This packet contains applicable standards, a copy of the problem, the answer check, our solutions, teaching suggestions, and samples of student work that we received when the problem ran in March 2007 (Library PoW #4075). For a print-friendly version use the Print this Problem link on the problem page.

We invite you to visit the PoW discussion groups to explore this problem with colleagues. From the Teacher Office use the link to PoW Members, or use this URL to go to funpow-teachers directly: http://mathforum.org/kb/forum.jspa?forumID=526 [Log in using your PoW username and password.]

Are you making the most of your PoW Membership? If you have an Individual Teacher Membership consider registering for one of our (free) Orientation Sessions to learn more about the features of your membership. Teachers with Class or School or District Memberships are welcome to take the free Orientation Session but also are encouraged to register for one of our online courses. View information, dates, and links to register here: http://mathforum.org/pd/

In Darlene's Dart Board students apply whole number computation skills and number sense to combine the numbers 15, 16, 17, 18, and 19 to arrive at a sum of 100.

If your state has adopted the Common Core State Standards, this alignment may be helpful:

Grade 3: Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
  1. Interpret products of whole numbers, e.g., interpret $5 \times 7$ as the total number of objects in 5 groups of 7 objects each.
  3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

- Multiply and divide within 100.
  7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

- Solve problems involving the four operations, and identify and explain patterns in arithmetic.
  8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Grade 4: Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
  3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Mathematical Practices

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Look for and make use of structure.

Additional alignment information can be found through the Write Math with the Math Forum service, where teachers can browse by NCTM and state standards, as well as popular textbook units, to find related problems.
The Problem

**Darlene's Dart Board**

Darlene challenged her friends to a game of darts on her new dart board. Players take turns throwing darts. A player wins by scoring exactly 100 points.

1. List one combination of scores that will win the game. Explain how you found it.
2. Is it possible to score 100 points with more darts or fewer darts than you listed? Explain how you know.

**Extra:** Find at least two other combinations that will win this game. List them and describe your strategy for finding them.

Answer Check

After students submit their solutions, we encourage them to check their answers by looking at the answer that we provide. Below is what they will see. You might use the accompanying questions as prompts to help students who are struggling, or to encourage those who have found a correct solution to improve their explanation.

1. There are at least 10 different ways to win. Make sure your score adds to 100.
2. A player needs to land exactly 6 darts on the board to get a winning score.

If your answer does not match ours,
- do you understand that a player can land more than one dart on a number?
- did you think about the digits in the ones place?
- did you try making a list of multiples of 15, 16, etc.?
- did you consider sums of odd and even scores?
- did you try to find the maximum (highest) score you could get with 5 darts? or the minimum (least) score you could get with 7 darts?
- did you check your arithmetic?

If your answer does match ours,
- is your explanation clear and complete?
- did you try the Extra?
- did you have an "Aha!" moment? Explain.

Our Solutions

Below are some ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. We often receive solutions from students who have used approaches we’ve not anticipated. I hope you will share such approaches on the funpow-teachers discussion board, along with any teaching strategies you found to be successful.

**Question #1: Strategy 1: Guess and check with one of each**

I tried adding one of each number first. Since they were all less than 20, I knew their sum would be less than 100.

\[15 + 16 + 17 + 18 + 19 = 85\]

I saw I could add one more 15 to reach 100. So \[15 + 15 + 16 + 17 + 18 + 19 = 100\]

**Question #1: Strategy 2: Guess and check with substitution**

I added \[15 + 16 + 17 = 48\]. That’s only two away from 50, which is half of 100. I traded the 17 for a 19.

\[15 + 16 + 19 = 50\]

So \[15 + 15 + 16 + 16 + 19 + 19 = 100\]

OR

I knew \[2 \times 15 = 30\], so six 15s = 90. That is 10 short of 100. I traded 5 of the 15s for 17s to make up five 2s.

\[15 + 17 + 17 + 17 + 17 + 17 = 100\]

**Question #2**

It is not possible to score 100 points with more than 6 darts because seven 15s (the smallest number) would be more than 100. \[7 \times 15 = 105\]
It is not possible to score 100 with fewer than 6 darts, because it takes five 20s to make 100, and 19 is the largest number we can use. \(5 \times 19 = 96\)

**Extra 1: Substitution**

Starting with my first combination, \(15 + 15 + 16 + 17 + 18 + 19 = 100\), I made trades to keep the total constant.

\[15 + 15 + 16 + 16 + 19 + 19 = 100\] [Trading a 17 for a 16 decreases the total by 1; trading an 18 for a 19 makes it up.]

\[15 + 15 + 15 + 17 + 19 + 19 = 100\] [Trading a 16 for a 15 decreases the total by 1; trading an 16 for a 17 makes it up.]

**Extra 2: List of multiples**

I made a list of the multiples of the numbers that could be combined with other multiples. I noticed that all the multiples of the even numbers, 16 and 18, were even numbers. The multiples of the odd numbers alternate even and odd.

<table>
<thead>
<tr>
<th># of Darts</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>32</td>
<td>34</td>
<td>36</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>48</td>
<td>51</td>
<td>54</td>
<td>57</td>
</tr>
<tr>
<td>4</td>
<td>64</td>
<td>68</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>85</td>
</tr>
</tbody>
</table>

I looked for combinations that added to 100. \(64 + 36 = 100\), so four 16s and two 18s add to 100.

**Extra 3: Parity**

Since I must use 6 darts and arrive at an even 100, I need to use an even number of odd scores, and therefore an even number of even scores also. I found \(4 \times 17 = 68\) and \(2 \times 16 = 32\). \(68 + 32 = 100\)

**Teaching Suggestions**

Darlene’s Dart Board does not make heavy cognitive demands on the solver. The only procedural skill it requires is addition of whole numbers. Unless your purpose in using the problem is to reinforce that skill, students may solve the problem with calculators, as long as they don’t use them randomly. It’s always advisable to have them record their attempts and results, even when using a calculator, including tests that don’t work, as it allows them to make adjustments and avoids trying the same incorrect combinations over and over. We want students to make use of the information you get from incorrect attempts in order to make a better next guess – achieving success through skill and understanding, not pure luck.

The problem can take advantage of and develop students’ number sense as well as their mental math skills. Encourage them to think about things like parity (odd/even) when adding numbers. Could a winning solution consist of 3 even numbers and 3 odd numbers? What do they know about the ones digits of multiples of 5? What combinations of numbers could result in a zero in the ones place?

The second question calls for some logical thinking. It’s not enough simply to try combinations and not find a way to reach 100 with more or fewer than 6 darts. Students need to develop a sound reason why it is not possible.

Consider introducing this problem by presenting students with the Scenario Only version (linked in the teacher’s blue box on the problem page). By having a rich discussion about what students notice before seeing the question, they have the opportunity to explore the context, practice math language, and develop a deeper understanding of the problem, which “primes” them for addressing the question. You might ask students to brainstorm a list of possible questions that could be answered with the given information.

The Extra asks solvers to generate more winning combinations. It is an opportunity to develop fluency with numbers and flexibility with different strategies. Consider making a class “collection” of as many different winning combinations as students can find. There are at least 10 different ways to win this game. All winning combinations involve 6 darts.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources. If you would like one page to find all of the 2010-2011 Current Problems as we add them throughout the season, consider bookmarking this page:

http://mathforum.org/pow/support/
Sample Student Solutions

Focus on Strategy

In the solutions below, I focus on the variety of strategies that students used to solve the problem. I have printed below the criteria for the different levels of Strategy from our problem-specific Scoring Rubric.

I have commented on what I noticed about each solution and suggested questions and prompts that might help the student move forward in their thinking.

<table>
<thead>
<tr>
<th>Novice</th>
<th>Apprentice</th>
<th>Practitioner</th>
<th>Expert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.</td>
<td>Tries a strategy that makes sense, but isn’t enough to solve the whole problem, OR doesn’t apply it systematically. OR Strategy is not explicit. Might verify a correct answer, but fail to explain how they found it.</td>
<td>Picks a sound strategy. Approaches the problem systematically and logically, achieving success through skill, not luck. A guess-and-test strategy is based on sound reasoning. Chosen strategy accounts for any answer(s) that changed after checking our answers.</td>
<td>Does one or more of these: Uses a sound strategy for the Extra. Uses two different strategies. Uses an unusual or sophisticated strategy, e.g., an effective and appropriate use of technology.</td>
</tr>
</tbody>
</table>

Chelsea  
Age 11  
Strategy: Novice

Their is no possible way that 19,18,17,16, and 15 equals 100.

It is not possible to get 100. First I added all the numbers together and got 535. Then I broke down the numbers and added them. Next I saw none of the numbers were close to 100. Last I found out it is impossible to get 100.

Heather  
Age 11  
Strategy: Apprentice

1. A combination that will win the game is 15+16+19+15+16+19.  
2. It is not possible to use more or less darts than six.

1. Fifteen plus sixteen equals thirty one, and thirty one plus nineteen equals fifty, so I knew that I could repeat that, and get exactly one hundred. So I repeated the process by adding 15+16+19 to equal one hundred.

2. It is not possible to score 100 points with more or less darts than 6 because if you add up all of the 5 numbers, then it will add up to 85, and you would only have used 5 darts. If you add up all of the five numbers, and re-add one of the numbers, it will add up to more than one hundred.

David  
Age 12  
Strategy: Practitioner

12 1. One combination of scores that will win the game is 17, 17, 17, 17, 17, and 15.  
2. It is not possible to score 100 points with more or less than 6 darts.

The first combination will work because if you add up all the numbers they equal 100. I found it by guessing and checking. I guessed that five 17s and one 16 would equal 100 and it came out to 101. I was 1 away from 100 so I changed 16 to 15 and got 100.

You can not have any more turns than 6 because if you multiply the lowest number on the board times 7 (turns) you get more than 100 so if that was the lowest number all the others wouldn’t be 100. You
can’t have less than 6 turns because if you use the highest possible score with 1 less turn (5) it will be less than 100 so all the others will be too because they are smaller than 19.

Jaime & Sara
Age 9, 10
Strategy: Practitioner

1. 15+15+16+19+19=100
2. No, it is not possible to score 100 points with more or fewer darts.
1. We found our answer by adding everything twice and seeing what they made.15+15=30, 16+16=32, 17+17=34, 18+18=36, 19+19=38.
Then we realized that 32+30+38=100.
2. No, it’s only possible to score 100 points if you use six darts, because if you use the smallest number 15 seven times it will equal more then 100 and thats the smallest number. If you use the biggest number 19 five times it will equal less then 100 and it is the biggest number.

Rachel & Maha
Age 10, 11
Strategy: Practitioner

1. 15+16+17+18+19+15=100
2. We wouldn’t be able to score exactly 100 points with more or less darts because 5 darts wouldn’t be enough, and 7 would be too much.
1. I added all the numbers together, which is 85. Then, I subtracted 100 and 85. That’s 15. Then I added 15 and 85. That’s 100. The strategy that I did was to guess and check.
2. Fewer: To find out, I would divide 100 and 5 to get an average. That’s 20. But all the numbers are less than 20, so fewer is impossible. More: To find out I would divide 100 with 7 to get an average. That is 14.29. But all the numbers are more than 14.29, so more is also impossible.
Strategy: I used logical math. I know i used logic, because I divided to get the answer.

Owen
Age 11
Strategy: Expert

One combination of scores that will win the game is:
15+15+15+17+19+19. 2. It is not possible to score 100 with more or less than 6 darts. Extra: 2 other combinations that = 100 are:
16+15+15+18+19+17 and 16+16+16+18+18+16.
1. I started with highest number - 19 - and multiplied by 5 and got 95, so I realized that there wasn’t a combination with 5 19’s that would equal 100. Then I skipped to 15, the lowest number, I multiplied by 5 and got 75 and I realized that there wasn’t a combination that I could get to 100 from there, either. I realized my combination would need both higher and lower numbers. I started with 15+15+15 and then added 19 +19 and got 83, so then I knew he last number had to be 17.
2. It is not possible to use 5 or 7 darts and add up to 100. The highest number is 19 and 19 times 5 = 95, which too low. Using 7 darts, I tried the lowest number - 15 - times 7 could be the lowest combination, but that is too high - it equals 105, so it is not possible to get 100 with 7 darts.
Extra: 2 other combinations that = 100 are: 16+15+15+18+19+17 and 16+16+16+18+18+16. The strategy I used to get different combinations was to look at the combination that I had already figured out and to increase one number and to decrease another by one each. So I knew it would still add up to 100!
15+16+18+19=85  85+15=100  6 darts

To find a combination that equals 100 you have to look at the one’s column for combinations that will give you a zero. By immediately adding all numbers you get 85 and it is easy to see that one more dart of 15 will equal 100. Also it is easy to see that the 5 in 15 will give you a zero if added to another 5...6+9 will give a 5 and 7+8 will give a five so you can discover other combos:

   18+17=35, 15+35=50  18+15+18+17+15=100
   19+16=35  15+35=50 19+16+15+19+16+15=100

Fewer darts would be 5 or less, 5 times the largest # is only 95 so less is impossible.

more darts would be 7 or more, seven times the smallest # is 105 so more is impossible.

Harry
Age 11
Strategy: Expert

Harry began testing by adding one of each of the possible numbers, and then found he could complete 100 points by adding one more 15. He used good number sense in paying attention to the ones digits in order to find combinations that will result in a 0 or five in the ones place.

Alaina
Age 12
Strategy: Expert

1. One combination is 16,16,16,18,18.
2. It is not possible.

To find my first combination I used the strategy of guess and check. I tried multiplying 16 by 4 and got the answer of 64. Then I added 18 to it twice because I knew that 64 plus 18 plus 18 is 100.

For the second question I figured out that it is only possible with a combination of 6 numbers. The biggest number that you could use and that times 5 is less than 100. I used the number five because it is one less number than six. If 19 times 5 is to small then 6 is the smallest number. 15 is the biggest number that you can use and 15 times 7 is more than 100. I multiplied 15 by 7 to find out if there could be a combination of 7 numbers. There could not.

Extra: 2 other combinations that I found are 19,19,15,16,16 and 17,17,18,18,15,15. My strategy for finding these combinations is that I picked three numbers that equaled 50 and put the numbers down twice because 50 times 2 is 100. I think my answers are reasonable because I’ve checked my answers. I’m confident that my answers are right because I have worked a lot with adding numbers.

Alaina used multiplication to find her first winning combination.

Her reasoning is sound for #2. I’d work with her to make the communication clearer, including using number models to accompany her text.

Knowing there had to be six darts, she found two different combinations of three numbers that added to 50, and then doubled them.

Scoring Rubric

The problem-specific scoring rubric we use to assess student solutions is a separate stand-alone document available from a link on the problem page. It is intended for teachers and mentors. We consider each category separately when evaluating students’ work, thereby providing more focused information regarding the strengths as well as weaknesses of the work. This in turn helps inform teachers’ instructional decisions.

A generic student-friendly rubric can be downloaded from the Teaching with PoWs link in the left menu (when you are logged in). We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

~ Claire
claire@mathforum.org