NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Procedural Fluency

Reasoning and Decision-Making, Not Rote Application of Procedures Position

> Procedural fluency is an essential component of equitable teaching and is necessary to developing mathematical proficiency and mathematical agency. Each and every student must have access to teaching that connects concepts to procedures, explicitly develops a reasonable repertoire of strategies and algorithms, provides substantial opportunities for students to learn to choose from among the strategies and algorithms in their repertoire, and implements assessment practices that attend to all components of fluency.

Introduction

Procedural fluency can be accomplished only when fluency is clearly defined and intentionally developed. Unfortunately, the term *fluency* continues to be (incorrectly) interpreted as remembering facts and applying standard algorithms or procedures. Procedural fluency is the ability to apply procedures efficiently, flexibly, and accurately; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another (NCTM 2014, 2020; National Research Council 2001, 2005, 2012; Star 2005). For example, to add 98 + 35, a person might add 100 + 35 and subtract 2 or change the problem to 100 + 33. Procedural fluency applies to the four operations and other procedures in the K–12 curriculum, such as solving equations for an unknown. For example, to solve for x in the equation $4(x + 2) = 12$, an efficient strategy is to use relational thinking, noticing that the quantity inside the parenthesis equals 3 and therefore x equals 1. As these examples illustrate, flexibility is a major goal of fluency, because a good strategy for one problem may or may not be as effective for another problem.

Declarations

The following declarations describe necessary actions to ensure that every student has access to and develops procedural fluency. These declarations apply to computational fluency across the K–12 curriculum, including basic facts, multidigit whole numbers, and rational numbers, as well as to other procedures throughout the curriculum such as comparing fractions, solving proportions or equations, and analyzing geometric transformations.

1. **Conceptual understanding must precede and coincide with instruction on procedures.** Learning is supported when instruction on procedures and concepts is explicitly connected in ways that make sense to students (e.g., Fuson, Kalchman, and Bransford 2005; Hiebert and Grouws 2007; Osana and Pitsolantis 2013) and iterative (e.g., Canobi 2009; Rittle-Johnson, Schneider, and Star 2015). Conceptual foundations lead to opportunities to develop reasoning strategies, which in turn deepens conceptual understanding; memorizing an algorithm does not. When students use a procedure they do not understand, they are more likely to make errors and fail to notice when the answer does not make sense (Kamii and Dominick 1998; Narode, Board, and Davenport 1993). Examples of explicitly connecting procedures and concepts can be found in the Additional Resources section.

2. **Procedural fluency requires having a repertoire of strategies.** Before students can flexibly choose an appropriate strategy, they must have strategies from which to choose. Strategies are flexible ways to solve a problem (e.g., compensation); algorithms are step-by-step procedures. Although both are important in mathematics, strategies should not be presented as rigid, step-by-step processes. Students should be able to flexibly use and adapt strategies and switch to a different strategy when their first choice is not working well (NCTM 2020). Every student must have the opportunity to learn more than one method. Limiting students to only one method puts them at a disadvantage, denying them access to more intuitive methods and the opportunity to flexibly choose a method that fits the problem at hand.

3. **Basic facts should be taught using number relationships and reasoning strategies, not memorization.** Students who learn fact strategies outperform students who learn through other approaches (e.g., Baroody et al. 2016; Henry and Brown 2008; Brendefur et al. 2015). Basic fact strategies use number relationships and benchmarks and thus support students, emerging conceptual understanding and flexibility (Bay-Williams and Kling 2019; Davenport et al. 2021). Strategies such as Making 10 build a foundation for strategies beyond basic facts, such as Make-a-Whole with fractions and decimals (Bay-Williams and SanGiovanni 2021).

4. **Assessing must attend to fluency components and the learner.** Assessments often assess accuracy, neglecting efficiency and flexibility. Timed tests do not assess fluency and can negatively affect students, and thus should be avoided (Boaler 2014; Kling and Bay-Williams 2021; NCTM 2020; Ramirez, Shaw, and Maloney 2018). Alternatives include interviews, observations, and written prompts.

The way in which fluency is taught either supports equitable learning or prevents it. Effective teaching of procedural fluency positions students as capable, with reasoning and decision-making at the core of instruction. When such teaching is in place, students stop asking themselves, "How did my teacher show me how to do this?" and instead ask, "Which of the strategies that I know are a good fit for this problem?" The latter question is evidence of the student's procedural fluency and mathematical agency, critical outcomes in K–12 mathematics.

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Additional Resources

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