How did that placement privilege or hinder your opportunities to engage with mathematics?
- How did the ability-grouping placement affect your identity as a learner of mathematics?
- How might that ability grouping have affected your peers?

Before You Read

After You Read

Action Steps

After reading chapter 3, engage in conversations centered on the following tasks and questions with the goal of challenging and disrupting unproductive beliefs and practices that uphold inequitable structures.

1. Revisit the following quote on page 25 from the position statement on advancing equity from the National Association of Education of Young Children (NAEYC):

Members of groups that have historically enjoyed advantages must be willing to recognize the often unintended consequences of ignorance, action, and inaction, and how they may contribute to perpetuating existing systems of privilege. (NAEYC 2019, p. 5)

Rephrase the quote in your own words and share with a partner. What is the quote communicating? In what ways has your school or district contributed to perpetuating existing systems of privilege in mathematics?

2. Revisit the bulleted list of three structural barriers listed on the top of page 26.
  - In what ways has your school or district taken action to disrupt these barriers?
  - Which of the barriers will be the most challenging for your school or district to dismantle? Why?
3. From any of the four “beliefs tables” in the chapter, identify an unproductive belief that you once held.
  - What experiences or learning occurred that resulted in a shift in your belief becoming more productive?
  - What lingering questions do you still have about some of the statements listed as unproductive beliefs?
4. What questions are emerging for you related to ability grouping and tracking? Reconsider the reflections you were asked to do before reading the chapter. How has your thinking changed related to ability group and tracking?
5. Consider the alternatives to high-stakes assessments provided on pages 35–36.
  - Which of these alternatives does your school or district use or encourage?
  - Which alternatives might you consider implementing more fully?
  - What might teachers and administrators need to do this successfully?
6. Revisit the section on “Mathematics Curriculum: From Fidelity to Integrity” on pages 37–40. Consider your school or district policy on curriculum implementation, including the use of pacing guides.
  - In what ways do the policies focus on implementation of mathematics curriculum with integrity rather than focusing on fidelity?
  - What work needs to be done to prioritize integrity over fidelity?

Before You Read

After You Read

Action Steps

1. Choose one *Conversation Starter* from the chapter. Use the question to engage in a discussion with an administrator, teacher, or parent in your school community.
2. As you interact with colleagues in your school or district, be listening to the language used to describe children.
  - Are children being labeled “low performing,” “below grade level,” or “at risk?”
  - What work might need to occur to support all stakeholders to reframe how they see and talk about children?
  - How might you begin a conversation to raise the issues associated with how words that perpetuate deficit views can be used to label children?

## Chapter 4

### Implementing Equitable Mathematics Instruction

**Key Recommendation:** Mathematics instruction should be consistent with research-informed and equitable teaching practices that nurture children’s positive mathematical identities and strong sense of agency.

Before You Read

After You Read

Action Steps

Think about your relationship with mathematics throughout your life.

- In what ways did your learning experiences and mathematical opportunities shape your relationship with mathematics?
- What role did your parents, caregivers, teachers, classmates, and friends play in shaping your current disposition toward mathematics?
- How has your own cultural and linguistic background shaped your experiences with mathematics?

Before You Read

After You Read

Action Steps

After reading chapter 4, engage in conversations centered on the following tasks and questions about equitable mathematics instruction and the development of mathematical identity and agency.

1. Pick a short passage in the section “What Mathematics and for Whom” (pp. 45–48) that made you pause, caused some discomfort in your thinking, or really resonated with you. Share the passage and your thinking about the passage.
2. How can we collaborate with families, caretakers, and communities to support children’s positive mathematical identities and strong sense of mathematical agency? What events or activities might we consider?

3. Reread the “Learning Alongside Jessica” vignette (pp. 52–53). Together, analyze the vignette for alignment to the eight Effective Mathematics Teaching Practices (p. 56).
  - Where in the vignette do you find evidence of the teaching practices?
  - What missed opportunities occurred to use teaching practices more fully?
  - What next steps might surface and strengthen enactment of the teaching practices?
4. What types of professional learning activities would best support efforts to help teachers in your district embrace and implement instruction reflective of the Mathematics Teaching Framework (p. 56)?
5. To what extent, and how consistently, do you and your colleagues implement mathematics instruction reflective of the ideas examined in this chapter? What work needs to be done to ensure that effective and equitable mathematics instruction occurs more consistently across your school or district?

Before You Read

After You Read

Action Steps

1. Identify three key takeaways for further conversations with your colleagues about the need to catalyze change.
2. Together, draft an action plan for how your school or district might use the Mathematics Teaching Framework to support equitable mathematics instruction. Include shared goals, action steps and tasks, and identify resources and supports needed to carry out the plan.

## Chapter 5

### Building a Foundation of Deep Mathematical Understanding

**Key Recommendation:** Early childhood settings and elementary schools should build a strong foundation of deep mathematical understanding, emphasize reasoning and sense making, and ensure the highest-quality mathematics education for each and every child.

Before You Read

After You Read

Action Steps

Children deserve rich opportunities to develop as strong learners, doers, and sense makers of mathematics. What would it take for every child to leave elementary school deeply understanding mathematics, enjoying mathematics, and believing they are capable of success in mathematics?

Before You Read

After You Read

Action Steps

After reading chapter 5, engage in conversations centered on the following tasks and questions for considering ways to deepen children’s understanding of mathematics.

#### 1. Practices and Processes

The four pairs of interrelated practices and processes that are particularly relevant for early childhood and elementary mathematics are (1) representing and connecting, (2) explaining and justifying, (3) contextualizing and decontextualizing, and (4) noticing and using mathematical structures.

- What might you see and hear in a lesson as evidence that children are engaged as doers of mathematics through the interrelated practices and processes?
- To what extent do children in your school have opportunities to develop the interrelated mathematical practices and processes?

## 2. Whole Number Concepts and Operations

- What important messages emerged for you in the sections on (a) number and operation sense, (b) subitizing, (c) place value, (d) basic number combinations and fluency, and (e) moving from additive to multiplicative thinking?
- How do the important messages support expectations for deepening children’s learning of whole number concepts and operations?
- What are some implications for examining the mathematics curriculum in your school or district and for supporting teachers in improving their mathematics instruction?

## 3. Fraction Concepts and Operations

- What important messages emerged for you in the sections on (a) unit fractions, (b) number line representations, and (c) real-world fraction contexts?
- How do the important messages support expectations for deepening children’s learning of fraction concepts and operations?
- What are some implications for examining the mathematics curriculum in your school or district and for supporting teachers in improving their mathematics instruction?

## 4. Early Algebraic Concepts and Reasoning

- What important messages emerged for you in the sections on (a) the equals sign, (b) properties of the operations, and (c) functional relationships?
- How do the important messages support expectations for deepening children’s learning of early algebraic concepts and developing algebraic reasoning?
- What are some implications for examining the mathematics curriculum in your school or district and for supporting teachers in improving their mathematics instruction?

## 5. Data Concepts and Statistical Thinking

- What important messages emerged for you in the sections on (a) engaging in statistical inquiry, (b) creating data displays, and (c) interpreting or telling the data story?
- How do the important messages support expectations for deepening children’s learning of data concepts and developing statistical thinking?
- What are some implications for examining the mathematics curriculum in your school or district and for supporting teachers in improving their mathematics instruction?

6. Geometry and Measurement Concepts and Spatial Reasoning

- What important messages emerged for you in the sections on (a) spatial reasoning, (b) shape attributes, and (c) co-constructing definitions?
- How do the important messages support expectations for deepening children’s learning of geometry and developing spatial reasoning?
- What are some implications for examining the mathematics curriculum in your school or district and for supporting teachers in improving their mathematics instruction?

Before You Read

After You Read

Action Steps

1. Choose one of the interrelated practices and processes to target for increased attention in your classroom or school. Identify your expectations for students and determine specific teaching actions that can be incorporated into daily mathematics instruction.
2. Collaboratively with other stakeholders, choose one content domain and critically examine the mathematics curriculum in your school or district for alignment to the ideas raised in the chapter and for areas to strengthen.

## Chapter 6

### Next Steps for Catalyzing Change

The recommendations highlighted in *Catalyzing Change in Early Childhood and Elementary Mathematics* call attention to the necessary and critical conversations that must occur to ensure the highest-quality mathematics education for each and every child. It is our responsibility to launch every child on their mathematical journey with confidence in themselves as knowers, doers, and sense makers of mathematics and with the realization that *each and every person belongs in mathematics*. (p. 128)

Before You Read

After You Read

Action Steps

*Catalyzing Change* calls for a just, equitable, and inclusive mathematics education system for each and every child. Critical conversations, self-reflections, and collaborative actions are needed among all stakeholders in early childhood and elementary mathematics. Which stakeholders might you want to engage in these conversations in your school, district, or community?

Before You Read

After You Read

Action Steps

After reading chapter 6, engage in conversations centered on the following tasks and questions to examine actions for stakeholders in early childhood and elementary mathematics.

1. Review the suggested actions for your role. Which actions can you take immediately, in the short-term, and in the long-term?
2. What additional actions would you add to the suggestions related to your role to enact the recommendations in *Catalyzing Change*? In what ways would these additional actions be beneficial for children’s learning of mathematics?
3. Suggested actions are provided for the stakeholder groups, including teachers, leaders, policy makers, and teacher educators. Which actions would you prioritize for each stakeholder group?
5. What additional actions would you add for stakeholders that would support enacting the recommendations in *Catalyzing Change in Early Childhood and Elementary Mathematics*?

6. Given your local context, what other stakeholders would you invite into these critical conversations in your school, district, or community?

Before You Read

After You Read

Action Steps

1. Identify and write down your most salient takeaway from the *Catalyzing Change* book study. Consider how this salient point is influencing you personally and professionally, and then set a goal toward making mathematics education more equitable for each and every child.
2. *Catalyzing Change in Early Childhood and Elementary Mathematics* raises issues for critical examination by all stakeholders in mathematics education and calls for systemic approaches that ensure access and equity for each and every child (p. 123). With other stakeholders, develop an action plan organized by the four key recommendations. For each recommendation, identify a target goal, operational steps, milestones, and resources needed for enacting the plan.