What is Principles and Standards for School Mathematics?

NCTM previously released three volumes of Standards documents—Curriculum and Evaluation Standards for School Mathematics (1989), Professional Standards for Teaching Mathematics (1991), and Assessment Standards for School Mathematics (1995). These documents have greatly influenced the way we think about teaching mathematics. Principles and Standards for School Mathematics strengthens and extends the messages of the original Standards documents. It provides a vision for mathematics education built on high achievement for all students in the classroom. This document and the NCTM’s previous Standards make recommendations about what mathematics students should learn, what classroom practice should be like, and what guidelines can be used to evaluate the effectiveness of mathematics programs.

Is Principles and Standards a curriculum?

No. Principles and Standards for School Mathematics, like NCTM’s original Standards documents, is not a curriculum. It provides local communities a focal point for constructive dialog and offers guidelines, to help in the development of rigorous, challenging mathematics curricula.

How can I use Principles and Standards for School Mathematics?

Its uses are many. For example, it can serve as a—

- tool for teachers, teacher educators, and staff developers to use in examining and improving the quality of mathematics instructional programs;
- professional resource for individual teachers to use in enhancing their classroom practices;
- guide for curriculum writers and publishers to help shape the development of curriculum frameworks, assessments, and instructional materials;
- basis for state and local school systems to develop their own guidelines;
- guide to set goals for assessment criteria at both the state and national levels;
- basis to stimulate ideas and ongoing conversations at the national provincial or state, and local levels about how best to help students gain a deep understanding of important mathematics.

How does Principles and Standards for School Mathematics differ from the original Standards documents?

Principles and Standards keeps the direction and core messages of the original Standards documents while updating the information to reflect current research on mathematics teaching and learning, the wisdom we have gained in more than 10 years of promoting Standards-based reform, and advances in technology. It expands and reorganizes the messages of the original Standards documents (without replacing them) by including the following:

- A common set of 10 Standards that articulate the growth of mathematical knowledge across the grades, rather than a different set and number of standards for each grade band
- Four grade bands (pre-K–2, 3–5, 6–8, 9–12) instead of three, allowing more focus on, and detail for, the elementary and middle grades
- Recommendations for the mathematical learning of preschool children
- A new Standard on representation that outlines the processes and outcomes of acquiring and demonstrating mathematical concepts mentally, symbolically, graphically, and by using physical materials
- The addition of Principles that outline particular characteristics of high-quality mathematics education that can be used as a guide for decision making
Significantly more citations from research to support the assertions made

An electronic edition (E-Standards) in addition to the print document, which includes the complete text of the document as well as electronic examples to enhance the discussions in the book

What are the basic messages of Principles and Standards for School Mathematics?

Overall, the NCTM Standards documents advocate a broader and more meaningful mathematics curriculum that meets the needs of a far greater proportion of the student population than has been true in the past. NCTM’s Principles and Standards for School Mathematics sets ambitious goals for the teaching and learning of mathematics, including the following:

- Learning mathematics with understanding and acquiring the skills and knowledge needed to solve mathematical problems
- Having an in-depth knowledge of the traditional basics of mathematics as well as the expanded basics—such as data analysis and statistics—needed for the technological world in which we live
- Developing reasoning skills that will engender flexible and resourceful problem solving

Achieving these goals requires solid mathematics curricula, competent and knowledgeable teachers who can integrate instruction with assessment, education policies that enhance and support learning, classrooms with ready access to technology, and a commitment to both equity and excellence.

What are the Principles?

The Principles highlight the basic characteristics of a high-quality mathematics instructional program and provide guidance for making educational decisions. The six Principles are as follows:

- Equity. Excellence in mathematics education requires equity—high expectations and strong support for all students.
- Curriculum. A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.
- Teaching. Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.
- Learning. Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.
- Assessment. Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.
- Technology. Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning.

What are the Standards?

The 10 Standards in Principles and Standards for School Mathematics describe the mathematical knowledge, understanding, and skills that students should acquire from prekindergarten through grade 12. Five Standards describe the mathematical content that students should learn to be successful, and five highlight the mathematical processes that students draw on to acquire and use their content knowledge. Together, these 10 Standards define the basic mathematics that all students should have the opportunity to learn—regardless of their decision to move on to higher education or to the workforce immediately after high school. The Content Standards are Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. The Process Standards are Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. For each of the Content Standards a list of specific grade-band expectations is provided.

What should I see in a mathematics classroom that reflects the vision of NCTM’s Standards?

First and foremost, you’ll see students doing mathematics. Students will be applying mathematics to real-world problems and developing strategies to solve more complex mathematics problems. They will be interacting with one another and using other resources along with textbooks.

Teachers will pose problems, ask questions that build on students’ thinking, and encourage students to explore different solutions. The classroom will have various mathematical and technological tools (such as physical materials, calculators, and computers) available for students to use when appropriate. There are many models of Standards-based classrooms, but the focus will always be on students’ learning, understanding, and doing high-quality mathematics.
Does *Principles and Standards* support teaching the “basics”? 

Absolutely. A major goal in the early grades—prekindergarten through grade 5—is the development of computational fluency with whole numbers. Fluency refers to having efficient, accurate, and generalizable methods (or algorithms) for computing that are based on well-understood properties and number relationships. Some of these methods are performed mentally, and others are carried out using paper and pencil to facilitate the recording of thinking.

*Principles and Standards* recognizes that computational proficiency alone is not enough. In today’s world students’ basic arithmetic skills must include the ability to choose what numbers to use and what operation is appropriate for carrying out the computation, deciding if the results make sense, and then making a decision about what to do next. Reasoning, problem solving, making connections, communicating, and using representations all come into play. Having both computational skills and conceptual understanding will enable students to solve problems that they encounter in their daily lives. *Principles and Standards* presents a stronger, bolder vision of the basics.

What does *Principles and Standards* say about the use of technology, such as calculators and computers, in the mathematics classroom? 

The Technology Principle unequivocally states the importance for all students to have access to a full range of technological tools. It also clearly recognizes the central role of the teacher in ensuring that technology is used appropriately to support students’ learning of mathematics, not as a crutch or a replacement for the mastery of important concepts and skills. Technological tools are helpful in doing and understanding mathematics and will be essential in the workplace and in the study of mathematics, science, or engineering in college. These tools furnish visual images of mathematical ideas, facilitate organizing and analyzing data, and can compute quickly, efficiently, and accurately so that students can focus on conceptual understanding.

What mathematics programs and textbooks does NCTM recommend? 

NCTM purposely does not endorse any mathematics curriculum, textbook, or instructional program. There is no one way to teach mathematics. Nor is there only one textbook or program that should be used.

However, there are certain requisites that teaching materials must meet if they are to move students closer to realizing the goals of the Standards. The sequence, timing, developmental appropriateness, and complexity of mathematical tasks described in these materials have a direct impact on the quality of the mathematical content students receive. To evaluate the mathematical content in instructional materials, we should ask the following questions:

- Do the teaching materials ask students to perform at high cognitive levels?
- Do the materials help teachers understand the content for themselves and foster a better understanding of the teaching and learning of mathematics?

The appendix of *Principles and Standards* also includes a detailed Table of Standards and Expectations across the four grade bands, which can be used to evaluate materials and programs.

What is the role of assessment in a Standards-based classroom? 

Assessment should be ongoing, continual, and multifaceted. It should be more than a single test at the end of instruction. Assessment should focus on conceptual understanding as well as procedural skills. Students should be given ample opportunities to demonstrate their mathematical understanding through a variety of methods, including portfolios, discussions, presentations, and projects, in addition to the traditional approach of written tests.

How can administrators make the vision set forth in *Principles and Standards* a reality in their schools? 

Administrators must create the school climate and energize teachers and students in ways that will challenge current expectations and set new goals for mathematics teaching and learning. Supporting the professional development of teachers, establishing mathematics teacher-leaders, providing time in the school day for teachers to collaborate and work together, developing effective processes for the analysis and selection of instructional materials, and examining the impact of high-stakes testing are all ways that administrators can contribute to the goal of a high-quality mathematics education for all students.
What about algebra in the eighth grade?

All students should have a solid background in algebra by the end of eighth grade, whether or not it is through a formal course. Principles and Standards provides guidelines for introducing algebraic concepts early and then reinforcing and strengthening them throughout the grades. By viewing algebra as a strand in the curriculum from prekindergarten on, teachers can help students build a solid foundation of understanding and experience as a preparation for more-sophisticated work in algebra in the middle grades and high school. Specifically, the Standards for grades 6–8 focus on algebra, as well as its connection to other important areas, such as geometry. By the end of eighth grade, students should have an algebraic background that enables them to enter substantive high school courses.

Do all students need four years of high school mathematics?

Yes. Principles and Standards clearly states that “all students are expected to study mathematics each of the four years that they are enrolled in high school.” The need to understand and be able to use mathematics in everyday life and in the workplace has never been greater and will continue to increase. For example:

- Mathematics for life. We live in a mathematical world. Everyday activities such as making purchasing decisions, choosing insurance or health plans, or planning for the future require quantitative understanding.
- Mathematics for the workplace. Mathematical thinking and problem solving are needed in a variety of jobs ranging from word processing to controlling machines, to analyzing complicated sets of data, to ensuring quality control in production processes. Providing a broad and deep mathematics program for all students ensures their ability to make, and be successful at, a wide range of career choices.
- Mathematics for the scientific and technical community. The challenges presented by the twenty-first century require more students to pursue an education path that will prepare them for lifelong work as mathematicians, statisticians, engineers, and scientists. We must move away from the notion that mathematics is only for a select few. Students with a deep interest in pursuing mathematical and scientific careers must continue to have their talents and interests engaged.

Since the previous Standards documents were published, what successes have we seen?

We’ve seen some promising improvements in mathematics education over the decade since the original NCTM Standards were released. Some examples are as follows:

- The National Assessment of Educational Progress (NAEP), which is designed to measure educational achievement trends over time, has shown that white, Hispanic, and African Americans have demonstrated steady and statistically significant improvement in their mathematical scores since the first NAEP report in 1990.
- Studies of Standards-based educational reform in Illinois, using data from 1992 to 1995, identified remarkable increases in mathematics achievement that were nearly one standard deviation above the state mean for schools with the greatest implementation of reform.
- Further evidence of students’ success in Standards-based mathematics programs can be found in states such as Pennsylvania, Massachusetts, North Carolina, Connecticut, Michigan, and Texas.

Other general improvements have been noted, such as more-focused and more-challenging curriculum materials, the revisions of state curriculum frameworks, and the strengthening of standards for the preparation of mathematics teachers.

What resources are available from NCTM to put the ideas and recommendations from Principles and Standards into action?

NCTM has many resources available to support the Standards.

- E-Standards is the electronic version of Principles and Standards that is available in CD-ROM or on the Web at standards.nctm.org. It provides access to the text of Principles and Standards and includes interactive applets and videos to help bring the text to life.
- The Illuminations Web site, illuminations.nctm.org, offers Standards-based lesson plans, including interactive applets for students to explore, learn, and apply mathematics.
- The NCTM Academy for Professional Development offers two- and five-day Institutes throughout the United States that are hands-on and designed to provide understanding and application of Principles and Standards.
- Navigations is a new series of grade-band books that contain practical, teacher-tested activities and materials. A CD-ROM with additional resources, such as electronic applets and selected articles, accompanies each book. When complete, the Navigations series will include more than 30 titles.
- Principles and Standards Outreach CD (2nd Edition) offers PowerPoint presentations, video clips, and additional material to support presentations and workshops on the messages of Principles and Standards for School Mathematics.

What is NCTM?

Founded in 1920, the National Council of Teachers of Mathematics is a nonprofit, nonpartisan education association with nearly 100,000 members and 250 Affiliates in the United States and Canada. It is the world’s largest organization dedicated to improving the teaching and learning of mathematics from prekindergarten through high school to ensure the highest-quality mathematics education for all students.

Contact NCTM to order Principles and Standards products or find out more information. Free copies of this brochure are also available.