What Is the Anatomy of a Good Review?

What makes a good review?

- The strengths and weaknesses of the article are indicated.
- The comments are as specific as possible.
- The mathematics and the pedagogy are examined carefully.
- The tone and substance of the review are respectful.
- The comments that you include are appropriate to forward to the author.

Just as you would guide your students, mention the strengths of the manuscript first and the weaknesses later. Frame the criticisms of the manuscript in a way that suggests steps that the author might take to improve the piece. *Each author needs specifics if he or she is to improve the content or style of the manuscript*. There is a place on the form to include confidential comments to the staff or Editorial Panel that you DO NOT want the author to see. However, what goes in the main review should be professional and offer constructive criticism, irrespective of the quality of the manuscript.

Although the Editorial Panel and journal editors will examine the mathematics, it is also the referee’s responsibility to check the mathematics thoroughly. We make every effort to produce a high-quality, error-free journal; that goal can only be accomplished with careful attention to accuracy by everyone involved in the process.

What are your recommendation options?

You will be asked to select one of these three categories for your final recommendation: reject, revise, or accept.

REJECT: Recommend that a manuscript be rejected if you feel the topic or its treatment is already well known by the journal readers. If the author’s ideas are difficult to understand or are not well executed and you feel that the author will be unable to improve the quality of the manuscript significantly, then it should be rejected. Also recommend REJECT if you feel that the ideas in the manuscript have the potential to make a contribution to the journal but the manuscript requires an extensive revision in style or organization.

REVISE: These manuscripts show strong promise but may need some stylistic improvement and some minor additions or deletions in the text. If you choose this option, the authors <u>**may be sent your comments**</u> and will be given a deadline by which to return the revision for a final review. The Editorial Panel may request that the revised manuscript be sent to you for a second review.

ACCEPT: Manuscripts in this category should address important topics in a clear, readable fashion. Minor editorial changes may be needed but these should be ones that the author can easily complete. Remember that comments from the referees are sent to the author as an aid in revising the article.

What does the review look like?

To help you get started, examine the items in this packet. They will give you guidelines for your review as well as an example of a published manuscript. The items include:

- Quick Reference Guide: Characteristics of a Helpful Review
- Manuscript Evaluation Form for Referees
- Case Study:
    - Part 1: Manuscript
    - Part 2: Referee Recommendations to Panel and Feedback to Author
    - Part 3: Editorial Panel Feedback to Author
    - Part 4: Published Article

Thank you for your interest in serving the mathematics education community by volunteering for this important task. If you have any questions, please e-mail the journal editor.

Teaching Children Mathematics: tcm@nctm.org.
Mathematics Teaching in the Middle School: mtms@nctm.org
Mathematics Teacher: mt@nctm.org

Quick Reference Guide: Characteristics of a Helpful Review

1. Read the review criteria and the questions on the evaluation form before reading the manuscript.

2. Summarize the intent and content of the article.

3. Provide a rationale for your recommendation to reject, revise, or accept.

4. Read the manuscript with these basic issues in mind:

   - Does the article add new information/ideas to the field or does it duplicate available materials?
   - Is the writing either too basic or too technical?
   - Did you identify any incorrect mathematics or inappropriate pedagogy?
   - Is the length adequate/appropriate for the topic?
   - Are all the figures necessary? Do the figures add something new to the text?
   - Is the manuscript appropriate for the audience? (Is it useful to teachers at various levels across grades, to mathematics supervisors, special educators, or teacher educators?)

5. Begin with positive comments. Point out both strengths and weaknesses of the article.

6. Include specific suggestions for improvements:

   - Correct mathematics as appropriate
   - Suggest deletions and/or additions
   - Consider reorganization of the manuscript as well as possible changes in title, subheadings, introduction, and conclusion.

7. Avoid personal biases.

8. Make comments that can be reproduced and shared with the author. Your comments and suggested revisions may be submitted in bulleted form. Where applicable, please refer to the manuscript page number and line number.

Mathematics Teaching in the Middle School
Manuscript Evaluation Form for Referees

This manuscript has been submitted for publication in MATHEMATICS TEACHING IN THE MIDDLE SCHOOL. Carefully check the mathematics, pedagogy, and the use of technology involved in the manuscript, where applicable. You will receive a copy of your review after you click the submit recommendation button.. If any comments you make do not show up fully in the box, please make notes as to the main idea. Sexual, ethnic, racial, or other stereotypes should be noted. Thank you for your thoughtful consideration of this material.

Are you a new referee for MTMS?

○ Yes ○ No

1. For which audience is this manuscript intended?

a. primary audience

○ Classroom Teachers ○ Teacher Educators ○ Supervisors ○ Consultants

b. secondary audience

○ Classroom Teachers ○ Teacher Educators ○ Supervisors ○ Consultants

2. What is the most likely contribution of this manuscript?

a. primary contribution

○ stimulates thinking about the practice of teaching

○ contributes to readers' understanding of mathematics

○ directly applicable in the classroom (Is there evidence of classroom use?)

○ directly applicable to teacher education programs (Is there evidence of use in these programs?)

○ adds to readers' understanding of appropriate pedagogy

○ addresses students with diverse needs

b. secondary contribution

○ stimulates thinking about the practice of teaching

○ contributes to readers' understanding of mathematics

○ directly applicable in the classroom (Is there evidence of classroom use?)

○ directly applicable to teacher education programs (Is there evidence of use in these programs?)

○ adds to readers' understanding of appropriate pedagogy

○ addresses students with diverse needs

3. Rate the manuscript by selecting your level of agreement with each statement below.

Use this scale:
1. disagree; 2. slightly disagree; 3. slightly agree; 4. Agree

a. The math content is significant and accurate.

○ 1 ○ 2 ○ 3 ○ 4 ○ N/A

b. The topic is appropriate for the middle school.

○ 1 ○ 2 ○ 3 ○ 4 ○ N/A

c. The ideas/activities are not widely known or practiced.

○ 1 ○ 2 ○ 3 ○ 4 ○ N/A

d. Major claims are documented.

1 2 3 4 N/A

e. The writing is of high quality (mechanics, grammar).

1 2 3 4

f. The ideas are well organized and clearly communicated.

1 2 3 4

g. The manuscript is consistent with equity concerns.

1 2 3 4

4. Is there evidence of student reasoning and sense-making in this article?

Yes No

5. What is your recommendation for this article?

Please Select

6. Comments, criticisms, suggestions that will be sent to the author(s). This should include specific directions for revisions if that is your recommendation.

Please do not reveal your identity. These comments will be sent as they are to the Author.

7. Additional comments for the Editorial Panel that explain or expand upon the recommendation.

8. Are you willing to mentor the Author?

Yes No

CASE STUDY
Part 1:
Manuscript

MTMS CASE STUDY
Part 1: Manuscript #10-05-071-3B

Filling fruit baskets using greatest common factor

As suggested by Stallings (2007), posing mathematics problems in different ways will raise the level of cognitive demand on students because it pushes them to think more deeply about mathematics. By engaging students in a task that requires them to determine their own solution strategies, students will gain a deeper understanding of the mathematical concept explored through the task. In this article, an activity is described in which a problem was posed to a class of seventh-grade students and the solution led to the development of the concept of greatest common factor.

The problem statement posed to the students along with an example, and the materials required for the activity are provided in Figure 1. The goal of the activity was for students to explore the concept of greatest common factor before the term "greatest common factor" was introduced. Students worked in groups of two or three to solve *The Fruit Basket Challenge* problem posed to them and used colored Popsicle sticks to model their work. The students then used colored pencils to draw a visual representation of their solutions to each *Fruit Basket Challenge* question. Students who were visual learners benefitted from this approach because they were able to understand both the number of fruit per basket as well as the number of baskets that could be made.

Prior to investigating *The Fruit Basket Challenge*, the students had explored prime numbers up to 100 using the Sieve of Eratosthenes so they were familiar with prime and composite numbers. The students were also familiar with the concepts of divisibility and prime factorization, and had available to them on the classroom wall an 8-foot by 10-foot grid of the prime factorizations of the counting numbers from 1 to 60 that was developed in a previous class period using the "Building Numbers from Primes" activity described by Burkhart (2009). The grid was used to identify patterns and relationships between prime factorizations of different numbers and was a good reference for *The Fruit Basket Challenge* problem.

The Fruit Basket Challenge:

The local food pantries are assembling food baskets, but they need your help. Each fruit basket **must** be <u>identical</u> so that it is fair to everyone. The volunteers want to make as many identical baskets as possible. <u>No fruit can be left over.</u> Please help the volunteers decide how to distribute the food by finding how many baskets they can make with the given fruit. **Use colored markers/pencils to model your work and show what should appear in a basket.

<u>Example</u>: 12 apples and 8 oranges
Answer: 4 baskets total. Each basket contains 3 apples and 2 oranges. I might draw 3 red circles for the apples and 2 orange circles for the oranges.

Materials Needed:

- Colored markers/crayons/pencils
- Four different colored manipulatives (e.g., Popsicle sticks, paper clips, bingo chips) – at least 25 of each color
- Fruit Basket Worksheet (see Appendix A)

Figure 1. The Fruit Basket Challenge Problem and Materials Needed for the Activity

After the students worked through an example problem with guidance from the teacher, they were allowed to explore on their own possible solutions to each of the *Fruit Basket Challenge* questions. As the questions began to involve more types of fruit, it was necessary for each group of students to create a key to indicate which Popsicle-stick color represented each fruit. For instance, most groups indicated that yellow was equivalent to a banana. An example of a key is provided in Figure 2.

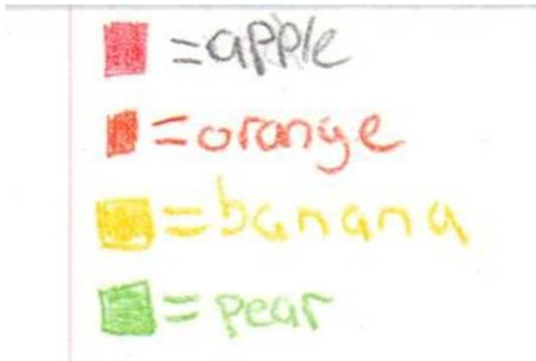


Figure 2. Example of a key showing which color represents each fruit.

There were a variety of solution strategies employed by the groups of students and almost all groups used the Popsicle sticks as an aid to finding a solution. Some of the advanced students understood that they were using the greatest common factor of the given numbers to determine the number of baskets and, since they were already familiar with the vocabulary, the students easily solved the problems using their prior knowledge of greatest common factor. Other students had to sort out the Popsicle sticks to solve each problem. They began by laying out the Popsicle sticks for the smallest number of fruit in piles of one and then distributed the Popsicle sticks representing the other fruit to those piles. The piles that did not have the same amount as the others were redistributed. For instance, in solving question 3 (20 apples and 15 oranges), initially 15 piles were created with each pile having one orange. The 20 apples were then distributed among the 15 piles. Since five piles had one orange and two apples and ten piles had one orange and one apple, the ten piles of one orange and one apple were redistributed among the five piles which resulted in each pile having four apples and three oranges. Thus, a total of five baskets were created with each basket having four apples and three oranges. Figure 3 shows a group of students solving question 3 (20 apples and 15 oranges). There are five piles of Popsicle sticks with each pile containing four red Popsicle sticks (apples) and three purple Popsicle sticks (oranges).

Figure 3. Students find a solution for 20 apples and 15 oranges.

Another solution strategy used by groups of students was to break up the same color of Popsicle sticks into equal groups and then compare the number of different colored piles to determine how to form the baskets of fruit. For instance, in solving question 2 (12 apples and 9 oranges), one group created four piles with three apples in each pile and another three piles with three oranges in each pile. Since there were at least three piles of each color (apples and oranges), three piles were created which used all the piles of oranges and three of the piles of apples. The remaining pile of three apples was then distributed to those newly formed piles. Thus, three fruit baskets were formed with each basket containing four apples and three oranges.

Probably the most difficult question for the students to solve was question 4 (7 apples and 13 oranges), since this problem requires that only one basket be made. Many of the groups questioned whether the problem was even possible to solve. However, with some questioning by the teacher, the students eventually recognized that 7 and 13 are prime numbers so they could not be divided into groups with equal numbers. In stating a solution, one student responded by saying, “Well, because we can’t split up the apples and oranges equally, we have to put them all into one basket.” Although this question consisted of two prime numbers, the students could have been given a question with relatively prime numbers, such as 7 apples and 4 oranges or 9 apples and 4 oranges, to get a solution of one basket.

Occasionally, students separated the fruit into piles using a common factor, but not the greatest common factor. For example, in question 1 (16 apples and 8 oranges), a group of students ended up with two baskets with each basket containing eight apples and four oranges. Although the students might have misunderstood the instructions to find the most baskets, it provided a good teachable moment because 8 and 4 have common factors so a different arrangement could be made to create more baskets. It was also a good opportunity to help students understand the distinction between common factors and the greatest common factor without having introduced this vocabulary. As an aid to distinguishing between these two concepts, the prime factorization grid on the classroom wall was used to compare factors of the given numbers. For instance, when students compared the factors of 16 and 8 from question 1 they noticed that both numbers have three 2s. Through questioning by the teacher, the students were able to understand the relationship between the number of fruit baskets and the prime factors that were common to the given numbers. The following dialogue is from the classroom discussion.

> TEACHER: Let's look at the chart. What do you notice about the factors of 16 and the factors of 8?
>
> STUDENTS: They each have three 2s.
>
> TEACHER: And, what do we do with the prime factorization to get the original number back – add, subtract, or what should we do?
>
> STUDENTS: We multiply!
>
> TEACHER: So what's $2\times2\times2$?
>
> STUDENTS: 8.
>
> TEACHER: And, how many fruit baskets did you find for question 1?
>
> STUDENTS: 8.
>
> TEACHER: Great! Let's look at question 2. Look at the chart and tell me what you notice about the factors of 12 and 9?

The dialogue continued in this manner as a lead-in to the first question at the end of the activity in which students were asked to identify any patterns that they noticed. The class discussion allowed students to relate the maximum number of baskets formed to the greatest common factor of the given numbers and saw that when each given number was divided by the greatest common factor that told them the number of pieces of that type of fruit to put in each basket.

Since this activity was done over a two-day period, the first day was devoted to exploration and investigation so that students had a chance to solve each question and write their final answer. Figure 4 provides an example of one group’s written solutions.

Figure 4. Final solutions provided by one group.

During the second day, a procedure for finding the greatest common factor was introduced. The activity was referenced as the procedure was explained to students. For instance, using the divide by a common prime method in question 1 (16 apples and 8 oranges) yields the following result:

2	16	8
2	8	4
2	4	2
	2	**1**

In this method, since 16 and 8 have a common prime factor of 2, we divide each number by 2 resulting in respective quotients of 8 and 4, which are written below 16 and 8. Again, since 8 and 4 have a common prime factor of 2, divide each number by 2 resulting in respective quotients of 4 and 2, which are written directly below 8 and 4. The procedure continues until the two numbers have no common prime factor remaining. Students can now see that the maximum number of baskets that can be assembled is found by multiplying the common prime numbers $2 \times 2 \times 2 = 8$ and each basket contains 2 apples and 1 orange, which are the numbers remaining after division by the greatest common factor. By relating this procedure for finding the greatest common factor to a real-life example, such as *The Fruit Basket Challenge*, students may be more likely to gain a better understanding of the outcome of the procedure.

If we are going to help students gain a deeper understanding of mathematical concepts and the procedures used with these concepts, we need to provide more opportunities for students to explore mathematics via tasks and engage students in problem-solving activities. Giving students the task of solving *The Fruit Basket Challenge* allowed them to explore the concept of greatest common factor before the term was introduced. It also provided a visual context and allowed students to develop their own solution strategies for solving problems involving the greatest common factor. Through this activity exploration, students were able to better understand the importance of finding the greatest common factor and how it related to a real-life example.

As educators, we are constantly striving to engage students, encourage learning, and answer that all-too-familiar question, "When are we ever going to use this?" This activity is designed to encourage student inquiry. Students must work with their partner to problem solve and determine a way to find how many of each fruit appear in each basket and how many baskets can be made. Most importantly, we have given the students a reason to learn the greatest common factor concept since helping food pantries is something that many people are familiar with or have done. This activity not only caters to several learning styles, including visual and kinesthetic, it also opens up discussions among students and allows the students to form some of their own conclusions before the teacher ever mentions *greatest common factor*.

References

Burkhart, J. (2009). Building numbers from primes. Mathematics Teaching in the Middle School, 15(3), 156 – 167.

Stallings, L. L. (2007). See a different mathematics. Mathematics Teaching in the Middle School, 13(4), 212 – 217.

Appendix A
Fruit Basket Worksheet

The Fruit Basket Challenge

The local food pantries are assembling food baskets, but they need your help. Each fruit basket **must** be <u>identical</u> so that it is fair to everyone. The volunteers want to make as many identical baskets as possible. <u>No fruit can be left over.</u> Please help the volunteers decide how to distribute the food by finding how many baskets they can make with the given fruit. **Use colored markers/pencils to model your work and show what should appear in a basket.

Example: 12 apples and 8 oranges

Answer: 4 baskets total. Each basket contains 3 apples and 2 oranges. I might draw 3 red circles for the apples and 2 orange circles for the oranges.

1. 16 apples and 8 oranges
2. 12 apples and 9 oranges
3. 20 apples and 15 oranges
4. 7 apples and 13 oranges
5. 7 apples and 14 oranges
6. 8 apples and 22 oranges
7. 6 apples, 8 oranges, and 10 bananas
8. 3 apples, 6 oranges, and 12 bananas
9. 10 apples, 15 oranges, and 25 bananas
10. 10 apples, 10 oranges, and 16 bananas

11. 9 apples, 15 oranges, and 6 bananas

12. 4 apples, 8 oranges, and 17 bananas

13. 4 apples, 6 oranges, 8 bananas, and 12 pears

14. 6 apples, 12 oranges, 15 bananas, and 9 pears

15. 3 apples, 3 oranges, 3 bananas, and 3 pears

16. 8 apples, 14 oranges, 4 bananas, and 8 pears

17. 9 apples, 6 oranges, 12 bananas, and 18 pears

18. 5 apples, 10 oranges, 15 bananas, and 10 pears

What patterns did you notice?

Did you ever end up with an answer in which just 1 fruit basket could be made? Why?

How did you decide how to group your items?

CASE STUDY Part 2: Referee Feedback

MTMS CASE STUDY Part 2: Referee Feedback

Referee/Reviewer #1 - 2010-06-14

General Recommendation	Revise and Resubmit-Minor
New Referee	No
Which primary audience is this manuscript intended?	Classroom Teachers
Which secondary audience is this manuscript intended?	Special Education
What is the most likely contribution?	directly applicable to classroom teaching (please indicate in the comments box below whether there is evidence of classroom use)
The math content is significant and accurate.	4
The math content is appropriate for the middle school.	3
The ideas/activities are not widely known or practiced.	3
The ideas are well organized and clearly communicated.	3
The manuscript is consistent with equity concerns.	4
Are you willing to mentor the Author?	Yes
Major claims are documented.	4
The writing is of high quality.	4

Likely Contribution Comments
There is evidence of classroom use in this article. I believe this would be a useful example of a hand-on way to introduce or reinforce greatest common factor.
Support for Rating
lines 6-8 show the target - greatest common factor lines 110-112 show the mathematical approach of using primes to divide for greatest common factor GCF is an important concept and this was a real-life application that help students discover the concept.
Appropriateness
In some middle schools, and at some grade levels, students would already have a firm grasp on

GCF using primes. However, I feel a large audience would appreciate having this approach available.
Support Appropriateness
Although many teachers create their own activities to achieve the same goal, providing an example, along with classroom examples, can be of help to less creative or more inexperienced teachers.
Examples of Clarity.
I found the second paragraph, lines 9 -17, a bit confusing. Perhaps establishing the problem posed first, with materials and strategies second would have been helpful.
Confidential Comments for the Editorial Panel
I did not see a list of the actual problems posed. That would have been helpful in understanding the student work.
Comments, criticisms, suggestions for Author
I would like to see the problems posed in a figure near the beginning of the article. I found the article straightforward with a great idea for introducing GCF. This type of activity can support many types of learners.

Referee/Reviewer #2 - 2010-06-28

General Recommendation	Revise and Resubmit-Major
New Referee	Yes
Which primary audience is this manuscript intended?	Classroom Teachers
Which secondary audience is this manuscript intended?	Teacher Educators
What is the most likely contribution?	directly applicable to classroom teaching (please indicate in the comments box below whether there is evidence of classroom use)
The math content is significant and accurate.	3
The math content is appropriate for the middle school.	4
The ideas/activities are not widely known or practiced.	3
The ideas are well organized and clearly communicated.	2
The manuscript is consistent with equity concerns.	4
Are you willing to mentor the Author?	No
Major claims are documented.	N/A

The writing is of high quality.	2

Likely Contribution Comments
• The use of manipulatives to help build understanding of greatest common factor is valuable. • Potential ties to real life situation (my students have packed fruit/food baskets for our local food bank, but distributed the fruit differently) • The practical tips for helping students to succeed with this activity are appreciated. • The author includes activities to be explored before engaging in The Fruit Basket Challenge (these included the sieve of Eratosthenes, familiarity with the concept of divisibility & prime factorization and a huge poster of the prime factorization made by the class). • An earlier MTMS article was recommended for creating the huge poster (Burkhart ~ 2009) -- good! • Evidence of classroom use: the manuscript includes dialogue between the teacher and students.
Support for Rating
• The students' strategies for solving the problems included different approaches. • I was concerned that readers might become bogged down in the solutions. Could they be presented in another way?
Support Appropriateness
• While this specific problem may not be familiar, the need for students to understand of greatest common factor is critical.
Examples of Writing Quality.
• The mechanics (spelling and punctuation) are fine. • The manuscript would benefit from editing to improve flow and readability. An example: One sentence (Lines 20 -28) contains nearly 60 words. Although the sentence contains good background info and a valuable suggestion, it could be expressed more clearly. • Some suggestions: Line 5 - 6 Awkward - perhaps instead 'deeper understanding of mathematical concepts.' Strike 'explored through the task' Line 6 - 7 Perhaps change to 'This article describes how an activity led seventh grade students to develop an understanding of the concept of greatest common factor through their solutions to different situations posed in a problem.' Line 9 Omit 'statement' in 'The problem statement posed' Line 12 -13 Omit 'posed to them and used' Change to 'to solve The Fruit Basket Challenge problem, using colored Popsicle sticks to represent different fruits.' Line 18 omit 'had' in 'the students had explored'
Examples of Clarity.
Ideas are presented in logical order, but need to be communicated more clearly.
Confidential Comments for the Editorial Panel I felt as if the writing was labored which made it feel as if too much time was spent describing parts of the lesson. I found myself wanting to rewrite the entire article. I wondered if the students wrote about what they'd learned. I question having students work on 18 problems with only one with a solution of just one basket.

I like the idea behind the manuscript. I think middle grade students would like the activity; it could help them understand the concept of greatest common factor. I think it would be a wonderful lesson to share via an online course with videos of the students working together or in a Poster Session at a conference.

Comments, criticisms, suggestions for Author
Copied from above: • The mechanics (spelling and punctuation) are fine. • The manuscript would benefit from editing to improve flow and readability. An example: One sentence (Lines 20 -28) contains nearly 60 words. Although the sentence contains good background info and a valuable suggestion, it could be expressed more clearly. • Some suggestions: Line 5 - 6 Awkward - perhaps instead 'deeper understanding of mathematical concepts.' Strike 'explored through the task' Line 6 - 7 Perhaps change to 'This article describes how an activity led seventh grade students to develop an understanding of the concept of greatest common factor through their solutions to different situations posed in a problem.' Line 9 Omit 'statement' in 'The problem statement posed' Line 12 -13 Omit 'posed to them and used' Change to 'to solve The Fruit Basket Challenge problem, using colored Popsicle sticks to represent different fruits.' Line 18 omit 'had' in 'the students had explored' • I like the idea behind the manuscript. I think middle grade students would like the activity and that it would help them understand the concept of greatest common factor. I think it would be a wonderful lesson to share via an online course with videos of the students working together or in a Poster session at a conference.

Referee/Reviewer #3 - 2010-06-09

General Recommendation	Accept; Publish
New Referee	No
Which primary audience is this manuscript intended?	Classroom Teachers
Which secondary audience is this manuscript intended?	
What is the most likely contribution?	directly applicable to classroom teaching (please indicate in the comments box below whether there is evidence of classroom use)
The math content is significant and accurate.	4
The math content is appropriate for the middle school.	4
The ideas/activities are not widely known or practiced.	N/A
The ideas are well organized and	4

clearly communicated.	
The manuscript is consistent with equity concerns.	4
Are you willing to mentor the Author?	
Major claims are documented.	4
The writing is of high quality.	4

Support for Rating
The article focuses on ways to help children understand factors and the greatest common factor. The students try to make equal baskets of fruit. For example they have 20 apples and 15 oranges. They can make 5 baskets with each basket containing 4 apples and 3 oranges. The tasks also lead children to understand prime numbers.
Appropriateness
Factoring numbers begins in 5th grade and middle school students will need to know the factors of numbers for algebra and other math concepts.
Where is Documentation needed?
The authors give examples of their students' thinking.
Examples of Clarity.
The article flowed and was easy to follow author's train of thought. Clear explanations of the tasks and the students' thinking were given.
Confidential Comments for the Editorial Panel
The author poses a task that provides a learning opportunity for all types of learners. The task can easily be modified for various levels of students.
Comments, criticisms, suggestions for Author
Well written.

CASE STUDY
Part 3:
Editorial Panel Feedback

MTMS CASE STUDY
Part 3: Editorial Panel Feedback

Comments (Panel Member)

Note	Comment
Likely Contribution Comments	
Related NCTM Resources	Orchestrating Discussions, MTMS
Recommendation Comments for the Staff	
Suggestions for possible discussion	
Comments, criticisms, suggestions for Author	As I read the Problem, I was wondering if it could have been left more open than suggesting drawing circles, etc. The one example of student solutions shows the students doing this. Did all students do this? Also, it seems like line 70 type questions should have been included in the activity as well as other 7 and 13 type problems. How was the number of problems determined? If students had the objective figured out after 5 problems, did they have to continue? Did some students need more problems to see any patterns? What decisions did you make as you ordered and wrote the problem set? I was intrigued by the variety of solution strategies that came about. It would have been interesting to see how students connected these strategies to finding gcf. For students that relied on manipulatives to sort out the fruit basket problem, were they eventually able to generalize? What is you gave them 500 apples and 400 oranges? It is great modeling for readers to see how a teachable moment was taken advantage of in line 72 - 79. Including student responses to the last three questions seems to be at the heart of learning and growth for MTMS readers. What patterns did students see? What moves did the instructor make to help students connect these to gcf or other

	mathematics? Did students have to describe their strategy? Focusing on how students solved the problem as opposed to their solutions seems to be very prudent. As a teacher, this is where the work begins. Figure 4 present solutions. Did other groups present their solutions differently? If so, a contrast would be nice. In line 108 'a procedure. . .was introduced'. If you had to introduce this procedure, how valuable was the lesson? Did any of the solution strategies seen in the classroom use this procedure? If so, make this connection for the reader. Do students have a strategy from the activity they can still rely on? If so, please elaborate on this. Line 122 - they may gain a better understanding of the outcome, but do they gain an understanding of the procedure? If students are to remember how to find the gcf, having something they can turn back to like their strategy from the Fruit Basket would be very useful. Useful comments from a reviewer: The manuscript would benefit from editing to improve flow and readability. An example: One sentence (Lines 20 -28) contains nearly 60 words. Although the sentence contains good background info and a valuable suggestion, it could be expressed more clearly. • Some suggestions: Line 5 - 6 Awkward - perhaps instead 'deeper understanding of mathematical concepts.' Strike 'explored through the task' Line 6 - 7 Perhaps change to 'This article describes how an activity led seventh grade students to develop an understanding of the concept of greatest common factor through their solutions to different situations posed in a problem.' Line 9 Omit 'statement' in 'The problem statement posed' Line 12 -13 Omit 'posed to them and used' Change to 'to solve The Fruit Basket Challenge problem, using colored Popsicle sticks to represent different fruits.' Line 18 - Rewrite In order to greatly improve the quality of what this manuscript has to offer the readers of MTMS, please consider: 1) Focus more on pedagogical considerations the instructor had to make a) teacher decisions creating worksheet b) how did instructor help student generalize beyond using procedure? c) how did instructor help students connect their fruit basket strategies to finding the gcf? d) did students share their strategies aloud? order of presentations? If the instructor made purposeful decisions about this, this should be elaborated on. 2) Include other solution posters that show student strategies for one or two of the problems

	3) include descriptions about how to find the gcf in the students own words. 4) describe the evidence the instructor has that student's understanding of gcf was improved by using this activity.

CASE STUDY
Part 4:
Published Article

A Fruitful

As students fill baskets with differing numbers of fruit, they develop the concept of GCF—often before learning the formal definition.

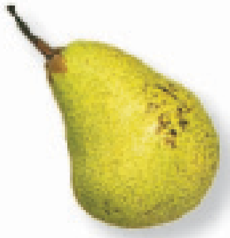
Carol J. Bell, Heather J. Leisner, and Kristina Shelley

MARTTI SALMELA/ISTOCKPHOTO.COM

Posing mathematics problems in different ways will raise students' level of cognitive demand because it will push them to think more deeply about mathematics (Stallings 2007). By engaging students in a task that requires them to determine their own solution strategies, students will gain a deeper understanding of the mathematical concept explored through the task. One activity, in particular, led seventh-grade students to develop an understanding of the concept of greatest common factor (GCF) through their solutions to different situations posed in a problem.

The problems posed to the students can be read in the **activity sheet**. The

Activity for Finding the Greatest Common Factor



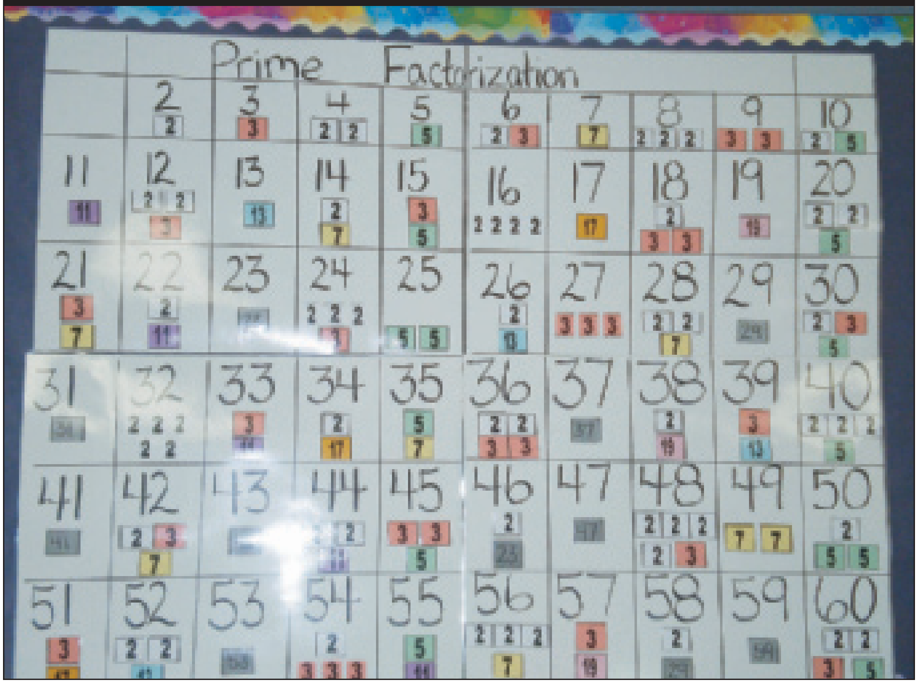
goal of the activity was for students to explore the concept of greatest common factor before the term was introduced. When developing the activity, smaller numbers were chosen for the questions so that manipulatives could easily be used. The **activity sheet** contains eighteen questions. Six questions each involving two, three, and four pieces of fruit allowed plenty of opportunity for students to explore sorting different quantities of fruit. As more numbers were included, the difficulty level increased; however, students could still see that the pattern they found working with fewer numbers held true regardless of the number of different types of fruit being sorted.

DIVIDING UP FRUIT BASKETS

Students worked in groups of two or three to solve the Fruit Basket Challenge problem, using colored Popsicle® sticks to represent different fruits. The students then used colored pencils to draw a visual representation of their solutions to each question. Visual learners benefited from this approach because they were able to understand both the number of fruit per basket as well as the number of baskets that could be made.

Most students had no knowledge of the concept of greatest common factor before investigating the Fruit Basket Challenge. The students had, however, explored prime numbers up

Fig. 1 A prime factorization grid helped students identify common factors in subsequent activities.



A Basket Analysis

The students and the teacher discuss their work thus far on filling baskets and finding the greatest common factor of 16 and 8:

Teacher: I noticed that you answered four baskets for the first question. Let's take a look at that. Here's some manipulatives from yesterday. Can you set up the problem again?
Students: Sure. We got four baskets with 4 apples and 2 oranges in each.
Teacher: Look at one of your baskets. You have 4 red sticks and 2 yellow sticks. Could you divide them into any more identical groups?
Students: I could divide this basket to make two baskets with 2 apples and 1 orange.
Teacher: Yes! Let's do that with all of them. What would that look like?
Students: If I had 2 apples and 1 orange in each basket, I'd have eight baskets altogether.
Teacher: Using the manipulatives, show me what that looks like.
[The student places the manipulatives into 8 stacks with 2 red sticks and 1 yellow stick in each stack.]
Teacher: Great. Let's look at the chart. What do you notice about the factors of 16 and the factors of 8?
Students: They each have three 2s.
Teacher: What do we do with the prime factorization to get the original number back: add, subtract, or what should we do?
Students: We multiply!
Teacher: What's $2 \times 2 \times 2$?
Students: Eight.
Teacher: How many fruit baskets did you find for question 1?
Students: Eight.

to 100 using the sieve of Eratosthenes, so they were familiar with prime and composite numbers. They were also familiar with the concepts of divisibility and prime factorization as well as the rules of divisibility. On the classroom wall an 8 ft. × 10 ft. grid listed the prime factorizations of the counting numbers from 1 to 60 (see **fig. 1**). This chart was developed in a previous class period using the Building Numbers from Primes activity described by Burkhart (2009). The grid was used to identify patterns and relationships between the prime factorizations of different numbers and was a good reference for the Fruit Basket Challenge problem.

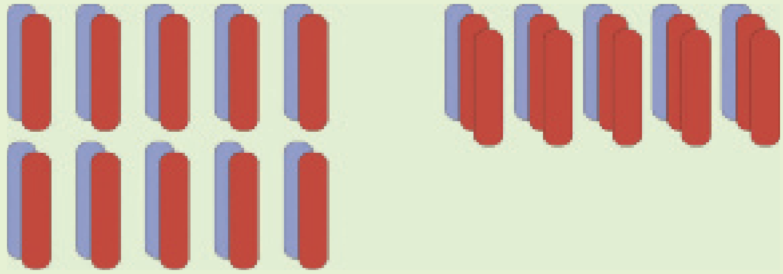
After the students worked through an example problem with the teacher, they individually explored possible solutions to each of the questions. As the questions began to involve more types of fruit, it was necessary for each group of students to create a key to indicate which Popsicle-stick color represented each fruit.

Students employed a variety of solution strategies. Almost all groups used the Popsicle sticks to help find a solution. A few of the advanced students understood that they were using the greatest common factor of the given numbers to determine the number of baskets. Since they were already familiar with the vocabulary, the students easily solved the problems using their prior knowledge of GCF. Other students had to sort the Popsicle sticks to solve each problem.

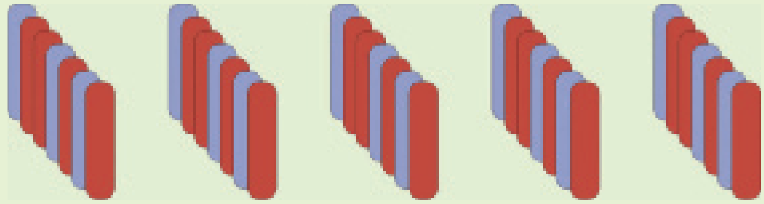
Students separated the Popsicle sticks into stacks of fruit, with the smallest number of fruit in one stack. The stacks that did not contain the same number of Popsicle sticks as the others were redistributed. For instance, in solving question 3 on the **activity sheet** (20 apples and 15 oranges), fifteen stacks were initially created, with each stack containing 1 orange (see **fig. 2a**). The 20 apples were then distributed among the fifteen stacks. Since five stacks had 1 orange and 2 apples and ten stacks had 1 orange and 1 apple, the ten stacks of 1 orange and 1 apple were redistributed among the five stacks, which resulted in each stack having 4 apples and 3 oranges (see **fig. 2b**). Thus, a total of 5 baskets were created, with each basket having 4 apples and 3 oranges. Of the five stacks of Popsicle sticks, each stack contained 4 red Popsicle sticks (apples) and 3 purple Popsicle sticks (oranges).

Another solution strategy used by groups of students was to break up the same color of Popsicle sticks into equal groups and then compare the number of different-color stacks to determine how to form the baskets of fruit. For instance, in solving question 2 (12 apples and 9 oranges), one group created four stacks with 3 apples in each stack and another

Fig. 2 Some students grouped and then regrouped their Popsicle sticks to determine the GCF of 20 apples (red) and 15 oranges (purple).



(a)
At left: Ten stacks of 1 orange and 1 apple;
at right: five stacks of 1 orange and 2 apples



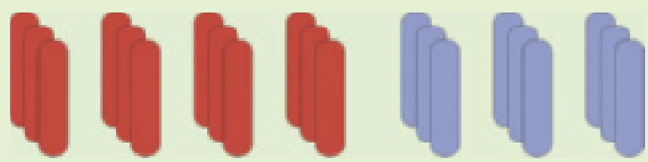
(b)
Five stacks of 3 oranges and 4 apples

By referencing the prime factorization chart, students were able to see how all the common factors of a set of numbers were used to form the GCF even though they had not been introduced to the term.

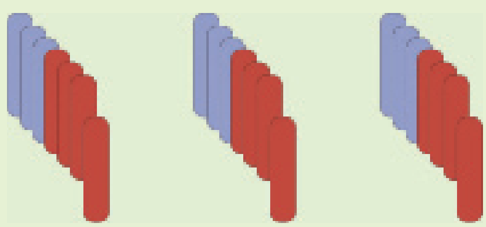
three stacks with 3 oranges in each (see **fig. 3a**). Since there were at least three stacks of each color (apples and oranges), three stacks were created that used all the stacks of oranges and three of the stacks of apples. The remaining stack of 3 apples was then distributed to those newly formed stacks (see **fig. 3b**). Thus, three fruit baskets were formed; each basket contained 4 apples and 3 oranges.

The most difficult question for students to solve was probably question 4 (7 apples and 13 oranges), since it required that only one basket be made. Many of the groups questioned whether the problem could be solved. However, with some questioning by the teacher, the students eventually recognized that 7 and 13 are prime numbers so they could not be divided into groups with equal numbers. In stating a solution, one student responded, "Because we can't split up the apples and oranges equally, we have to put them all into one basket." Although this question consisted of two prime numbers, the students could have been given a question with relatively prime numbers, such as 7 apples and 4 oranges or 9 apples and 4 oranges, to get a solution of one basket. In question 12, students observed that all three numbers (4, 8, and 17) have no common factor even though 4 and 8 have a greatest common factor of 4. The solution is thus one basket for the three different types of fruit.

Fig. 3 Other students grouped their Popsicle sticks and distributed the extras to determine the GCF of 12 apples (red) and 9 oranges (purple).



(a)
Four stacks of 3 apples and three stacks of 3 bananas



(b)
Three stacks of 3 bananas and 3 apples and an additional apple on each stack

Occasionally, students separated the fruit into stacks using a common factor, but not the *greatest* common factor. For example, in question 1 (16 apples and 8 oranges), a group of students produced four baskets; each basket contained 4 apples and 2 oranges. In this case, the students identified a common factor but not the *greatest* common factor.

Although the students might have misunderstood the instructions for finding the most baskets, it provided a good teachable moment because 4 and 2 have a common factor. A different arrangement could be made to create more baskets. It was also a good opportunity to help students understand the distinction between common factors and the greatest common factor without having introduced this vocabulary.

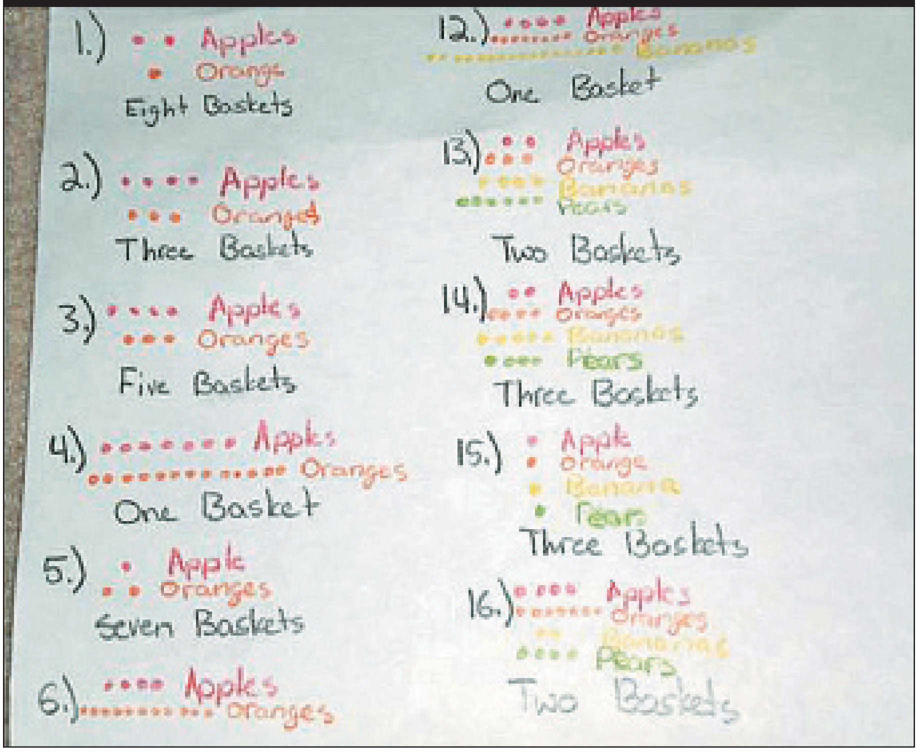
To distinguish between these two concepts, the prime factorization chart on the classroom wall was used to compare factors of the given numbers. For instance, when students compared the factors of 16 and 8 from question 1, they noticed that both numbers have three 2s. Through questioning by the teacher (see the **sidebar** on p. 224) and using the prime factorization chart on the wall (see **fig. 1**), students were able to understand the relationship between the number of fruit baskets and the prime factors that were common to the given numbers.

As the dialogue continued, the students in this group were better able to understand how the common factors in the chart were used to determine the number of baskets needed. This connection helped students when they were asked to identify any patterns that they noticed. Class discussion allowed students to relate the maximum number of baskets formed to the GCF of the given numbers. The students concluded that when each given number was divided by the GCF, that amount told them the number of pieces of that type of fruit to put in each basket.

BUILDING A DEFINITION OF GREATEST COMMON FACTOR

Since this activity was done over a two-day period, the first day was devoted to exploration and investigation so that students had a chance to solve each question and write their final answer. One group's written solutions are shown in **figure 4**. As homework

Fig. 4 One group presented a colorful visual representation of their final solutions.



from the first day, students reflected on the patterns they discovered so that they could discuss their ideas with their group members and other classmates the next day. During the second day, students used their work to develop a method for finding the greatest common factor.

When asked to share their techniques for solving the first few questions in the activity, some students explained that they were familiar with the term *factor* and that filling the baskets was related to finding a common factor. Other students had not yet been able to move away from the manipulatives, so a document camera was used to visually explain the solutions. All students then saw the connection. By referencing the prime factorization chart on the wall during the class discussion, some were able to see how all the common factors of a set of numbers were used to form the greatest common factor even though they had not been introduced to the term. Students were asked questions involving larger numbers on the grid, such as 40 and 60, so that they could use the patterns they found to develop a procedure for calculating the GCF.

Since students were familiar with the rules of divisibility, it was natural for them to use the divide-by-a-common-prime method when developing a procedure for calculating the greatest common factor. For instance, using this prime method in question 1 (16 apples and 8 oranges) yields the result in **figure 5**. In this method, since 16 and 8 have a common prime factor of 2, we divide each number by 2, resulting in respective quotients of 8 and 4, which are written below 16 and 8. Again, since 8 and 4 have a common prime factor of 2, dividing each number by 2 results in respective quotients of 4 and 2, which are written directly below 8 and 4. The procedure continues until the two numbers have no common prime factor remaining. Students could see that the maximum number of baskets that could be assembled was found by multiplying the common prime numbers $2 \times 2 \times 2 = 8$ and that each basket contained 2 apples and 1 orange, which were the numbers remaining after division by the GCF.

Fig. 5 This table demonstrated the divide-by-a-common-prime method to find the GCF of 16 and 8.

2	16	8
2	8	4
2	4	2
	2	**1**

BUNDLING THE WORK TOGETHER

By using a real-life example, such as the Fruit Basket Challenge, students may be more likely to remember how to calculate the GCF and why it is important. If we are going to help students gain a deeper understanding of mathematical concepts and the procedures used with these concepts, we need to provide more opportunities for students to explore mathematics through specific tasks and engage students in problem-solving activities. Giving students the Fruit Basket Challenge allowed them to explore the concept of greatest common factor before the term was introduced. It also provided a visual context and allowed them to develop their own solution strategies for solving problems involving the GCF. Through this activity exploration, students were able to better understand the importance of finding the GCF and how it related to a real-life example.

As educators, we are constantly striving to engage students, encourage learning, and answer that all-too-familiar question, "When are we ever going to use this?" This activity is designed to encourage student inquiry. Students must work with a partner to problem solve and determine a way to find how many of each fruit should be in each basket and how many baskets can be made. Most important, we have given students a reason to learn the concept of the greatest common factor since helping food pantries is something that many people are familiar with or have done. This activity not only caters to several learning styles, including visual and kinesthetic, but also opens up discussions among students and allows them to form their own conclusions before the teacher mentions the term *greatest common factor*.

REFERENCES

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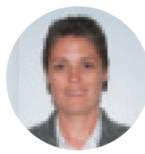

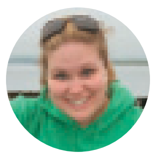

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Name ______________________________

THE FRUIT BASKET CHALLENGE

Local food pantries are assembling food baskets for donation. Determine how many baskets volunteers can make using the given fruit, so that each basket contains the same number of each kind of fruit. The goal is to make as many identical baskets as possible, with no fruit leftover. Use colored markers or pencils to model your work, and show what should appear in a basket.

Example: 12 apples and 8 oranges
Answer: 4 baskets total. Each basket contains 3 apples and 2 oranges. Draw 3 red circles for the apples and draw 2 orange circles for the oranges.

1. 16 apples and 8 oranges
2. 12 apples and 9 oranges
3. 20 apples and 15 oranges
4. 7 apples and 13 oranges
5. 7 apples and 14 oranges
6. 8 apples and 22 oranges
7. 6 apples, 8 oranges, and 10 bananas
8. 3 apples, 6 oranges, and 12 bananas
9. 10 apples, 15 oranges, and 25 bananas
10. 10 apples, 10 oranges, and 16 bananas
11. 9 apples, 15 oranges, and 6 bananas
12. 4 apples, 8 oranges, and 17 bananas
13. 4 apples, 6 oranges, 8 bananas, and 12 pears
14. 6 apples, 12 oranges, 15 bananas, and 9 pears
15. 3 apples, 3 oranges, 3 bananas, and 3 pears
16. 8 apples, 14 oranges, 4 bananas, and 8 pears
17. 9 apples, 6 oranges, 12 bananas, and 18 pears
18. 5 apples, 10 oranges, 15 bananas, and 10 pears
19. What patterns did you notice?
20. Did you have an answer in which just 1 fruit basket could be made? Why?
21. How did you decide how to group your items?