

Catalyzing Change in High School Mathematics:

Initiating Critical Conversations

Executive Summary

The need for *Catalyzing Change* is clear: The steady improvement in mathematics learning seen since 1990 at the elementary and middle school levels has not been shared at the high school level, underscoring the critical need for change in mathematics education at the high school level. The purpose of *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* is to open serious discussions among the key stakeholders in high school mathematics education to engage in resolving the vexing barriers that have long impeded meaningful and necessary change in high school mathematics education.

Background

The last three decades have seen significant progress in the teaching and learning of mathematics in the United States. The standards-based reform era began in 1989 when the National Council of Teachers of Mathematics (NCTM) published *Curriculum and Evaluation Standards for School Mathematics*, which introduced standards promoting rigorous mathematics content and the development of important mathematical processes and practices. NCTM soon followed this publication with closely related ones that outlined the teaching and assessment practices needed to support those standards (NCTM 1991, 1995). Subsequent iterations of Standards publications, including *Principles and Standards for School Mathematics* (NCTM 2000), *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM 2006), and *Focus in High School Mathematics*: *Reasoning and Sense Making* (NCTM 2009), continued to promote this focus on content and practices, and the Common Core State Standards for Mathematics (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center and CCSSO] 2010 later built on them.

Rigorous standards, along with increased knowledge and implementation of research-informed instructional practices summarized in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014) have contributed to a positive long-term trend since 1990 in mathematics learning at both the elementary and the middle school levels, as measured by the National Assessment of Educational Progress (NAEP; National Center for Education Statistics [NCES] 2015).

Despite an increase in the percentage of high school students enrolling in upper-level mathematics courses over the last three decades (Dossey, McCrone, and Halvorsen 2016), high school NAEP scores have remained essentially flat for the past decade (NCES 2016), and fewer than 50 percent of U.S. high school graduates in 2016 were considered ready for college-level mathematics work, as measured by their ACT mathematics scores (ACT 2016). Even more disturbing, the Programme for International Student Assessment (PISA) showed that U.S. high school students trailed their international peers, on average, in mathematical literacy, defined as the "capacity to formulate, employ, and interpret mathematics in a variety of contexts … to describe, explain, and

predict phenomena (Organisation for Economic Co-operation and Development [OECD] 2016, p. 28). In addition, evidence indicates that U.S. young adults lack not only the quantitative and problem-solving skills necessary for success in the workplace and postsecondary education but also the numeracy and problem-solving skills necessary for "meaningful participation in our democratic institutions" (Goodman, Sands, and Coley 2015, p. 5).

Most critically, a significant percentage of high school students do not have access to the mathematics that they need either for their personal or for their professional adult lives. This issue of inequity in mathematics education—seemingly intractable over many years—makes it essential to initiate a serious discussion among a variety of stakeholders to achieve the critical mass necessary to catalyze change in high school mathematics. All students must have the opportunity to obtain an education in mathematics that will serve them well, regardless of their interests and ambitions.

Mathematics education at the high school level is part of a complex system of policies, traditions, and societal expectations. This system and its structures—school and district policies, practices, and conditions that either support or impede student learning of mathematics—are what need to be critically examined and improved. Therefore, improvements to high school mathematics will necessitate the engagement of all stakeholders to reexamine longstanding beliefs, practices, and policies that are impeding progress. *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* is a call to action to all individuals with a stake in high school mathematics, inviting and urging them to embrace and participate in the serious conversations that must take place to bring about and give support to necessary changes in high school mathematics. These individuals include school, district, and state administrators; instructional leaders and coaches; classroom teachers; counselors and curriculum and assessment developers; high education administration and faculty; and policymakers at all levels.

Key Recommendations

Catalyzing Change identifies and addresses critical challenges in high school mathematics to ensure that each and every student has the mathematical experiences necessary for his or her future personal and professional success. To address these critical challenges, *Catalyzing Change* makes four key recommendations and presents the Essential Concepts for high school mathematics to identify the mathematics needed be each and every student.

Key Recommendation: Each and every student should learn the Essential Concepts in order to expand professional opportunities, understand and critique the world, and experience the joy, wonder, and beauty of mathematics.

Much of the discourse surrounding mathematics education and standards has centered on students' preparation in mathematics and statistics for postsecondary education or a career. Although these goals are clearly important and will remain so, *Catalyzing Change* underscores the fact that other purposes of mathematics education are also important. Students should leave high school with the quantitative literacy and critical thinking processes needed to make wise decisions in their personal lives. Students should be able to determine whether or not claims made in scientific, economic, social, or political arenas are valid. Students should have an appreciation for the beauty and usefulness of mathematics and statistics. And students should see themselves as capable lifelong learners and confident doers of mathematics and statistics. Never have the broader aims of mathematics education been more important than they are today, when mathematics underlies much of the fabric of society, from polling and data mining in politics, to algorithms targeting advertisements to groups of people on social media, to complex mathematical models of financial instruments and policies that affect the lives of millions of people. To support these purposes for learning mathematics, *Catalyzing Change* offers forty-one Essential Concepts in the domains of number, algebra and functions, statistics and probability, and geometry and measurement that each and every student should learn. As shown in figure 1, the Essential Concepts are organized into areas of focus within the domains, except in the domain of number, since at the high school level, concepts in number are typically woven into content and instruction in the other domains.

Essential Concepts in High School Mathematics
Essential Concepts in Number
Essential Concepts in Algebra and Functions
Focus 1: Algebra
Focus 2: Connecting Algebra to Functions
Focus 3: Functions
Essential Concepts in Statistics and Probability
Focus 1: Quantitative Literacy
Focus 2: Visualizing and Summarizing Data
Focus 3: Statistical Inference
Focus 4: Probability
Essential Concepts in Geometry and Measurement
Focus 1: Measurement
Focus 2: Transformations
Focus 3: Geometric Arguments, Reasoning, and Proof
Focus 4: Solving Applied Problems and Modeling in Geometry

Fig. 1. Areas of focus in the Essential Concepts within the domains of high school mathematics

The Essential Concepts do not represent yet another set of standards or a list of disconnected topics to be covered. The Essential Concepts represent a distillation of the critical concepts and skills that, regardless of a state's, province's, or district's standards, students should acquire, retain, and be able to use long after high school. In outlining this critical content, *Catalyzing Change* highlights the roles of technology, reasoning and proof, and modeling, as well as the connections among the content areas. The Essential Concepts specifically address the widespread concern that high school mathematics standards lack the focus of the K–8 mathematics standards—a shortcoming that increases the difficulty that teachers have in supporting students in developing a deep foundational understanding of mathematics.

Key Recommendation: High school mathematics should discontinue the practice of tracking teachers as well as the practice of tracking students into qualitatively different or dead-end course pathways.

Current reform efforts that focus largely on standards, with some attention to improved instructional practice, are unlikely to address and alleviate equity concerns unless they also address and dismantle the conditions and system structures that stand as barriers to the creation of positive mathematical experiences for students. *Catalyzing Change* outlines three structural barriers to achieving equitable learning outcomes within educators' influence: tracking students into course pathways that do not prepare students for the continued study of mathematics; tracking teachers in ways that deny certain students access to high-quality instruction; and providing inadequate instructional supports before and during high school.

Catalyzing Change is clear that tracking students and tracking teachers are both unjust practices. Student tracking is insidious because it places some students into qualitatively different or lower levels of a mathematics course and, in some cases, puts students into terminal mathematics pathways that are not mathematically meaningful and do not prepare them for any continued study of fundamental mathematics or effective participation in democratic society. *Catalyzing Change* draws a distinction between tracking and acceleration, arguing that acceleration of students through shared content may be appropriate if a student has demonstrated deep understanding of grade-level or course-based mathematics standards beyond his or her current level. Furthermore, *Catalyzing Change* defines what constitutes demanding mathematics courses that should count toward high school graduation credits in mathematics.

Like mathematics students, mathematics teachers themselves are often tracked, with the most experienced teachers, or those who are perceived to be most effective, assigned to upper-level mathematics courses and the least experienced assigned to entry-level mathematics courses. *Catalyzing Change* recommends that, whenever possible, high school mathematics teachers in the same department have teaching assignments that are balanced to include both upper-level and entry-level mathematics courses. By balancing teacher assignments, teachers' knowledge of the overall curriculum is deepened, burnout among new teachers is reduced, and collaborative teams can be established with experienced teachers, thereby improving the overall learning experiences of students.

Rigorous standards, coupled with research-informed effective teaching practices, can help ensure that students who enter high school have the mathematics foundation necessary to succeed in the Essential Concepts. However, it would be naïve to assume that every student who enters high school has had the mathematical experiences necessary for immediate success. Therefore, *Catalyzing Change* outlines effective support structures that need to be in place, both in middle school and in high school, to help ensure that all students can successfully acquire the Essential Concepts as well as mathematics beyond the Essential Concepts.

Key Recommendation: Classroom instruction should be consistent with research-informed and equitable teaching practices.

Providing students with more rigorous instruction requires improvements in teaching practices. Because mathematics teaching involves not only supporting students to learn concepts and develop skills and understanding but also empowering students to see themselves as capable of participating in and being doers of mathematics, improvements in teaching practices must address not only ways to make mathematics more accessible to students but also ways to support students in seeing themselves as knowers and doers of mathematics.

Therefore, improvements in teaching practices must consider mathematical identity and agency as essential constructs for raising the mathematical achievement and strengthening the mathematical disposition of each and every student. The ways in which students experience mathematics have a significant impact on the ways in which they identify themselves as doers of mathematics. By implementing research-informed and equitable instructional practices, teachers can significantly improve the experiences and learning outcomes of students. To support teachers in these improvement efforts, *Catalyzing Change* uses the eight Mathematics Teaching Practices articulated in *Principles to Actions* (NCTM 2014) as a framework for making connections between high-leverage teaching practices and the development of mathematical identity, agency, and competence.

Key Recommendation: High schools should offer continuous four-year mathematics pathways with all students studying mathematics each year, including two to three years of mathematics in a common shared pathway focusing on the Essential Concepts, to ensure the highest-quality mathematics education for all students.

To maximize students' opportunities after high school and prepare them to actively engage in democratic society, *Catalyzing Change* recommends that high schools require students to enroll in mathematically demanding (non-terminal) mathematics courses every year in high school, completing four years of high school mathematics, including a mathematics course during their last year of high school. The expectation in *Catalyzing Change* is that a single curricular model would deliver the common pathway to all students in a single school setting, to ensure each and every student's access to a high-quality mathematics education while avoiding the creation of separate and unequal tracks. *Catalyzing Change* envisions that the set of Essential Concepts in a common shared pathway will provide the equitable educational experience that each and every student deserves in mathematics. *Catalyzing Change* further stresses the importance of ensuring that students' mathematics coursework beyond the Essential Concepts be determined by each student's own needs, goals, interests, and aspirations, rather than by any difference in mathematical ability perceived by anyone else.

Next Steps

Catalyzing Change in High School Mathematics: Initiating Critical Conversations is designed to open serious conversations and sustained efforts on multiple levels to engage all stakeholders in the system of high school mathematics education in the work of improving learning experiences and outcomes for each and every high school student. *Catalyzing Change* offers a number of specific recommendations that can be used to begin these critical conversations and a number of initial actions that teachers, schools, and districts; policymakers; and postsecondary educators can undertake.

Catalyzing Change is offered as the beginning of a long-term process. Progress toward the vision that it sets out will require sustained effort by the Council and numerous other groups over many years. NCTM will continue to support this collaborative effort through professional learning opportunities and additional publications that will provide tools and additional resources to deepen understanding of, and begin taking action on, the recommendations put forward in this publication. This work is critical for all of us to undertake, but it will not be easy, since the challenges are real and longstanding. Making a difference will require collaboration, communication, and work across diverse groups and communities. The need for change and improvement is urgent. We owe this effort not only to our students but also to ourselves as we work together to create and nurture the society we wish to inhabit.

References

- ACT. The Condition of College and Career Readiness 2016: National. Iowa City, Iowa: ACT, 2016.
- Dossey, John A., Sharon Soucy McCrone, and Katherine Taylor Halvorsen. *Mathematics Education in the United States,* 2016: A Capsule Summary Fact Book. Reston, Va.: National Council of Teachers of Mathematics, 2016.
- Goodman, Madeline J., Anita M. Sands, and Richard J. Coley. *America's Skills Challenge: Millennials and the Future*. Princeton, N.J.: Educational Testing Service, 2015.
- National Center for Education Statistics (NCES). *The Nation's Report Card: 2015 Mathematics and Reading at Grades 4 and* 8 (NCES 2015-136). Washington, D.C.: Institute of Education Sciences, U.S. Department of Education, 2015.
- *2015 Results: Mathematics and Reading, Grade 12.* Washington, D.C.: Institute of Education Sciences, U.S. Department of Education, 2016. https://www.nationsreportcard.gov/rea
- National Council of Teachers of Mathematics (NCTM). An Agenda for Action: Recommendations for School Mathematics of the 1980s. Reston, Va.: NCTM, 1980. http://www.nctm.org/flipbooks/standards/agendaforaction/index.html
- -------. Curriculum and Evaluation Standards for School Mathematics. Reston, Va.: NCTM, 1989.
- -------. Professional Standards for Teaching Mathematics. Reston, Va.: NCTM, 1991.
- . Assessment Standards for School Mathematics. Reston, Va.: NCTM, 1995.
- -------. Principles and Standards for School Mathematics. Reston, Va.: NCTM, 2000.
- ———. Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence. Reston, Va.: NCTM, 2006.
- -------. Focus in High School Mathematics: Reasoning and Sense Making. Reston, Va.: NCTM, 2009.
- -------. Principles to Actions: Ensuring Mathematical Success for All. Reston, Va.: NCTM, 2014.
- National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO). Common Core State Standards for Mathematics. Common Core State Standards (College- and Career-Readiness Standards and K–12 Standards in English Language Arts and Math). Washington, D.C.: NGA Center and CCSSO, 2010. http://www.corestandards.org

Organisation for Economic Co-operation and Development (OECD). *PISA 2015 Results (Volume I): Excellence and Equity in Education*. Paris: OECD Publishing, 2016.



NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

15699