Principles to Actions:
Ensuring Mathematical Success for All
A 25-year History of Standards-Based Mathematics Education Reform

1989 *Curriculum and Evaluation Standards for School Mathematics*

2000 *Principles and Standards for School Mathematics*

2006 *Curriculum Focal Points*

2010 *Focus in High School Mathematics*

2010 *Common Core State Standards for Mathematics*
Standards Have Contributed to Higher Achievement

• The percent of 4th graders scoring proficient or above on NAEP rose from 13% in 1990 to 42% in 2013.

• The percent of 8th graders scoring proficient or above on NAEP rose from 15% in 1990 to 36% in 2013.

• Between 1990 and 2012, the mean SAT-Math score increased from 501 to 514 and the mean ACT-Math score increased from 19.9 to 21.0.
Trend in Grade 4 NAEP Mathematics Scores

Although We Have Made Progress, Challenges Remain

• The average mathematics NAEP score for 17-year-olds has been essentially flat since 1973.

• Among 34 countries participating in the 2012 Programme for International Student Assessment (PISA) of 15-year-olds, the U.S. ranked 26th in mathematics.

• While many countries have increased their mean scores on the PISA assessments between 2003 and 2012, the U.S. mean score declined.

• Significant learning differentials remain.
Trend in Grade 4 Mathematics By Race/Ethnicity

High-Quality Standards are Necessary for Effective Teaching and Learning, But Insufficient

The Common Core does not describe or prescribe the essential conditions required to make sure mathematics works for all students.
The primary purpose of *Principles to Actions* is to fill the gap between the adoption of rigorous standards and the enactment of practices, policies, programs, and actions required for successful implementation of those standards.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.
The overarching message is that effective teaching is the non-negotiable core necessary to ensure that all students learn mathematics. The six guiding principles constitute the foundation of PtA that describe high-quality mathematics education.

NCTM. (2014). *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: NCTM.
Guiding Principles for School Mathematics

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism

Essential Elements of Effective Mathematics Programs
For Each Principle

- Productive and Unproductive Beliefs are Listed
- Obstacles to Implementing the Principle are Outlined
- Overcoming the Obstacles
- Taking Action
  - Leaders and Policymakers
  - Principles, Coaches, Specialists, Other School Leaders
  - Teachers
Teaching and Learning Principle

Teaching and Learning

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.
Obstacles to Implementing High-Leverage Instructional Practices

Dominant cultural beliefs about the teaching and learning of mathematics continue to be obstacles to consistent implementation of effective teaching and learning in mathematics classrooms.
## Beliefs about teaching and learning mathematics

<table>
<thead>
<tr>
<th>Unproductive beliefs</th>
<th>Productive beliefs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics learning should focus on practicing procedures and memorizing basic number combinations.</td>
<td>Mathematics learning should focus on developing understanding of concepts and procedures through problem solving, reasoning, and discourse.</td>
</tr>
<tr>
<td>Students need only to learn and use the same standard computational algorithms and the same prescribed methods to solve algebraic problems.</td>
<td>All students need to have a range of strategies and approaches from which to choose in solving problems, including, but not limited to, general methods, standard algorithms, and procedures.</td>
</tr>
<tr>
<td>Students can learn to apply mathematics only after they have mastered the basic skills.</td>
<td>Students can learn mathematics through exploring and solving contextual and mathematical problems.</td>
</tr>
<tr>
<td>The role of the teacher is to tell students exactly what definitions, formulas, and rules they should know and demonstrate how to use this information to solve mathematics problems.</td>
<td>The role of the teacher is to engage students in tasks that promote reasoning and problem solving and facilitate discourse that moves students toward shared understanding of mathematics.</td>
</tr>
<tr>
<td>The role of the student is to memorize information that is presented and then use it to solve routine problems on homework, quizzes, and tests.</td>
<td>The role of the student is to be actively involved in making sense of mathematics tasks by using varied strategies and representations, justifying solutions, making connections to prior knowledge or familiar contexts and experiences, and considering the reasoning of others.</td>
</tr>
<tr>
<td>An effective teacher makes the mathematics easy for students by guiding them step by step through problem solving to ensure that they are not frustrated or confused.</td>
<td>An effective teacher provides students with appropriate challenge, encourages perseverance in solving problems, and supports productive struggle in learning mathematics.</td>
</tr>
</tbody>
</table>
Eight High-Leverage Instructional Practices

- Establish mathematics goals to focus learning
- Implement tasks that promote reasoning and problem solving
- Use and connect mathematical representations
- Facilitate meaningful mathematical discourse
- Pose purposeful questions
- Build procedural fluency from conceptual understanding
- Support productive struggle in learning mathematics
- Elicit and use evidence of student thinking
Establish mathematics goals to focus learning.

Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses goals to guide instructional decisions.
### Establish mathematics goals to focus learning

#### Teacher and student actions

<table>
<thead>
<tr>
<th><strong>What are teachers doing?</strong></th>
<th><strong>What are students doing?</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit.</td>
<td>Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?)</td>
</tr>
<tr>
<td>Identifying how the goals fit within a mathematics learning progression.</td>
<td>Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices.</td>
</tr>
<tr>
<td>Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning.</td>
<td>Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going.</td>
</tr>
<tr>
<td>Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.</td>
<td>Assessing and monitoring their own understanding and progress toward the mathematics learning goals.</td>
</tr>
</tbody>
</table>
Eight High-Leverage Instructional Practices

Implement tasks that promote reasoning and problem solving.

Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and that allow for multiple entry points and varied solution strategies.
<table>
<thead>
<tr>
<th>What are teachers doing?</th>
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</thead>
<tbody>
<tr>
<td>Motivating students’ learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding.</td>
<td>Persevering in exploring and reasoning through tasks.</td>
</tr>
<tr>
<td>Selecting tasks that provide multiple entry points through the use of varied tools and representations.</td>
<td>Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas.</td>
</tr>
<tr>
<td>Posing tasks on a regular basis that require a high level of cognitive demand.</td>
<td>Using tools and representations as needed to support their thinking and problem solving.</td>
</tr>
<tr>
<td>Supporting students in exploring tasks without taking over student thinking.</td>
<td>Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.</td>
</tr>
<tr>
<td>Encouraging students to use varied approaches and strategies to make sense of and solve tasks.</td>
<td></td>
</tr>
</tbody>
</table>
Eight Research-Informed Instructional Practices

Use and connect mathematical representations.

Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Use and connect mathematical representations
Teacher and student actions

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<tr>
<td>Selecting tasks that allow students to decide which representations to use in making sense of the problems.</td>
<td>Using multiple forms of representations to make sense of and understand mathematics.</td>
</tr>
<tr>
<td>Allocating substantial instructional time for students to use, discuss, and make connections among representations.</td>
<td>Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations.</td>
</tr>
<tr>
<td>Introducing forms of representations that can be useful to students.</td>
<td>Making choices about which forms of representations to use as tools for solving problems.</td>
</tr>
<tr>
<td>Asking students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
<td>Sketching diagrams to make sense of problem situations.</td>
</tr>
<tr>
<td>Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
<td>Contextualizing mathematical ideas by connecting them to real-world situations.</td>
</tr>
<tr>
<td>Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.</td>
<td>Considering the advantages or suitability of using various representations when solving problems.</td>
</tr>
</tbody>
</table>
Facilitate meaningful mathematical discourse.

Effective teaching of mathematics facilitates discourse among students in order to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
## Facilitate meaningful mathematical discourse
Teacher and student actions

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<tr>
<th>What are teachers doing?</th>
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<tbody>
<tr>
<td>Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations.</td>
<td>Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse.</td>
</tr>
<tr>
<td>Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion.</td>
<td>Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments.</td>
</tr>
<tr>
<td>Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches.</td>
<td>Seeking to understand the approaches used by peers by asking clarifying questions, trying out others’ strategies, and describing the approaches used by others.</td>
</tr>
<tr>
<td>Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.</td>
<td>Identifying how different approaches to solving a task are the same and how they are different.</td>
</tr>
</tbody>
</table>
Eight Research-Informed Instructional Practices

Pose purposeful questions.

Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships.
## Pose purposeful questions

**Teacher and student actions**

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<tr>
<td>Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking.</td>
<td>Expecting to be asked to explain, clarify, and elaborate on their thinking.</td>
</tr>
<tr>
<td>Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification.</td>
<td>Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly.</td>
</tr>
<tr>
<td>Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion.</td>
<td>Reflecting on and justifying their reasoning, not simply providing answers.</td>
</tr>
<tr>
<td>Allowing sufficient wait time so that more students can formulate and offer responses.</td>
<td>Listening to, commenting on, and questioning the contributions of their classmates.</td>
</tr>
</tbody>
</table>
Eight Research-Informed Instructional Practices

Build procedural fluency from conceptual understanding.

Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
## Build procedural fluency from conceptual understanding

**Teacher and student actions**

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<tr>
<td>Providing students with opportunities to use their own reasoning strategies and methods for solving problems.</td>
<td>Making sure that they understand and can explain the mathematical basis for the procedures that they are using.</td>
</tr>
<tr>
<td>Asking students to discuss and explain why the procedures that they are using work to solve particular problems.</td>
<td>Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.</td>
</tr>
<tr>
<td>Connecting student-generated strategies and methods to more efficient procedures as appropriate.</td>
<td>Determining whether specific approaches generalize to a broad class of problems.</td>
</tr>
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Eight High-Leverage Instructional Practices

Support Productive Struggle in Learning Mathematics.

Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
## Support productive struggle in learning mathematics

### Teacher and student actions

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<tr>
<td>Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle.</td>
<td>Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle.</td>
</tr>
<tr>
<td>Giving students time to struggle with tasks, and asking questions that scaffold students’ thinking without stepping in to do the work for them.</td>
<td>Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks.</td>
</tr>
<tr>
<td>Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles.</td>
<td>Persevering in solving problems and realizing that is acceptable to say, “I don’t know how to proceed here,” but it is not acceptable to give up.</td>
</tr>
<tr>
<td>Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.</td>
<td>Helping one another without telling their classmates what the answer is or how to solve the problem.</td>
</tr>
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Eight Research-Informed Instructional Practices

Elicit and use evidence of student thinking.

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.
## Elicit and use evidence of student thinking
### Teacher and student actions

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<tr>
<td>Identifying what counts as evidence of student progress toward mathematics learning goals.</td>
<td>Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse.</td>
</tr>
<tr>
<td>Eliciting and gathering evidence of student understanding at strategic points during instruction.</td>
<td>Reflecting on mistakes and misconceptions to improve their mathematical understanding.</td>
</tr>
<tr>
<td>Interpreting student thinking to assess mathematical understanding, reasoning, and methods.</td>
<td>Asking questions, responding to, and giving suggestions to support the learning of their classmates.</td>
</tr>
<tr>
<td>Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend.</td>
<td>Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.</td>
</tr>
<tr>
<td>Reflecting on evidence of student learning to inform the planning of next instructional steps.</td>
<td></td>
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</table>
Effective teaching and learning, while the non-negotiable core of successful mathematics programs, are part of a system of essential elements of excellent mathematics programs.
Five Essential Elements of Effective Mathematics Programs

Access and Equity
Curriculum
Tools and Technology
Assessment
Professionalism
Guiding Principles for School Mathematics: Curriculum

**Curriculum**

An excellent mathematics program includes curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.
Curriculum Obstacle

Grade level mathematics curriculum standards are often treated as a checklist of topics. When conceptualized as such, mathematics content becomes nothing more than a set of isolated skills, often without a mathematical or real-world context and disconnected from related topics.
Guiding Principles for School Mathematics: Assessment

Assessment

An excellent mathematics program ensures that assessment is an integral part of instruction ... and informs feedback to students, instructional decisions, and program improvement.
Traditionally assessment tends to emphasize the evaluation of student achievement (e.g., assigning grades), and more recently, the rating of schools and the performance of teachers – the cultural perception that links assessment to grading and rating ...
Teacher Action Steps to Support the Assessment Principle

Work in collaborative teams, grade level or subject-based, to develop common assessments that will be used formatively, commit to their use, and use the results to advance student learning and improve instruction.
Guiding Principles for School Mathematics: Access and Equity

**Access and Equity**

An excellent mathematics program requires that all students have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.
Guiding Principles for School Mathematics: Tools and Technology

**Tools and Technology**

An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.
Obstacles to Tools and Technology

Often students do not actually engage with technologies or tools in ways that promote mathematical reasoning and sense making. Having students watch a computer presentation or tutorial in which mathematical facts and examples appear, no matter how visually engaging, is not significantly different from having students watch a teacher write the same information on a white board.
Guiding Principles for School Mathematics: Professionalism

**Professionalism**

In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for their personal and collective professional growth toward effective teaching and learning of mathematics.
Professionalism Obstacle

In too many schools, professional isolation severely undermines attempts to significantly increase professional collaboration ... some teachers actually embrace the norms of isolation and autonomy. A danger in isolation is that it can lead to teachers developing inconsistencies in their practice that in turn can create inequities in student learning.
Collaboration Should Include

• An examination and prioritization of the mathematics content and mathematics practices students are to learn.

• The development and use of common assessments to determine if students have learned the agreed-on content and related mathematical practices.

• The use of data to drive continuous reflection and instructional decisions.
Collaboration Should Include

• The setting of both long-term and short-term instructional goals.

• Development of action plans to implement when students demonstrate they have or have not attained the standards.

• Discussion, selection, and implementation of common research-informed instructional strategies and plans.
Start Small, Build Momentum, and Persevere

The process of creating a new cultural norm characterized by professional collaboration, openness of practice, and continual learning and improvement can begin with a single team of grade-level or subject-based mathematics teachers making the commitment to collaborate on a single lesson plan.
Thus, teachers of mathematics must take the following actions:

For the Teaching and Learning Principle:

- Consistently implement the eight Mathematics Teaching Practices.
- Elicit, value, and celebrate varied approaches and solution paths that students take to solve mathematics problems, explain their thinking, and critique the arguments of others.
- Give priority to the mathematical practices, including problem solving, reasoning, and constructing viable arguments in every aspect of classroom practice—including teaching, assessment, curriculum decisions, and the use of tools and technology.
For the Assessment Principle:

- Work in collaborative grade-level or subject-based teams to develop common assessments to be used formatively, commit to their use, and analyze and apply the results to advance student learning and improve instruction.
- Evaluate students’ mathematics learning on the basis of multiple measures to make more reliable and valid judgments about what students know and are able to do.
- Provide students with descriptive, accurate, and timely feedback on assessments, including strengths, weaknesses, and next steps for progress toward the learning targets.
- Recognize that effective instruction and ongoing review are the best high-stakes “test prep” strategies.
- View assessment results as supplying part of the picture of instructional effectiveness and use them to drive instructional decision making, focus personal professional growth, and make program improvements.

For the Professionalism Principle:

- Continually grow in knowledge of mathematics for teaching, mathematical pedagogical knowledge, and knowledge of students as learners of mathematics.
- Demand opportunities for professional development and collaboration that strengthen mathematics content knowledge and the implementation of the Mathematics Teaching Practices.
- Collaborate with colleagues on issues of access and equity, curriculum, instruction, tools and technology, assessment, and professional growth.
- Assume collective responsibility for the learning of all students in the school.
- Join and participate in local, state, or national professional organizations.
Guiding Principles for School Mathematics

1. Teaching and Learning
2. Access and Equity
3. Curriculum
4. Tools and Technology
5. Assessment
6. Professionalism
The primary purpose of *Principles to Actions* is to fill the gap between the adoption of rigorous standards and the enactment of practices, policies, programs, and actions required for successful implementation of those standards.
Principles to Actions: Ensuring Mathematical Success for All

- Describes the **supportive conditions, structures, and policies** required to give all students the power of mathematics
- Focuses on **teaching and learning**
- Engages students in **mathematical thinking**
- How to ensure that mathematics achievement is maximized for every student
- Not specific to any standards; it’s universal