Continuing the Journey: Mathematics Learning 2021 and Beyond

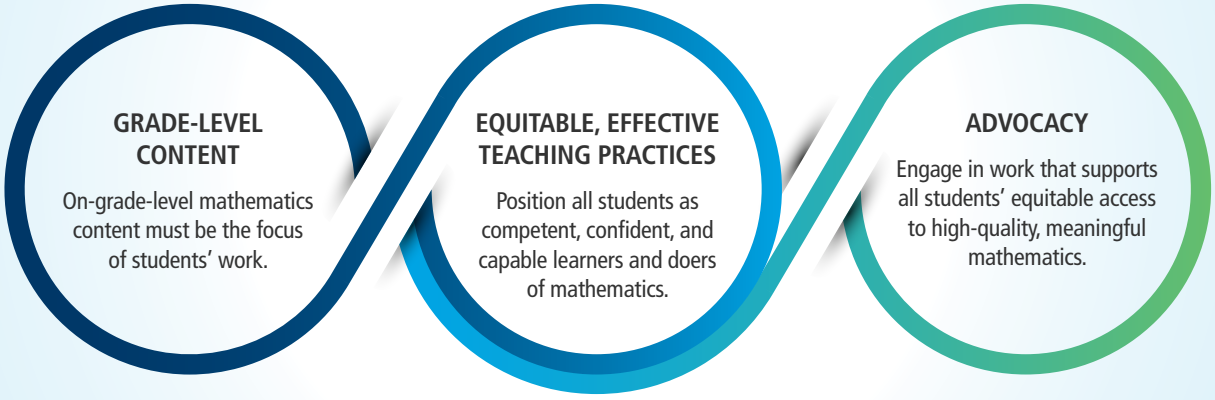
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Moving forward into the 2021–2022 school year seems like a daunting task. After all of the disruptions and challenges accompanying the COVID-19 pandemic, supporting mathematics students and teachers involves rethinking our priorities and resources. How do we attend to perceived potential learning and opportunity gaps? How do we foster social and emotional well-being and cultivate a safe learning environment? How can we best understand and value the varied learning each of our students has experienced during the past school year? And the list goes on.

With so many interruptions and variations in student experiences during the COVID-19 pandemic, there are those who assume that students will have lost a year or more of learning. But students *did* learn during the pandemic. They learned through virtual lessons; in-person classroom instruction; and interactions with teachers, other students, and family members. As in past years, students will come into the classroom with mathematical learning experiences, and we need to honor the knowledge they have gained from those experiences. We acknowledge that not all students had the same experiences, and many have learning needs that may not have been met. As a community, mathematics educators must commit to provide equitable learning experiences so that all students "have access to high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential" (NCTM 2014, p. 5). Reaching this goal at this unique moment in time is not easy. Even before the pandemic, numerous programs were not meeting the needs of many marginalized students. The disruptions of the past 18 months, which affected students inequitably, make the goal of meeting the needs of all students even more challenging. The process of continuing the journey in mathematics learning is complex work and a long-term investment that calls for diligence and an ongoing commitment.

Continuing the Journey: Mathematics Learning 2021 and Beyond is the result of a joint effort of the Association of State Supervisors of Mathematics (ASSM), NCSM: Leadership in Mathematics Education, and the National Council of Teachers of Mathematics (NCTM); it presents considerations, questions, and potential solution processes to educators and school leaders to address the challenges highlighted by the COVID-19 pandemic. *Continuing the Journey: Mathematics Learning 2021 and Beyond* addresses three key areas that have implications for equitable access to high-quality mathematics teaching and learning: (1) a focus on grade-level content; (2) instruction through equitable, effective teaching practices; and (3) planning for advocacy.

AREAS WITH SERIOUS IMPLICATIONS FOR EQUITABLE ACCESS TO HIGH-QUALITY MATHEMATICS TEACHING AND LEARNING



As you read through this document, we ask that you consider your role in mathematics education and your sphere of influence, engage in reflection, and identify actions you and those you serve can take as advocates. How can you advocate for starting the year with on-grade-level content? How can you advocate for equitable, effective instruction with the teachers and leaders you serve?

Adhering to Grade-Level Content

Supporting each and every student is essential in developing a deep mathematical understanding; in understanding and critiquing the world through mathematics; and in experiencing the wonder, joy, and beauty of mathematics—all of which contribute to a positive mathematical identity (NCTM 2018, 2020a, 2020b). To support students in this desired outcome, we must design learning opportunities for students to make connections within and across grade levels, emphasizing reasoning and sense making to ensure the highest-quality mathematics education for each and every student (NCTM 2020a, 2020b).

Regardless of what has come before, on-grade-level mathematics content must be the focus of our work with students. This includes on-grade-level mathematics content for PreK-8 and on-course-level mathematics content for high school. Particularly for the coming school year, mathematics educators must begin with building coherence through grade-level content and shifting from remediation to supporting and scaffolding to facilitate student understanding.

Building Coherence through Grade-Level Content

Achieving the goal of organizing grade-level content around big ideas while attending to learning progressions will require that district and state or provincial curriculum leaders provide sufficient guidance for grade-level and vertical teams to collaborate and create greater coherence using content standards, standards for mathematical practice, priority grade-level and course content, and individual units of study. Mathematical coherence occurs when concepts are connected within and across grades while incorporating attention to mathematical processes and practices. That students see how the mathematics they are currently learning builds on their prior knowledge is important. Making coherence explicit becomes the Velcro™—that is, how the content sticks together—connecting concepts to move student understanding forward.

Mathematics includes a wide range of concepts and skills that are intricately connected. Leveraging those connections and learning progressions will move student understanding forward. The analogy of using Velcro provides a window into how mathematical ideas should be developed, connected, and organized in grade-level course content for the ultimate benefit of students. This will ensure equal access and create opportunities for students to learn and develop a deeper mathematical understanding as teachers draw out essential features of mathematical ideas while cultivating the mathematical potential in a student's thinking.

Use tools such as concept maps to bring coherence to prior knowledge and on-grade-level content. They provide a visual organizer to connect standards, learning outcomes, practices and processes, and the instructional design of lessons. Such tools may serve as the Velcro to support students in connecting their ideas both within and across grade levels.

When mathematics lessons lack coherence, they appear to be stand-alone lessons or units with little attention to connections and coherence to other lessons or units of instruction. Coherence should include connected topics during lessons and units. For example, many students believe that decimals are completely different from fractions. When they think that way, they do not generalize their understanding from one representation to the other, so everything seems to be a brand-new idea. We can leverage a deeper understanding of the content by creating intentional connections across the content.

As important as the content students learn are the opportunities students have to engage in mathematical processes and practices as they learn the content. Students should be encouraged to make sense of mathematical problems and persevere in solving them; reason abstractly and quantitatively; construct viable arguments and critique the reasoning of others using precise language; use mathematics to model the world; and develop generalizations (NGA Center and CCSSO 2010, pp. 6–8). The processes and practices must be viewed as coequal partners with the specific content priorities for a course or grade. One cannot be fully realized without the other, and both are essential in helping students reach the broader mathematical goals for student learning.

Attending to mathematical coherence by incorporating big ideas and essential understandings and engaging in mathematical processes and practices with support can leverage students' current thinking to cultivate a deeper mathematical understanding. When a coherent and connected approach is taken, content from previous grades that students may seem to be "missing" can be taught along with grade-level content or just-in-time support. This moves the conversation away from a focus on "right or wrong answers" toward leveraging different student perspectives and incorporating different strategies used by their peers into their own thinking. On the basis of the strength of both teacher and student inputs, these reasoning pathways help move student learning forward with grade-level content. See additional recommendations from in the Determining Necessary Prior Knowledge section of *Moving Forward*. The Call to Action section below also provides reflection questions and resources to guide this work.

Shifting from Remediation to Supporting and Scaffolding

Acknowledging and building on student strengths is essential. "Many students who are challenged by mathematics never 'get to' the more engaging mathematics and therefore never experience the beauty and joy of doing mathematics" (Kobett and Karp 2020, p. 12). The advancement of student learning requires building on students' strengths and understanding the beliefs teachers hold about what content is taught, how it is delivered, and who receives it (Aguierre, Ingram, and Martin 2013). Teachers' beliefs influence the decisions that they make regarding how they teach mathematics and can be productive or unproductive (NCTM 2014). Unproductive beliefs hinder the implementation of effective instructional practice and limit student access to important mathematics content and practices, whereas productive beliefs support each and every student having access to high-quality mathematics learning opportunities that build on student strengths. Lessons and units designed with student strengths in mind include the following:

- Intentional balance of time for students to acquire conceptual as well as procedural mathematics knowledge
- Experiences for students to access prior knowledge that can be vehicles for developing conceptual and procedural knowledge
- Opportunities to provide learning experiences that allow each student to feel capable and safe (Aguirre, Ingram, and Martin 2013, p. 58)

Scaffolding and supporting students are key components of an equitable approach. So, what is scaffolding? Scaffolding is defined by Holton and Clarke (2006) as "an act of teaching that (a) supports the immediate construction of knowledge by the learner; and (b) provides the basis for the future independent learning of the individual" (p. 131). Going back and addressing every part of every standard is unnecessary. In fact, scaffolding may not happen in just one lesson—scaffolding can occur throughout multiple lessons. Scaffolding and supporting students make tasks accessible without lowering the cognitive demand, the type of questioning used, or ways of promoting mathematical discourse. Additionally, support during a lesson can come from other students when all students are placed in visibly randomized groups (Liljedahl 2020). See additional recommendations from *Moving Forward* in the Determining What Students Know section.

Supports and scaffolds should not be framed as remediation. Research shows that just-in-case remediation increases, rather than decreases, gaps in student achievement, whereas just-in-time scaffolding supports and values student knowledge while being intentional to ensure success with grade-level content (TNTP and ZEARN, 2021). Begin the year with grade-level content and instruction and include opportunities for just-in-time scaffolds and support. Develop scope and sequences that focus on grade-level content and include scaffolding supports that address content from prior grade levels as needed. A coherent and connected approach to content, along with scaffolds and support, ensures students with different learning needs have access to and can be successful with grade-level content. See additional recommendations from *Moving Forward* in the Structures That Organize Students for Instruction section and the NCTM *Back to School Guides* for additional resources.

This is our time to humanize mathematics by creating greater opportunities to learn for all students and ensure that all students have access to grade-level content.

Call to Action: What Can I Do Now?

Consider the following productive ideas for content (see Table 1) that are true for every school year (NCSM and NCTM 2020; NCTM 2018, 2020a, 2020b) and are an important part of success for 2021 and beyond. Then consider the questions and resources as you and your leadership team are making decisions about intentional connections across content.

This is our time to humanize mathematics by creating greater opportunities to learn for all students and ensure that all students have access to grade-level content. Creating greater coherence and providing just-in-time support for grade-level content provide a lever that helps students develop a deeper mathematical understanding, increase their motivation for learning because they see the connections, and enhance their identities as confident and effective doers of mathematics. Each student knows different things and will need different types of support. We can design continual mathematical on-ramps by listening to students' reasoning and considering learning pathways that connect concepts in ways that are helpful and accessible (Powell and Gray 2021).

TABLE 1 PRODUCTIVE IDEAS FOR CONTENT

Ideas for Adhering to Grade-Level Content	Questions to Consider	Resources
Keep joy and humanity in teaching and learning.	• How do you find ways to enjoy your work? • In what ways can you design lessons and tasks to facilitate wonder, joy, and beauty in the learning of mathematics? • What strategies are used to explore your own mathematical identity? • How do you support students in their positive mathematics identity?	*Rehumanizing Mathematics for Black, Indigenous, and LAtinx Students* *Humanizing Disability in Mathematics Education: Forging New Paths* *Mathematics for Human Flourishing*
Focus on and teach to student strengths.	• How do you notice, name, and document student strengths? • How can you use strengths to design lesson plans and scaffolding? • What strategies are used to build student mathematics identity and agency?	*Strengths-Based Teaching and Learning in Mathematics: 5 Teaching Turnarounds for Grades K–6* *Teaching Math to Multilingual Students: Positioning English Learners for Success* *Acorns to Oaks: Nurturing Growth through Strengths-Based Practices* *A Wonder-Full Task Leads to a Wonder-Full Intervention*
Connect to the major work of the grade/course learning.	• Will this learning be needed in future grades or courses? • Is this standard or concept connected to important ideas in previous grades or courses? (*Moving Forward* 2020, p. 6) • Will it elevate student reasoning and sense making with explicit connections to grade-level content?	*Learning Progressions* *Catalyzing Change Series* *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* *Mathematics Unit Planning in a PLC at Work (PK–2, 3–5, 6–8, HS)* *Essential Understandings series*
Work collaboratively in grade-level teams or mathematics departments.	• How does my team work together to develop a shared understanding of the essential learning? • What process do we use to identify a variety of strategies and representations to support student learning? • What prerequisite skills can be an on-ramp to the essential learning? • How does my team create common formative assessments that align to essential learning? How does my team work together to build understanding of connections across content? • What process do we use to determine the depth of the standards and recognize to what degree concepts and skills need to be mastered for student understanding?	*The Math Pact: Achieving Instructional Coherence within and across Grades, (Elementary, Middle, High)* *Mathematics Assessment and Intervention in a PLC at Work* *Mathematics Instruction and Tasks in a PLC at Work* *Mathematics Homework and Grading in a PLC at Work* *Jumpstart Formative Assessment Toolkit* *Mathematics Formative Assessment, Vol. 2* *The Formative 5: Everyday Assessment Techniques for Everyday Classroom K–8* *Strategies for Formative Assessment* *Benefits of Formative Assessment*
Scaffold learning to grade-level standards by using prerequisite knowledge.	• How do we intentionally use prerequisite knowledge standards or progressions to scaffold the lesson for students to access grade-level content? • How do we intentionally scaffold during instruction to provide access to grade-level content?	*Mathematics Unit Planning in a PLC at Work (PK–2, 3–5, 6–8, HS)*
Eliminate tracking of students on the basis of perceived abilities.	• How do we use just-in-time supports and scaffold student learning so every student has access to more engaging mathematics during which they experience the beauty and joy of doing mathematics? • How do we use ongoing formative assessments to build on student strengths?	*Detracking School Mathematics to Ensure Equitable and Empowering Programs and Opportunities* *Closing the Opportunity Gap: A Call for Detracking Mathematics* *NCTM Catalyzing Change Series*
Inform families of the essential learning and major work of the grade.	• How do we communicate with families the expectations for grade-level content? • In what ways do we partner with families and communities to support on grade-level learning?	*NCTM Back-to-School Guide for Families for resources*

Foundations for Equitable, Effective Teaching Practices

As we continue the mathematics teaching journey in 2021 and beyond, focusing on equitable instructional practices is imperative, as is being mindful of instructional strategies employed to support students in on-grade-level learning. We need to position all students as competent, confident, and capable learners and doers of mathematics, while affirming their strengths every day in ways that cultivate positive mathematical identities and support students in developing agency (Aguirre, Mayfield-Ingram, and Martin 2013). We want to empower our learners and cultivate equitable instructional mindsets.

Resist the temptation to revert to what we know are ineffective teaching practices. One of the most equitable things you can do for all of your students, including those who have been previously marginalized, is to adhere strongly to NCTM's eight effective mathematics teaching practices (NCTM 2014). Students with different learning needs are capable learners of grade-level content when presented with equitable, effective teaching practices that humanize students' mathematics experience. Supporting students with strengths-based Individual Education Plans (IEP's) that position students as doers of mathematics promotes individual confidence while enhancing the learning for all (Tan et al. 2019).

We should recognize, believe in, and build on the strengths of our students.

It is important that we cultivate equitable instructional mindsets and practices, build students' positive mathematical identities, create an equitable classroom culture, support making connections through effective instructional practices, and engage teachers and teams to create a culture of collaboration. Discussion of each area follows.

Cultivating Productive Instructional Mindsets and Practices

Each and every student needs and deserves access to high-quality, meaningful mathematics. We should recognize, believe in, and build on the strengths of our students. In too many schools, unproductive mindsets and practices have harmed many students and particularly marginalized students such as Black, Brown, and Indigenous students; students who are economically disadvantaged; students who are learning the English language; and students with special needs and disabilities. In the pandemic-disrupted school years, marginalized students may have taken the brunt of the impact of disrupted learning when schools went virtual. Because of the challenges of mitigation strategies, IEPs may have been temporarily disregarded, some students may have had inequitable access to technology at home, and language acquisition supports may have evaporated in many places.

As we continue the learning journey into the 2021–2022 school year, we must cultivate more productive mindsets that support *all* students, including those who are typically marginalized. This cultivation requires educators at all levels within the system to disrupt the unproductive mindsets and practices that inhibit learning. Productive mindsets and practices and suggestions for disrupting unproductive mindsets and practices are provided in Table 2.

The suggestions in Table 2 are just a few of the mindsets and practices needed to ensure each and every student is provided the opportunity to engage in high-quality and meaningful mathematics every year. Another important element of setting a foundation for equitable, effective teaching practices is building students' positive mathematical identities.

Building Positive Mathematical Identities

Students and teachers have experienced many varied teaching and learning opportunities in both formal and informal spaces. Learning continued during the pandemic and should be recognized and valued. We also need to acknowledge that students and teachers experienced losses and challenges—including personal, milestone, social, and emotional—during this time. Now more than ever, teachers should learn about who their students are, identify their strengths, and teach to build on their strengths (Kobett and Karp 2020). This is a time to make sure students see themselves as capable thinkers and doers of mathematics.

The media uses terms like *learning loss* and *learning gaps* and makes other deficit-based statements. These terms devalue the incredible work that students, educators, and families have done this year. Using such terms as learning loss and gaps can lead to a developed sense of inadequacy and damage. Terms such as *unfinished learning*, *opportunity to learn*, and *accelerated learning* are strength-based terms that should be used to affirm what students bring to the learning experience and to value existing knowledge. One role of mathematics teachers and leaders is to ensure continual affirmation of students' positive mathematical identities through their learning experiences, feedback, and building on their strengths that will support them in developing strong and resilient positive identities.

An essential step to promoting positive mathematical identity is to start with knowing the learners (Brown and Seda 2021). Each and every student is unique, with different experiences. As teachers learn about who students are, they can also learn about their strengths, interests, cultures, and hobbies. As teachers, we should get to know our students as individuals so we can plan intentional instructional experiences that create access to grade-level content. As leaders, we need to know those whom we serve as individuals so we can plan intentional learning experiences to support access to grade-level content.

TABLE 2 DISRUPTING UNPRODUCTIVE MINDSETS AND PRACTICES THAT INHIBIT LEARNING

Productive Mindsets and Practices	Unproductive Mindsets and Practices to Disrupt
Create a mindset that views mathematics as an interconnected web of concepts, knowledge, and skills.	• Disrupt a linear view of learning that creates barriers between content by suggesting a particular skill or concept must be "mastered" before students can access other content.
Believe that all students are mathematically brilliant. Assessing, activating, and building on prior knowledge are a normal part of the teaching process. Students become confident in their mathematics ability when teachers believe in their ability.	• Disrupt deficit language and views that negatively affect students' mathematical identity, agency, and authority in learning mathematics. • Disrupt a view that perceived learning gaps are problematic, treating students as less capable, and lowering students' sense of efficacy.
Deeply understand standard(s) and learning goals in a unit. Focus instruction around learning goals that consider the concepts, language, and representations students are developing in the unit. Support connections between concepts across the grade levels so that students can strengthen their understanding of grade level content as well as prior and future content.	• Disrupt learning organized around performance goals that suggests students learn concepts in isolation and cannot move forward in their learning until they have mastered a particular set of skills and procedures.
Elicit and examine your knowledge of students. Value the knowledge and skill students bring to each topic and signal this value through asset-based language. Examine student work (e.g., diagnostic, formative) to consider which concepts, language, and representations students already show evidence of using on previous tasks and which concepts, language, and representations might need additional just-in-time support during the unit.	• Disrupt language around "learning loss" that could lead to remediation practices. • Disrupt making assumptions about what students can and cannot do based on limited evidence that could lead to remediation.
Invite students into the mathematics of the unit. Use instructional routines that support student access to low-floor, high-ceiling tasks and ensure students can bring their breadth of knowledge and experience in understanding and engaging in the task.	• Disrupt instruction that values only one solution pathway and only some types of knowledge.
Make connections explicit. When planning a unit, consider the concepts, language, and representations within the unit and how they connect to one another. How will these connections be developed? Which connections will be important to name during instruction? What connections to mathematical ideas and real-world contexts will strengthen and support student learning?	• Disrupt instruction that initially teaches concepts in ways that send the message that mathematics is a list of isolated procedures.
Value reflection and revision. Have students grapple with cognitively demanding tasks without the fear of everything being graded. Model reflection as a teacher so students see learning as a journey. Allow students to revisit or redo assignments so they develop a deep understanding of concepts (Safir and Dugan 2021).	• Disrupt grading all assignments. This practice sends the message that mathematics is procedural and having the right answer means the destination has been reached or the learning has stopped.
Establish a culture of feedback. Create a culture in which you provide students both written and verbal feedback to help them grow as learners. Create a protocol for students to provide each other with feedback. Conference with students to better understand how they are thinking about a concept. Co-create rubrics with students so they are clear on the expectations that are rooted in language of behaviors we are trying to see (Safir and Dugan 2021).	• Disrupt using feedback that is a single score, percentages, or grades that are given to students afterward and only represent a token system of points.
Practice responsive teaching. Use formative assessment throughout a lesson to respond to the needs of learners, knowing when to scaffold or when to allow students to productively struggle. Provide opportunities for students help other students, share mathematical ideas, or help to work through a misconception (Acceleration Workshop, CT Center for School Change 2021).	• Disrupt the use of single assessments or summative assessment as the sole assessment practice. • Disrupt the use of traditional methods that emphasize evaluation of student achievement and school accountability rather than student learning and understanding. • Disrupt the view of assessment as limited to testing students.

Another benefit of knowing students is so teachers and teams can make intentional connections between students and high-quality, relevant instructional tasks. Engage students in meaningful grade-level mathematics and observe how they interact, listen to the questions they have, the reasoning they use, how they navigate challenges, and how they communicate their thinking. This level of engagement will open doors, invite students into the mathematics, and help them develop a sense of belonging. As students see themselves as doers of mathematics, they build confidence and develop mathematical agency and authority. As teachers and students continue to develop mathematical understanding, ensure that teachers and students are developing productive beliefs regarding the teaching and learning of mathematics.

Creating an Equitable Classroom Culture

A positive, supportive classroom community is necessary for advancing student learning. Classroom communities bond through trust, relationship, and experience. Teachers have worked very hard during the pandemic in various ways to engage students to create a virtual equitable classroom culture by using breakout rooms, screen sharing, the chat box, and other technology tools this past year. One positive that has come out of the pandemic is that many virtual lessons have started with more concerted efforts to strengthen community as students have worked in a somewhat isolated fashion. Many lessons that both students and teachers learned during pandemic-era instruction should be carried forward into subsequent school years. A focus on building an equitable classroom community is one of those lessons.

We must continue to cultivate and support an equitable classroom culture that strengthens and supports positive mathematics identities. Aguirre, Mayfield-Ingram, and Martin (2013) identify five equity-based instructional practices that support this goal: (1) going deep with mathematics, (2) leveraging multiple mathematical competencies, (3) affirming mathematics learners' identities, (4) challenging spaces of marginality, and (5) drawing on multiple resources of knowledge (p. vi). Together with the eight mathematical teaching practices described in *Principles to Actions: Ensuring Mathematical Success for All (2014)*, these practices provide a strong foundation for strengthening the teaching and learning of mathematics for each and every student, providing an equitable classroom culture.

As we begin this school year, we must be intentional with building a positive, collaborative learning community with our students. Those who have been to school likely have not had opportunities to engage with the full complement of their classmates. Working to rebuild classroom communities is a critical step for mathematics teachers and teams to take at the beginning of the year, as is working to maintain and strengthen them throughout the year. Take time during mathematics class to co-create class norms for engagement and interaction and implement tasks that support and promote a community of learning. Have students help craft the norms and identify behaviors and actions that are examples of the norms after working together as a group. Throughout the year, continuously revise and reinforce norms as a group. In addition, having students work and gather as a group helps to create classrooms where all viewpoints are seen and heard and form a community feel for students and the teacher alike in the space (Safir and Dugan 2021).

Ensure that you are being a warm demander (Hammond 2015) in your learning spaces. A warm demander continuously makes time to build a classroom culture in which teachers honor their students as individuals, build supportive relationships, and cultivate safe spaces where students can be themselves so they bring all of their mathematical brilliance into the learning space (Berry 2021). All the while, support students' engagement in rigorous mathematics that ensures they are reasoning, sharing ideas, and struggling productively. Equitable classrooms are places where all students feel welcome and get the support they need while striving to meet high expectations.

Making Connections through Instruction

NCTM defines mathematical connections as the ability to "recognize and use connections among mathematical ideas; understand how mathematical ideas interconnect and build on one another to produce a coherent whole; recognize and apply mathematics in contexts outside of mathematics" (2000, p. 64). To ensure that students truly understand and make sense of mathematics, we must plan instructional experiences that provide students with regular opportunities to make connections.

Copley, Karp, and Dougherty (2017) remind us that "tasks that focus only on procedural aspects may not help students learn the mathematical idea deeply" (p. 7) nor make important connections between and within mathematical representations that can assist in deepening conceptual understanding. Thus, it is important that leaders and teachers collaborate to create a coherent plan that includes a well-organized curriculum as well as instructional tasks that support students as they make sense of mathematics and make connections among important mathematical ideas. A coherent plan, along with in-depth teacher content knowledge and implementation of research-informed equitable teaching practices (NCTM 2014), influences the manner in which the content is taught, the ideas that are shown as important, and the type of tasks selected.

Planning authentic classroom activities that allow students to explore a mathematical concept once it is introduced, extending that knowledge, and connecting it to other concepts within and across grades will deepen students' understanding of mathematics concepts. Connections that draw together key ideas and topics within a domain help students develop

a coherent understanding of the concept or process they are learning. Engaging students in real-world problems in context around areas of interest that affect them individually and, in their communities, gives mathematics meaning. The goals are to teach on-grade-level content and provide support and scaffolding as needed to foster students' sense making. Table 3 shows three practices with areas for consideration that every teacher can use to achieve these goals.

For additional suggestions related to making connections through instruction, see the Teaching Practices Supporting Mathematics Teaching and Learning section in *Moving Forward*.

Creating a Culture of Collaboration

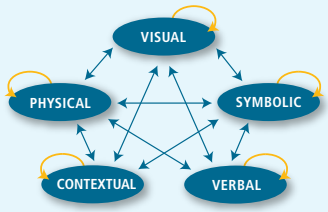
For mathematics educators to collaborate with others is vital to meeting the needs of each and every learner. The collective wisdom of a mathematics team is much greater than a single individual member of a team. Mathematics leaders and teachers must ensure that structures are created for intentional collaboration and must identify policies and other structures that might impede collaboration and the work of teachers.

As noted in *Moving Forward*, teachers need dedicated time during their daily schedule to collaborate with their peers who teach the same grade or same course, including special education and English language learner specialists. To create coherence across grade levels and mathematics course matriculation, teacher teams should collaborate vertically. Working with the teacher teams in grades before or after a specific grade level promotes more effective activation of prior knowledge strategies, more focus on the essential content of each grade, more focus on the academic language and strategies used to develop mathematical rigor, and less repetition of content from year to year. Vertical collaboration will promote identifying content connections between grade levels and between concepts, which in turn will help teacher teams guide their students to understand the same connections.

Teacher teams will need additional time and support to ensure that students are provided with access to grade-level content through horizontal and vertical collaboration. Collaboration will offer opportunities for teams to develop effective strategies to support diverse learning needs, specifically developing robust and intentional scaffolds and questions to support or extend learning. Maximizing access to grade-level content by knowing student strengths, interests, and experiences is crucial.

To avoid undercutting students' interest or enthusiasm for learning, math teacher teams should focus on strengths and avoid over assessing students to identify gaps during the first days and weeks of school (*Moving Forward*). Teacher teams can focus on developing an understanding of student strengths to implement strategies that tap into prerequisite knowledge during a new unit of instruction and provide opportunities for students to engage in tasks to access their funds of knowledge and highlight their ideas, strategies, and representations. For more ideas, see the Most Effective Teaching Practices, Determining What Students Already Know, and Introducing New Learning sections of *Moving Forward*.

TABLE 3 SUPPORT AND SCAFFOLDING TO TEACH ON-GRADE-LEVEL CONTENT

Instructional Practice	Considerations
Integrate activities that allow students to make connections between mathematics topics and concepts within and across grade levels.	• Intentionally plan activities that allow students to relate new topics to concepts or skills previously learned and connect mathematical ideas to one another. • Encourage students to use multiple strategies to show and explain their thinking. • Have students articulate connections by asking how they use previous learning in the new task or to describe how the new topic relates to a previous topic. • Engage in collaborative planning with colleagues about the ways specific concepts are taught and connections between the concept both within grade levels and across grade levels.
Use and connect multiple representations.	• Engage students in the use of multiple representations. • Encourage students to use and select representations purposefully to foster sense making and explore relationships and connections between various representations.  Huinker's (2015) diagram (in NCTM 2020b, p. 51) indicates important connections among various representation forms.
Develop students' abilities to recognize and identify mathematical structures from previous experiences and apply understanding in new situations.	• Select tasks and facilitate class discussions that provide opportunities for students to explore and focus on mathematical structure. • Looking for and making use of structure can support students in making connections. • Students should recognize and identify structures from prior learning experiences and apply this understanding in a new situation (NCTM 2014).

Educators and leaders must continue to focus on creating access to grade-level content and refrain from the use of tracking, ability grouping, or other ineffective models to try to meet the needs of each and every learner. See *Moving Forward* for more ideas on structures in the Structures That Organize Students for Instruction section or the Structures That Support Teachers section.

Call to Action: What Can I Do Now?

Consider the following productive ideas for equitable effective instruction (see Table 4). Then consider the questions and resources as you and your leadership team create a culture of learning for all, implement equitable effective instructional strategies, plan grade-level learning experiences, and cultivate productive mindsets and practices.

As we continue to move forward, educators need to position all students as competent, confident, and capable learners and doers of mathematics, while affirming their strengths every day. As educators focus on grade-level content and equitable instruction, we are now called upon to advocate for a system that confronts problems and engages in advocacy strategies.

TABLE 4 PRODUCTIVE IDEAS, QUESTIONS, AND RESOURCES FOR EFFECTIVE EQUITABLE INSTRUCTION

Productive Ideas for Equitable Effective Instruction	Questions to Consider	Resources
Invite students into the mathematics.	• What intentional actions do teachers and teams take to understand their learners? • What reflection opportunities are created for teachers and teams to self-assess any potential deficit views or language that is used to hinder developing positive mathematical identities? • How do teachers and teams create an effective classroom culture?	*Creating Classroom Communities – Mathematics Teacher Focus Issue* *Strengths-Based Teaching and Learning in Mathematics: 5 Teaching Turnarounds for Grades K–6*
Make connections explicit.	• How do the teacher and teams analyze current and previous grade-level content to identify connections across grade levels? • What protocols or processes are employed to support vertical coherence? • How do teachers and teams support vertical connections with the process standards? • How do we intentionally plan for the use of physical (manipulatives), visual (drawings, charts, etc.), contextual (provide a context/real-world scenario/application), symbolic (use of numbers, expressions, and equations), and verbal (have student dialogue/explain thinking)?	*The Math Pact Series: Achieving Instructional Coherence within and across Grades* *Taking Action Series* *Reasoning and Sense Making in the Mathematics Classroom Series* *5 Practices for Orchestrating Productive Mathematics Discussions, 2nd edition*
Ensure grade level instructional experiences reflect the rigor of the standards.	• What opportunities are included in lessons for students to show responses in more than one way or using more than one representation? • How do students reflect on different strategies in each lesson?	*Principles to Actions: Ensuring Mathematical Success for All* *Taking Action Series*
Connect mathematical content with mathematical processes.	• How are teacher teams intentionally planning for students' engagement in the process standards? • What learning experiences are included to develop problem solving? • What tasks are used to create access to the content (i.e. low-floor, high-ceiling tasks)? • What opportunities are intentionally created in a lesson for student-to-student discourse?	*S3D: Fostering and Improving Small-Group, Student-to-Student Discourse* *5 Practices for Orchestrating Productive Mathematics Discussions, 2nd edition* *Good Questions* *Mathematical Thinking*
Cultivate instructional mindset and practices.	• What resources are useful in planning for supporting productive mindsets and practices? • How are teacher teams addressing ways to disrupt potential biases and deficit thinking and language? • What opportunities are intentionally planned to create lessons and tasks that build on student strengths? • How do teacher teams work together to rethink and reimagine mathematics classrooms that engage all students in meaningful mathematics, particularly those traditionally marginalized in their school, district, state, or province?	*The Living Tree of Mathematics* *The Culturally Relevant Cognitively Demanding (CRCD) Mathematical Task Framework* *Principles to Actions* *Taking Action Series* *Reimagining the Mathematics Classroom* *Mathematics Instruction and Tasks in a PLC at Work*

Planning for Advocacy

As we continue our journey following a year of disruption, we must rethink PK–12 mathematics education. Mathematics educators and leaders have a unique opportunity to critically examine historic and entrenched practices and advocate for policies, structures, and approaches to teaching that support and promote high-quality, equitable teaching and learning of mathematics for each and every student. Current systems marginalize too many students. Therefore, dismantling inequitable structures and racism must be foundational to our advocacy efforts. In *Moving Forward* (NCTM and NCSM 2020), mathematics teachers and leaders are called on to advocate for teachers, students, and "humanizing the development and implementation of education policies and practices" (p. 1).

We must ensure that voices from all stakeholders are included in decision-making for a renewed approach to convey real challenges associated with our systems, build understanding, and generate solutions to meet the needs of marginalized students. Too often, decisions are made by homogeneous groups of adults who are not representative of students in terms of racial, ethnic, or gender identity. Our voices are important, individually and collectively. Building alliances among different groups of stakeholders effectively allows us to engage in advocacy work that supports all students' equitable access to high-quality, meaningful mathematics.

Advocacy Strategies and Practices

As you engage in advocacy in specific situations, reflecting on questions and considering responses to them are crucial actions:

- What is the purpose of advocacy in this situation? What do you want to accomplish?
- What communication is needed to achieve your purpose?
- Which audiences need to hear your messages?
- What are the key messages that you need to convey to your audiences?
- What are the most appropriate methods to communicate your messages?
- What resources and professional networks can you draw on to support your efforts?

Responses to these questions help you frame your direction for advocacy and next steps. We propose Mathematics Advocacy Practices (see Table 5) to complement the Mathematics Teaching Practices from *Principles to Actions* (NCTM 2014), positioning advocacy parallel to one's role as an educator. At the core of the Mathematics Advocacy Practices is the acknowledgment that advocacy is akin to teaching and learning more so than political theatrics and persuasion.

In the same way that a teacher's aim to build confidence and capability in students can benefit from positioning learners as sense makers who bring their own experiences to bear as they create understanding, so too might advocates benefit from appreciating those with opposing views as having common aims built on their own realities that can emerge as new and shared understandings about how the system works (or should work). As you read table 5, review the comparison between the Mathematics Teaching Practices and the Mathematics Advocacy Practices. Consider your current efforts and identify additional action steps to become a more effective advocate.

At the core of the Mathematics Advocacy Practices is the acknowledgment that advocacy is akin to teaching and learning more so than political theatrics and persuasion.

For further recommendations on advocacy from *Moving Forward*, see the section labeled Teachers, Students, and Humanizing the Development and Implementation of Education Policies and Practices. Call to Action: What Can I Do Now?

Consider the following productive ideas for advocacy (see table 6). Then consider the questions and resources as you and your leadership team are planning for the upcoming school year. In addition to these resources, be sure to investigate NCTM's *Principles to Actions* and books in the Catalyzing Change series or NCSM's *Framework for Leadership in Mathematics Education* for guidance about key ideas for which you might need to advocate in your context.

Federal and state or provincial programs support much of the work described in this section. In light of the COVID-19 pandemic, additional funding sources may be available to support schools to help students continue learning on grade level while supplementing the foundational skills necessary for that on-grade-level learning. This should include funding to support teachers in their learning and planning.

It is time to build allies and empower all voices to be heard. Seek opportunities to advocate, such as talking with the media, testifying at the state level, writing letters to policy makers, presenting to local school boards, or communicating with parents and families about mathematics education. Resources such as the NCTM *Advocacy Toolkit* are available to assist you and your colleagues in this work.

Consider the following productive ideas (see Table 6) as you plan to advocate. Reflect on the questions and identify possible resources to support your journey.

As educators and leaders, we can strengthen our advocacy efforts to address shared concerns and understandings about grade-level content and equitable teaching practices. Our voices are important, individually and collectively. If we do not engage now in the decision making and take action, then others who may not have a clear understanding of mathematics education will determine for us what is valued, supported, and funded. The stakes are high and have implications for teachers, administrators, students, and families in every community.

TABLE 5 MATHEMATICS TEACHING AND ADVOCACY PRACTICES

Mathematics Teaching Practices	Mathematics Advocacy Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.	**Establish advocacy goals to focus efforts**. Effective advocacy establishes clear goals to promote mathematics education, situates goals within equity frameworks and evidence, and uses the goals to guide approaches and accomplish purposes.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.	**Implement communication and approaches that promote mathematics**. Effective advocacy engages stakeholders in understanding problems that need to be solved. It also requires inciting inspiration to pursue action.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.	**Use and connect mathematical representations**. Effective advocacy engages stakeholders in considering different perspectives, making connections, finding common ground, and co-creating solutions. Using mathematical representations is useful to illustrate problems, make arguments, and deepen understanding.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.	**Facilitate meaningful discourse**. Effective advocacy facilitates discourse among stakeholders to build shared understanding of problems that need action by analyzing and comparing data and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.	**Pose purposeful questions**. Effective advocacy uses purposeful questions to assess situations, highlight inequities, bring awareness to issues, challenge perspectives, and advance action.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.	**Build fluency from conceptual understanding**. Effective advocacy builds fluency with creating conditions to identify problems, analyze data, generate representations, build understanding, and pursue action.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.	**Support productive struggle in advocacy efforts**. Effective advocacy provides stakeholders, individually and collectively, with opportunities and support to engage in productive struggle as they grapple with problems, challenges, and constraints related to promoting mathematics education.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.	**Elicit and use evidence of student thinking**. Effective advocacy uses evidence of student thinking to assess access and opportunity to learn and identify systems and structures that serve as barriers to mathematical learning and understanding.

TABLE 6 PLANNING TO ADVOCATE

Productive Ideas for Advocacy	Questions to Consider	Resources
Identify possible challenges or barriers.	• What are you passionate about? • What are obstacles that hinder mathematics teaching, learning, and leading? • What problems, challenges, or inequities need action in your setting?	*NCTM Position Statement: High Expectations* *NCTM Catalyzing Change Series* *NCTM Social Justice and Equity Resources*
Build alliances.	• Who are colleagues, both locally and nationally, who can join or support your efforts? • How do you build relationships with district data/assessment and strategic communication personnel? • Who are the key teachers in your state who have access and are influencing state policy?	*NCTM Advocacy Toolkit* *NCTM Position Statement: Access and Equity* *Catalyzing Change Series*
Use the Mathematics Advocacy Practices to reflect and begin conversations in your communities.	• What do you want to accomplish in your context? • What communication is needed to achieve your purpose? • Which audiences need to hear your messages? • What are the key messages that you need to convey to your audiences? • What are the most appropriate methods to communicate your messages? • What resources and professional networks can you draw on to support your efforts?	*Moving Forward* *NCTM Position Statement: Closing the Opportunity Gap* *NCTM Position Statement: Mathematics in Early Childhood* *NCSM Position Statement: Formative Assessment*
Pursue professional development opportunities to continue to develop advocacy competencies.	• What book studies can you lead or participate in related to advocacy in mathematics education? • What resources and professional development offered by NCTM, NCSM, and ASSM can you pursue?	*Back to School resources* *Catalyzing Change and Principles to Action Book Studies* *NCTM Position Statement: Curricular Coherence and Open Education Resources* *NCSM Position Statement: Tier 1 Instruction* *NCSM Position Statement: RtI and Tier 2 Instruction* *NCTM Webinars for PD* *Videos and Tasks for PD*
Identify resources that need to be compiled or created to support your particular advocacy needs.	• What are talking points that would be useful? • What representations, images or student work would illustrate the issues?	*NCTM President's Message: Advocacy as a Mathematics Education Community—The Time Is Now* *NCTM Advocacy Resources*
Use your voice and talent. It matters.	• Who needs to hear your message? • How might you invite them to the table?	*NCTM Advocacy Toolkit* *NCSM and TODOS Joint Position Statement: Social Justice and Equity in Mathematics Education*

Your Role in Continuing to Move Forward

The current system of mathematics education is not working for all students. The global pandemic in 2020 and 2021 highlighted the persistent inequities within the system. Mathematics educators and leaders continue to face pressures, challenges, changes, and opportunities that influence the teaching and learning of mathematics (e.g., the adoption of new mathematics curricula, technology tools, state or provincial standards and assessments, or district policies).

Continuing the Journey: Mathematics Learning 2021 and Beyond focuses on three key ideas that mathematics educators and leaders must address. As we adhere to on-grade-level content through equitable, effective teaching practices and engage in effective advocacy, we can ensure equitable outcomes for each and every student.

As we continue the journey emerging from the pandemic, these types of situations will require educators and leaders to build strong alliances as we work together to dismantle inequitable structures and racism in mathematics education that have hindered too many students for far too long. Strong advocates are equipped with the knowledge and the resources to engage in important conversations. It is imperative that we all work together in advocating for high-quality mathematics for each and every student and support for classroom teachers, families, and communities in this work.

For mathematics educators to collaborate with others is vital to meeting the needs of each and every learner.

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