NCTM Draft Standards for Middle Grades Mathematics Teacher Preparation
January 21, 2019

1. Brief Introduction to the program standards

Attached are the 2018 revised standards for the preparation of middle grades teachers of mathematics. The standards have been modified and streamlined, with the content preparation of teachers reflecting the major areas of study of middle grades mathematics. Increased attention has been given to equity, rigorous mathematics understanding for middle grades students, and the development of positive mathematical identity.

This revision reflects current conversation in the mathematics community regarding not just standards of content, but the deepening urgency to address the nature of effective mathematics teaching and learning. These standards attend to current proposals for the preparation of teachers and related recent calls for reform of the high school mathematics. They also take into account the need to prepare teachers to engage students with the college and career ready standards.

These standards are influenced by the complementary work of other organizations who are concerned about the preparation of mathematics teachers and were informed heavily by the standards developed by AMTE and CAEP. While the AMTE standards are aspirational (AMTE, 2017), these standards are designed to assist mathematics education programs in providing evidence of high-quality teacher education for middle grades mathematics teachers in today's public schools.

The adoption of the last revision of standards moved from 17 standards to 7. A maximum of 8 assessments were allows and included a required content licensure exam and the student teaching assessment. In planning what became the 2012 standards, national dialog was strongly focused on the importance of deep content knowledge for teachers. The mathematics competencies for standard 1 in number, algebra, geometry, probability and statistics, calculus, and discrete mathematics as well as the mathematical processes described in standard 2 reflected the mathematics community's specific and detailed responses. Standards 3-6 provided expanded standards and elements that focused on effective mathematics teaching that engages students in reasoning, communicating, analyzing and applying the mathematics they were learning. Because the 2012 standards and elements existed as a stand-alone document, the elements became quite dense in an effort for clarity and emphasis.

Five years later (3 ½ years with all mathematics programs using the standards after a transition), it has become evident that the level of specificity in the 2012 standards has made the designing of assessments and subsequent alignment with the standards and

elements a more cumbersome and disconnected process. In many cases, the level of specificity required extensive development of assessments that typically extended beyond what a beginning teacher is able to demonstrate and achieve.

The current revisions of standards and components (2018) were developed with a more holistic focus on mathematics content and processes, and an expanded description of the teaching practices as articulated by NCTM in the last 4 years. Based upon the formative review in August 2018 standards, components, and rubrics have been revised to ensure that the components and rubrics reflect the language of the standards and make explicit what candidate performances would be required for a program to receive national recognition. We will continue to complete the application throughout the spring as we seek feedback from our professional colleagues in NCTM, AMTE, and mathematics education program faculty.

Draft Proposed Standards:

Preamble:

The 2019 revision of the standards for the preparation of teachers have been developed to reflect the current climate and research regarding the teaching of mathematics. This revision reflects current conversation in the mathematics community regarding not just standards of content, but the deepening urgency to address the nature of effective mathematics teaching and learning for each and every student. These standards attend to current proposals for the preparation of teachers and the increasing calls for reform of the high school mathematics curriculum. They also take into account the need to prepare teachers to engage students with the college and career ready standards.

These standards build upon the work of many others who have sought to ensure that the power and beauty of mathematics are made fully evidence throughout each student's public school and higher education experiences. Guidance for this particular revision relied on MET II (2012, CBMS), GAISE (2005), Statistical Education of Teachers (ASA), Common Core State Standards for Mathematics (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), .Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME) (SIAM, 2016), K-6 Elementary Teacher Standards (CAEP, 2018), and the International Literacy Association (2018). Authors of this revision included members of the team that developed the AMTE Standards, Standards for Preparing Teachers of Mathematics (2017) and brought perspective from the development of those aspirational standards.

Using the best of what we know of the importance of the teacher in the individual classroom and the repertoire of knowledge, skills, and commitments needed to teach, the standards are heavily influenced by the growing consensus highlighted in *Principles to Actions: Ensuring Mathematical Success for All.* These principles, Teaching and Learning, Access and Equity, Curriculum, Tools and Technology, Assessment, and Professionalism, help articulate the standards and components developed for those preparing beginning teachers and supervisors of mathematics.

While there have been successes in increasing mathematics achievement as measures by NAEP, SAT, ACT and AP exams, many challenges remain for many students, including many from underrepresented groups. It is imperative that new teachers provide all students access to high quality mathematics instruction that focuses on meaning, understanding, and the application of procedural knowledge (NCTM 2014).

Ongoing challenges for the teaching profession and the continuing challenges of attracting able candidates to teach mathematics have created an urgency to refine the standards for beginning teachers. Accountability for student achievement is reflected in more specific standards and indicators regarding assessment. Candidates will enter schools where "fewer than 50 percent of U.S. high school graduates in 2016 were considered ready for college-level mathematics work"(NCTM 2018, p. xii) and where these statistics limit the personal and professional opportunities for students. Many of these same concerns also are manifested in middle schools.

Standard 1: Knowing and Understanding Meaningful Mathematics

Standard 1: Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications within and among mathematical domains of Number and Operations; Algebra and Functions; Statistics and Probability; Geometry, Trigonometry, and Measurement.

Supporting Explanation:
Standard 1 requires that middle school mathematics teacher education candidates be knowledgeable about mathematics content they will be responsible to teach. The candidates will be able to situate this knowledge by demonstrating and applying conceptual understanding, procedural fluency, and factual knowledge among the major mathematical domains: Number; Algebra and Functions; Statistics and Probability; Geometry, Trigonometry, and Measurement.

According to the National Research Council (2001) effective programs of teacher preparation support future teachers to understand the mathematics they teach, how their students learn that mathematics, and how to facilitate student learning. As one can see from the recommendation, content knowledge is one of the core components of the effective teaching of mathematics. To this point, the Association of Mathematics Teachers Educators (2017) emphasizes that a "well-prepared" beginning teacher of mathematics attain a robust understanding of mathematics. The AMTE standards highlight that a mathematics teacher entering the profession must possess both the underling mathematics and statistical content knowledge for teaching as well as engage in the mathematical and statistical practices that have been highlighted in other documents including essential mathematics concepts outlined in *Catalyzing Change* (NCTM, 2018), college and career readiness standards (e.g., Common Core State Standards for Mathematics) and research-informed practices outlined in Principles to Actions: Ensuring Mathematical Success for All (National Council of Teachers of Mathematics, 2014).

To deepen conceptual understanding, candidates will be looking forward and backward, challenging the knowledge of mathematics that they bring to their college career (MET 2) as well as acquiring deeper and more complex understanding of the mathematical domains. By revisiting domains of mathematics and expanding their knowledge, these standards require that candidates deepen their conceptual understanding. (moving beyond procedural fluency) and build their confidence in the modeling and applications of the mathematics that they are learning. Enhanced problem solving skills and further experience with multiple representations and models are emphasized as well. This standard is designed to guide best practice in the preparation of mathematics teachers who will have a mission to make accessible and facilitate the learning of high-quality mathematics for all students, first, through their own deep understand of mathematics.

1a) Essential Concepts in Number and Operations. Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of number including flexibly applying procedures, and using real and rational numbers in contexts, attending to units, developing solution strategies and evaluating the correctness of conclusions. *Major mathematical concepts in Number include number systems (particularly rational numbers); algorithmic and recursive thinking; number and set theory; ratio, rate of change, and proportional reasoning; and structure, relationships, operations, and representations.*

Supporting Explanation:
As prospective mathematics teachers begin their preparation, they typically have had success learning mathematics themselves and have confidence in their ability to learn. Some teachers of middle grades mathematics operate with the beliefs that middle grades students come with a fully operational understanding of whole numbers, operations, and the types of situations that they represent. Yet their own understanding may often be incomplete and similar to secondary mathematics teachers with the "often unstated assumption of high school mathematics that the real numbers exist and satisfy the same properties of operations as the rational numbers. Teachers need to know how to prove what is unstated in high school in order to avoid false simplifications and to be able to answer questions from students seeking further understanding" MET2. While procedural fluency is often emphasized in their earlier mathematics experiences, the exploration of university mathematics includes reexamining essential concepts of number in the context of higher levels of mathematics.

The re-exploration of the number system (from early number through the real number system) ensures that candidates will have a fully operational conceptual understanding as well as procedural knowledge of the entire real number system, including rational, irrational numbers with multiple representations. (MET2). Extended experience with the use of numbers in multiple contexts including those that represent units, necessitates ability to use strategies to select current units, scalings, and determine appropriate levels of accuracy (Catalyzing Change, 2018). Candidates are able to re-explore number theory with additional insights and understandings.

Candidates should be able to move fluidly between multiple representations of number and understand how concrete materials and technology can assist in the development of conceptual knowledge. Candidates can use tools from sketching through modeling technology to explore how tools can enhance and broaden their understanding. The re-exploration of number provides candidates the opportunity to revisit the learning progressions that come with the building of a number system from countable to infinite, continuous number systems. Candidates are engaged with properties of number systems and explore the differences in the properties of rational numbers and real numbers.

As candidates gain facility with each number set, they should also be engaged in persevering with problems that apply the various number systems to real-life, engaging problem solving.

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1b) Essential Concepts in Algebra and Functions. Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of algebra and functions including how mathematics can be used systematically to represent patterns and relationships among numbers and other objects, analyze change, and model everyday events and problems of life and society. *Essential Concepts in Algebra and Functions include algebra that connects mathematical structure to symbolic, graphical, and tabular descriptions; connecting algebra to functions; induction; and develops families of functions of discrete and continuous variables as a fundamental concept of mathematics*

1c) Essential Concepts in Statistics and Probability. Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of statistics and probability including how statistical problem solving and decision making depend on understanding, explaining, and quantifying the variability in a set of data to make decisions. They understand the role of randomization and chance in determining the probability of events. *Essential Concepts in Statistics and Probability include quantitative literacy; visualizing and summarizing data; statistical inference; probability; exploratory data analysis and applied problems and modeling.*

1d) Essential Concepts in Geometry, Trigonometry, and Measurement Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of geometry including using visual representations for numerical functions and relations, data and statistics, and networks, to provide a lens for solving problems in the physical world. *Essential Concepts in Geometry, Trigonometry, and Measurement include measurement; transformations; scale; graph theory; geometric arguments; reasoning and proof; applied problems and modeling; development of axiomatic proof; and the Pythagorean theorem.*

Standard 2 Knowing and Using Mathematical Processes

Candidates demonstrate their ability to use mathematical processes of problem solving; reasoning and explaining; modeling and using tools; and seeing structure and generalizing across mathematical domains. They understand that these practices intersect with mathematical content and contexts and that understanding relies on the ability to demonstrate these practices within and among mathematical domains and in their teaching.

2a) Problem Solving:
Candidates demonstrate {a range of mathematical problem solving} strategies to make sense of and solve non-routine problems (both contextual and non-contextual) across mathematical domains.

2b) Reasoning and Communicating:
Candidates organize their mathematical thinking and use the language of mathematics to express ideas precisely, both orally and in writing to multiple audiences.

2c) Modeling and Using Tools:
Candidates formulate, represent, analyze, and interpret mathematical models using a variety of tools including technology.

Standard 3: Knowing Students and Planning for Mathematical Learning
Candidates use knowledge of students and mathematics to plan rigorous and engaging mathematics instruction supporting students' access and learning.

Sample Supporting Explanation:

As candidates begin to transition in their roles from learners of mathematics to those who will guide the learning of others, they must begin the process of understanding the learning of others and the structure of the mathematics that they will teach.

Planning is complex for candidates; they must learn to identify and use equitable teaching strategies that support meaningful mathematics. They must move beyond their own learning to consider the diversity of individual students and groups of students that they will serve. They musts consider what they know about students' beliefs and attitudes about learning mathematics as well as how student understanding develops. They must begin to anticipate how students will make sense of mathematics based upon their understandings, their previous experience, their interaction with each other, and their beliefs about how they can learn mathematics.

Planning articulates and supports tasks, language, contexts, learning trajectories that enable and motivate students to participate meaningfully and make sense of the mathematics they are exploring and learning. It includes learning and using connections that exist among standards, curriculum documents, instructional materials, and assessment frameworks. Candidates must begin to identify, access and use supplemental resources.

Well prepared beginning middle grades teachers of mathematics apply knowledge of standards, pedagogical content knowledge, knowledge of curriculum and effective and equitable teaching practices to support students understanding, elicit and use evidence of students' thinking.

3a) Student Diversity:
Candidates identify and use students' individual and group differences to plan rigorous and engaging mathematics instruction that supports students' meaningful participation and learning.

Sample Supporting Explanation:
Candidates must work to understand the critical and deliberate professional learning necessary to acknowledge and build upon the diversity of individual students and groups of students, particularly those whose learning experiences and needs are different from their own. For the purposes of these standards, NCTM builds upon the CAEP definition of diversity: (1) Individual differences (e.g., personality, interests, learning modalities, and life experiences), and (2) group differences (e.g., race, ethnicity, ability, gender identity, gender expression, sexual orientation, nationality, language, religion, political affiliation, and socio-economic background).

In fact, these standards seek to ensure a programmatic responsibility to ensure each candidate attends specifically to a full range of students, with particular attention to students who are traditionally underrepresented and/or underserved in the learning of mathematics. Additionally, candidates must ensure that students with exceptionalities have the appropriate accommodations to ensure their success.

3b) **Students Mathematical Strengths:**
Candidates identify and use students' mathematical strengths to plan rigorous and engaging mathematics instruction that supports students' meaningful participation and learning.

3c) Positive Mathematical Identities:
Candidates understand that teachers' interactions impact individual students by influencing and reinforcing student's mathematical identities, positive or negative, and plan experiences and instruction to develop and foster students' positive mathematical identities.

Standard 4: Teaching Meaningful Mathematics

Candidates implement effective and equitable teaching practices using knowledge of mathematics content, mathematical learning progressions, mathematical processes, and students.

4a) **Rigorous Mathematics Learning Goals:**
Candidates establish rigorous mathematics learning goals for students situated within learning progressions, mathematics standards and practices, and the purposes for learning mathematics.

4b) **Engaging Students in High Cognitive Demand Learning:**
Candidates analyze, modify, sequence, and implement tasks to engage students in high cognitive demand mathematical learning experiences that promote reasoning and sense making.

4c) Mathematics-Specific Tools:
Candidates select mathematics-specific tools, including technology, to support students' learning, understanding, and application of mathematics and integrate tools into instruction.

4d) Mathematical Representations:
Candidates select mathematical representations to support students' learning, understanding and application of mathematics and implement and connect representations during instruction.

4e) Using Student Responses:

Candidates anticipate multiple student responses, potential challenges and misconceptions, and use this information to orchestrate student discussions that support and extend learning.

4f) Conceptual Understanding and Procedural Fluency:
Candidates use conceptual understanding to build procedural fluency for students through instruction that includes explicit connections between concepts and procedures.
4g) Facilitate Discourse:
Candidates facilitate discourse among students to build shared understanding of mathematical ideas and ensure that students engage in rigorous mathematics.

Standard 5: Assessing Impact on Student Learning
Candidates assess and use evidence of students' progress towards rigorous mathematics learning goals.

5a) **Assessing for Learning:**

Candidates use both informal and formal assessments to elicit students' progress toward rigorous mathematics learning goals.

5b) Candidates use data from informal and formal assessments to analyze students' progress toward rigorous mathematics learning goals.

5c) Candidates use evidence of student learning to analyze effectiveness of his/her instruction and propose adjustments to instruction to improve student learning.

Standard 6: Social and Professional Context of Mathematics Teaching and Learning

Candidates are reflective mathematics educators who collaborate with colleagues and other stakeholders to grow professionally, to support student learning, and to create more equitable mathematics learning environments.

6a) Candidates reflect on their impact on students' mathematical identities and develop professional learning goals that promote students' positive mathematical identities.

6b) Candidates communicate with families to share and discuss strategies for ensuring the mathematical success of their children.

6c) Candidates collaborate with colleagues to grow professionally and support student learning of mathematics.

6d) Candidates identify beliefs and classroom practices that produce equitable and inequitable mathematical learning experiences and outcomes for students and seek to create more equitable learning environments.

Draft Assessment Rubrics

]1a) Essential Concepts in Number and Operations. Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of number including flexibly applying procedures, and using real and rational numbers in contexts, attending to units, developing solution strategies and evaluating the correctness of conclusions. *Major mathematical concepts in Number include number systems (particularly rational numbers); algorithmic and recursive thinking; number and set theory; ratio, rate of change, and proportional reasoning; and structure, relationships, operations, and representations.*

Level 1 **The Beginning Candidate**	Level 2 **The Developing Candidate**	Level 3 **The Competent Candidate**	Level 4 **The Accomplished Candidate**
Candidate does not demonstrate conceptual understanding, procedural fluency, and/or factual knowledge for major mathematical concepts in Number.	Candidate demonstrates conceptual understanding, procedural fluency, and/or factual knowledge, but is not able to apply the major mathematical concepts in Number.	Candidate demonstrates and applies conceptual understanding, procedural fluency, and factual knowledge of major mathematical concepts in Number. Candidate uses technology to enhance their learning of Number.	Candidate demonstrates and applies conceptual understanding, procedural fluency, and factual knowledge of major mathematical concepts in Number. Candidate uses technology to enhance their learning of Number. Candidate makes connections within and among mathematical domains.

1b) Essential Concepts in Algebra and Functions. Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of algebra and functions including how mathematics can be used systematically to represent patterns and relationships among numbers and other objects, analyze change, and model everyday events and problems of life and society. *Essential Concepts in Algebra and Functions include algebra that connects mathematical structure to symbolic, graphical, and tabular descriptions; connecting algebra to functions; induction; and develops families of functions of discrete and continuous variables as a fundamental concept of mathematics*

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate does not demonstrate conceptual understanding, procedural fluency, and/or factual knowledge for major mathematical concepts in Algebra and Functions.	Candidate demonstrates conceptual understanding, procedural fluency, and/or factual knowledge, but is not able to apply the major mathematical concepts in Algebra and Functions.	Candidate demonstrates and applies understandings of major mathematics concepts, procedures, knowledge, and applications of algebra and functions. Candidate uses technology to enhance their learning of Algebra and Functions.	Candidate demonstrates and applies conceptual understanding, procedural fluency, and factual knowledge of major mathematical concepts in Algebra and Functions. Candidate uses technology to enhance their learning of Number. Candidate makes connections within and among mathematical domains.

1c) Essential Concepts in Statistics and Probability. Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of statistics and probability including how statistical problem solving and decision making depend on understanding, explaining, and quantifying the variability in a set of data to make decisions. They understand the role of randomization and chance in determining the probability of events. *Essential Concepts in Statistics and Probability include quantitative literacy; visualizing and summarizing data; statistical inference; probability; exploratory data analysis and applied problems and modeling.*

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate does not demonstrate conceptual understanding, procedural fluency, and/or factual knowledge for major mathematical concepts in Statistics and Probability.	Candidate demonstrates conceptual understanding, procedural fluency, and/or factual knowledge, but is not able to apply the major mathematical concepts in Statistics and Probability.	Candidate demonstrates and applies understandings of major mathematics concepts, procedures, knowledge, and applications of Statistics and Probability. Candidate uses technology to enhance their learning of Statistics and Probability.	Candidate demonstrates and applies conceptual understanding, procedural fluency, and factual knowledge of major mathematical concepts in Statistics and Probability. Candidate uses technology to enhance their learning of Statistics and Probability. Candidate makes connections within and among mathematical domains.

1d) Essential Concepts in Geometry, Trigonometry, and Measurement Candidates demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of geometry including using visual representations for numerical functions and relations, data and statistics, and networks, to provide a lens for solving problems in the physical world. *Essential Concepts in Geometry, Trigonometry, and Measurement include measurement; transformations; scale; graph theory; geometric arguments; reasoning and proof; applied problems and modeling; development of axiomatic proof; and the Pythagorean theorem.*

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate does not demonstrate conceptual understanding, procedural fluency, and/or factual knowledge for major mathematical concepts in Number.	Candidate demonstrates conceptual understanding, procedural fluency, and/or factual knowledge, but is not able to apply the major mathematical concepts in Number.	Candidate demonstrates and applies understandings of major mathematics concepts, procedures, knowledge, and applications of algebra and functions. Candidate uses technology to enhance their learning of Algebra and Functions.	Candidate demonstrates and applies conceptual understanding, procedural fluency, and factual knowledge of major mathematical concepts in Number. Candidate uses technology to enhance their learning of Number. Candidate makes connections within and among mathematical domains.

2a) Problem Solving:

Candidates demonstrate {a range of mathematical problem solving} strategies to make sense of and solve non-routine problems (both contextual and non-contextual) across mathematical domains.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate solves non-routine problems (contextual or non-contextual) when given a strategy.	Candidate solves non-routine problems (contextual and non-contextual) when given a strategy.	Candidate demonstrates use of mathematical problem solving strategies to make sense of and solve contextual and non-contextual problems in more than one mathematical domain.	Candidate demonstrates coordination and unprompted use of multiple mathematical problem solving strategies when making sense of and solving contextual and non-contextual problems across mathematical domains. Candidate can compare strategies and make connections across domains.

2b) Reasoning and Communicating:
Candidates organize their mathematical thinking and use the language of mathematics to express ideas precisely, both orally and in writing to multiple audiences.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate is unable to organize their own	Candidate is able to organize their own mathematical	Candidate is able to organize their own mathematical	Candidate is able to organize their own mathematical

mathematical thinking and does not use the language of mathematics.	thinking using the language of mathematics with prompting and support. Candidates are able to express their ideas orally or in writing.	thinking and use the language of mathematics to express ideas precisely, both orally and in writing to multiple audiences.	thinking and use of the language of mathematics to express ideas precisely, both orally and in writing to multiple audiences. Candidate seeks out opportunities to share their thinking with professors, peers, and colleagues.

2c) Modeling and Using Tools:
Candidates formulate, represent, analyze, and interpret mathematical models using a variety of tools including technology.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate does not demonstrate the ability to use models and tools OR Is unable to formulate and interpret mathematical models.	Candidate uses mathematical models and tools AND Formulates and represents, but needs assistance in analyzing and interpreting models.	Candidate uses mathematical models and tools, including technology, to formulate, represent, analyze, and interpret mathematical models using a variety of tools including technology from real-world contexts or mathematical problems.	Candidate uses mathematical models and tools including technology, to formulate, represent, analyze and interpret mathematical models derived from real-world context and mathematical problems. The candidate seeks opportunities to extend and reformulate models based upon analysis.

3a) Student Diversity:

Candidates identify and use students' individual and group differences to plan rigorous and engaging mathematics instruction that supports students' meaningful participation and learning.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate does not use students' individual differences or group differences in planning rigorous and engaging mathematics instruction.	Candidate uses students' individual or group differences in planning rigorous and engaging mathematics instruction for a subset of students.	Candidate uses students' individual and group differences in planning rigorous and engaging mathematics instruction that supports meaningful participation and learning by across a full range of students.	Candidate uses students' individual and group differences in planning rigorous and engaging mathematics instruction that supports meaningful participation and learning by each and every student.

3b) **Students Mathematical Strengths:**
Candidates identify and use students' mathematical strengths to plan rigorous and engaging mathematics instruction that supports students' meaningful participation and learning.

Level 1 The Beginning Candidate	Level 2 The Developing Candidate	Level 3 The Competent Candidate	Level 4 The Accomplished Candidate
Candidate does not use students' mathematical strengths in planning rigorous and engaging mathematics instruction.	Candidate uses students' mathematical strengths in planning rigorous and engaging mathematics instruction for a subset of students.	Candidate uses students' mathematical strengths in planning rigorous and engaging mathematics instruction that supports meaningful participation and learning by across a full range of students.	Candidate uses students' mathematical strengths in planning rigorous and engaging mathematics instruction that supports meaningful participation and learning by each and every student.

3c) Positive Mathematical Identities:
Candidates understand that teachers' interactions impact individual students by influencing and reinforcing student's mathematical identities, positive or negative, and plan experiences and instruction to develop and foster students' positive mathematical identities.

Level 1 The Beginning Candidate	Level 2 The Developing Candidate	Level 3 The Competent Candidate	Level 4 The Accomplished Candidate
Candidate does not recognize that teachers' interactions impact individual students by influencing and reinforcing student's mathematical identities, positive or negative; or candidate does	Candidate understands that teachers' interactions impact individual students by influencing and reinforcing student's mathematical identities, positive or negative.	Candidate understands that teachers' interactions impact individual students by influencing and reinforcing student's mathematical identities, positive or negative.	Candidate understands that teachers' interactions impact individual students by influencing and reinforcing student's mathematical identities, positive or negative.

not plan experiences and instruction to develop and foster students' positive mathematical identities for a subset of students.	Candidate plans experiences and instruction to develop and foster students' positive mathematical identities for a subset of students.	Candidate plans experiences and instruction to develop and foster students' positive mathematical identities across a full range of students.	Candidate plans experiences and instruction to develop and foster students' positive mathematical identities for each and every student.

4a) Rigorous Mathematics Learning Goals:
Candidates establish rigorous mathematics learning goals for students situated within learning progressions, mathematics standards and practices, and the purposes for learning mathematics.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate establishes mathematics learning goals for students which lack rigor.	Candidate establishes mathematics learning goals for students which demonstrate some level of rigor but are not situated within learning progressions, mathematics standards and practices, or the purposes for learning mathematics.	Candidate establishes rigorous mathematics learning goals for students situated within learning progressions, mathematics standards and practices, and the purposes for learning mathematics.	Candidate establishes rigorous mathematics learning goals for students situated within learning progressions, mathematics standards and practices, and the purposes for learning mathematics and recognizes and uses connections when establishing goals.

4b) Engaging Students in High Cognitive Demand Learning:

Candidates analyze, modify, sequence, and implement tasks to engage students in high cognitive demand mathematical learning experiences that promote reasoning and sense making.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate selects tasks without regard to engaging students in in high cognitive demand mathematical learning experiences.	Candidate analyzes, modifies, and sequences tasks which would engage students in high cognitive demand mathematical learning experiences, but implementation fails to maintain a high cognitive demand when the tasks and/or engage students in reasoning and sense making.	Candidate analyzes, modifies, sequences, and implements tasks to engage a full range of students in high cognitive demand mathematical learning experiences that promote reasoning and sense making.	Candidate analyzes, modifies, sequences, and implements tasks to engage each and every student in high cognitive demand mathematical learning experiences that promote reasoning and sense making.

4c) Mathematics-Specific Tools:
Candidates select mathematics-specific tools, including technology, to support students' learning, understanding, and application of mathematics and integrate tools into instruction.

Level 1 The Beginning Candidate	Level 2 The Developing Candidate	Level 3 The Competent Candidate	Level 4 The Accomplished Candidate
Candidate selects tools without regard to supporting students' learning, understanding, and application of mathematics.	Candidate selects mathematics-specific tools, including technology, to support students' learning, understanding, and application of mathematics and is unable or unsuccessful in integrating tools into instruction.	Candidate selects mathematics-specific tools, including technology, to support a full range of students' learning, understanding, and application of mathematics and integrates tools into instruction.	Candidate selects mathematics-specific tools, including technology, to support each and every students' learning, understanding, and application of mathematics and integrates tools into instruction.

4d) Mathematics Representations:
Candidates select mathematical representations to support students' learning, understanding and application of mathematics and implement and connect representations during instruction.

Level 1 The Beginning Candidate	Level 2 The Developing Candidate	Level 3 The Competent Candidate	Level 4 The Accomplished Candidate
Candidate selects mathematical **representations** without regard to supporting students' learning, understanding and application of mathematics.	Candidate selects mathematical **representations** to support students' learning, understanding and application of mathematics and is unable or unsuccessful in implementing or connecting	Candidate selects mathematical **representations** to support students' learning, understanding and application of mathematics and implements and connects representations during instruction.	Candidate selects and connects mathematical **representations** to support students' learning, understanding and application of mathematics and implements and facilitates

	representations during instruction.		students making connections between representations.

4e) Using Student Responses:
Candidates anticipate multiple student responses, potential challenges and misconceptions, and use this information to orchestrate student discussions that support and extend learning.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate anticipates student responses, potential challenges, or misconceptions. Candidate is unable to use this information to inform instruction.	Candidate anticipates multiple student responses. Candidate anticipates potential challenges or misconceptions. Candidate does not use this information to organize and facilitate student discussions or discussion does not support and extend student learning.	Candidate anticipates multiple student responses, potential challenges and misconceptions, and uses this information to orchestrate student discussions that support and extend learning.	Candidate considers individual and group differences when anticipating multiple student responses, potential challenges and misconceptions, and uses this information to orchestrate student discussions that support and extend learning for each and every student.

4f) **Conceptual Understanding and Procedural Fluency:**
Candidates use conceptual understanding to build procedural fluency for students through instruction that includes explicit connections between concepts and procedures.

Level 1 The Beginning Candidate	Level 2 The Developing Candidate	Level 3 The Competent Candidate	Level 4 The Accomplished Candidate
Candidate designs instruction that does not include both conceptual understanding and procedural fluency.	Candidate designs instruction that includes both conceptual understanding and procedural fluency but the conceptual understanding does not serve as a foundation for or is connected to developing procedural fluency.	Candidate designs and implements instruction which uses conceptual understanding to build procedural fluency including explicit connections between concepts and procedures.	Candidate designs and implements instruction which uses conceptual understanding to build procedural fluency including explicit connections between concepts and procedures. Candidate facilitates students making connections between procedures and concepts.

4g) Facilitate Discourse:
Candidates facilitate discourse among students to build shared understanding of mathematical ideas and ensure that students engage in rigorous mathematics.

Level 1 The Beginning Candidate	Level 2 The Developing Candidate	Level 3 The Competent Candidate	Level 4 The Accomplished Candidate
Candidate is unable to facilitate discourse among students in support of building shared understanding of mathematical ideas.	Candidate facilitates discourse among students to build shared understanding of mathematical ideas. Discourse is limited to a subset of students.	Candidate facilitates discourse among students to build shared understanding of mathematical ideas and ensure that a full range of students engage in rigorous mathematics.	Candidate facilitates discourse among students to build shared understanding of mathematical ideas and ensures that each and every student engage in rigorous mathematics.

5a) **Assessing for Learning:**

Candidates use both informal and formal assessments to elicit students' progress toward rigorous mathematics learning goals.

Level 1 **The Beginning Candidate** Candidate uses informal and/or formal assessments, but assessments do not measure rigorous mathematics learning goals.	**Level 2** **The Developing Candidate** Candidate uses informal or formal assessments to elicit progress towards rigorous mathematics learning goals.	**Level 3** **The Competent Candidate** Candidate selects, creates or adapts assessments and uses both informal and formal assessments to elicit progress towards rigorous mathematics learning goals for a full range of students.	**Level 4** **The Accomplished Candidate** Candidate selects, creates or adapts assessments and uses both informal and formal assessments to elicit progress towards rigorous mathematics learning goals for students' individual learning needs.

5b) Candidates use data from informal and formal assessments to analyze students' progress toward rigorous mathematics learning goals.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**

Candidate does not use data from assessments to analyze students' progress toward rigorous mathematics learning goals.	Candidate uses data from informal or formal assessments to analyze students' progress toward rigorous mathematics learning goals.	Candidate uses data from informal and formal assessments to analyze a full range of students' progress toward rigorous mathematics learning goals.	Candidate uses data from informal and formal assessments to analyze each individual student's progress toward rigorous mathematics learning goals.

5c) Candidates use evidence of student learning to analyze effectiveness of his/her instruction and propose adjustments to instruction to improve student learning.

Level 1 **The Beginning Candidate** Candidate does not use evidence of student learning to analyze effectiveness of their instruction, or they analyzed effectiveness of instruction without proposing adjustments to instruction.	**Level 2** **The Developing Candidate** Candidate uses evidence of student learning to analyze effectiveness of their instruction and proposes adjustments to instruction, but those adjustments are not explicitly connected to the analysis of the data.	**Level 3** **The Competent Candidate** Candidate uses evidence of student learning to analyze effectiveness of their instruction and proposes adjustments to instruction that are explicitly connected to the analysis of the data and address the full range of students.	**Level 4** **The Accomplished Candidate** Candidate uses evidence of student learning to analyze effectiveness of their instruction and propose adjustments to instruction that are explicitly connected to the analysis of the data and address the learning needs of individuals and groups of students. [Candidate spontaneously looks for patterns, aberrations, and unexpected performances across data sources to plan subsequent instruction. (supporting explanation)]

6a) Candidates reflect on their impact on students' mathematical identities and develop professional learning goals that promote students' positive mathematical identities.

Level 1 **The Beginning Candidate**	Level 2 **The Developing Candidate**	Level 3 **The Competent Candidate**	Level 4 **The Accomplished Candidate**
Candidate reflects on their impact on students' mathematical identities but does not develop professional learning goals to better promote students' positive mathematical identities.	Candidate reflects on their impact on students' mathematical identities and develops professional learning goals that promote students' positive mathematical identities, but without identifying specific strategies or resources.	Candidate reflects on their impact on students' mathematical identities and develops professional learning goals that promote students' positive mathematical identities, including specific strategies for meeting these goals.	Candidate reflects on their impact on individual student's mathematical identities and develops professional learning goals that promote students' positive mathematical identities, including specific strategies and professional resources for meeting these goals.

6b) Candidates communicate with families to share and discuss strategies for ensuring the mathematical success of their children.

Level 1 **The Beginning Candidate**	Level 2 **The Developing Candidate**	Level 3 **The Competent Candidate**	Level 4 **The Accomplished Candidate**
Candidate communicates information to families about	Candidate communicates information to families about	Candidate communicates with families about the	Candidate communicates with families about the

mathematical ideas and processes.	mathematical ideas and processes, and suggests good mathematics resources for families to contribute to the mathematical success of their children.	mathematical ideas and processes that students are exploring, suggests good mathematics resources, and provides opportunities for the candidate and families to discuss strategies for ensuring the mathematical success of their children.	mathematical ideas and processes that students are exploring, suggests good mathematics resources, and provides opportunities for the candidate and families to discuss strategies for ensuring the mathematical success of their children. Candidate seeks out opportunities in the community to understand and interact with families.

6c) Candidates collaborate with colleagues to grow professionally and support student learning of mathematics.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate identifies potential collaboration or professional learning opportunities that focus on learning and teaching in mathematics education.	Candidate collaborates with colleagues or participates in professional development and/or learning communities that focus on learning and teaching in mathematics education.	Candidate collaborates with colleagues to support student learning of mathematics. Candidate participates in professional development and/or learning communities	Candidate identifies opportunities based on targeted professional learning needs; Candidate collaborates with colleagues to support student learning of mathematics;

		that focus on learning and teaching in mathematics education.	Candidate participates in professional development and/or learning communities that focus on learning and teaching in mathematics education.

6d) Candidates identify beliefs and classroom practices that produce equitable and inequitable mathematical learning experiences and outcomes for students and seek to create more equitable learning environments.

Level 1 **The Beginning Candidate**	**Level 2** **The Developing Candidate**	**Level 3** **The Competent Candidate**	**Level 4** **The Accomplished Candidate**
Candidate is unable to identify beliefs and practices that produce inequitable mathematical learning experiences and outcomes for students.	Candidate identifies beliefs and classroom practices that produce inequitable mathematical learning experiences and outcomes for students. Candidate identifies beliefs that produce equitable mathematical learning experiences and outcomes for students.	Candidate identifies beliefs and classroom practices that produce equitable and inequitable mathematical learning experiences and outcomes for students. Candidate seeks out information to increase equitable practices and/or eliminate inequitable practices to further mathematical learning.	Candidate identifies personal beliefs, classroom practices, and systemic structures that produce equitable and inequitable mathematical learning experiences and outcomes for students. Candidate seeks out information to increase equitable practices and/or eliminate inequitable practices to further mathematical learning for individual students.

			Candidate describes ways to help traditionally marginalized students experience success.